2017 Higher Paper 1

① $f(x) = 5x \qquad g(x) = 2\cos x$

a) $f(g(0))$

$= f(2\cos 0)$

$= f(2)$

$= \underline{\underline{10}}$

b) $g(f(x))$

$= g(5x)$

$= \underline{\underline{2\cos 5x}}$

② $x^2 + y^2 - 8x - 6y - 15 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -8 \qquad 2f = -6$

$g = -4 \qquad f = -3$

centre $(4, 3)$ $\qquad P(\overset{a}{-2}, \overset{b}{1})$

$$m_{\text{radius}} = \frac{3-1}{4--2} = \frac{2}{6} = \frac{1}{3}$$

$\therefore m_{\perp} = -3$

$y - 1 = -3(x+2)$

$y - 1 = -3x - 6$

$\underline{\underline{y = -3x - 5}}$

③ $y = (4x-1)^{12}$

$$\frac{dy}{dx} = 12(4x-1)^{11} \times 4$$

$$= \underline{\underline{48(4x-1)^{11}}}$$

(4) $x^2 + 4x + (k-5) = 0$

$b^2 - 4ac = 0$ for equal roots

$4^2 - 4(1)(k-5) = 0$

$16 - 4k + 20 = 0$

$36 - 4k = 0$

$4k = 36$

$\underline{\underline{k = 9}}$

(5) a) $u = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ $v = \begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ $u.v = 15 - 8 - 6 = \underline{\underline{1}}$

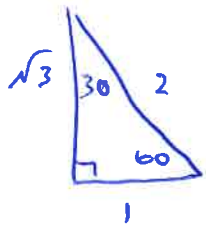
$|u| = \sqrt{25+1+1} = \sqrt{27}$

b) $u.w = |u||w| \cos\theta$

$= \sqrt{27} \times \sqrt{3} \times \cos\frac{\pi}{3}$

$= \frac{\sqrt{27}}{2} \sqrt{3}$

$= \frac{\sqrt{81}}{2} = \underline{\underline{\frac{9}{2}}}$



$\cos\frac{\pi}{3} = \frac{1}{2}$

(6) $h(x) = x^3 + 7$

Let $y = x^3 + 7$

$x^3 = y - 7$

$x = \sqrt[3]{y-7}$

$\therefore f^{-1}(x) = \underline{\underline{\sqrt[3]{x-7}}}$

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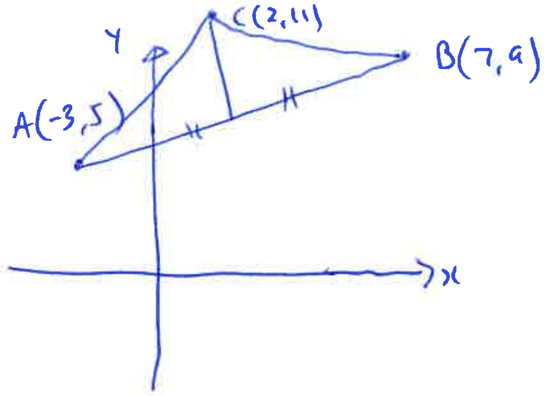


midpoint of $AB = \left(\frac{7+-3}{2}, \frac{9+5}{2}\right)$

$= (2,7)$

$m_{median} = \frac{11-7}{2-2} = \frac{4}{0}$ = vertical line.

$\therefore$ equation is $\underline{\underline{x = 2}}$

8 $d(t) = \frac{1}{2t} = \frac{1}{2}t^{-1}$

$d'(t) = -\frac{1}{2}t^{-2} = -\frac{1}{2t^2}$

$d'(5) = -\frac{1}{2(5)^2} = \underline{\underline{-\frac{1}{50}}}$

9 a) $U_{n+1} = mU_n + 6$

$U_1 = 28$, $U_2 = 13$

$U_2 = mU_1 + 6$

$13 = 28m + 6$

$28m = 7$

$\underline{\underline{m = \frac{1}{4}}}$

b) i) $-1 < \frac{1}{4} < 1$ so sequence approaches a limit.

ii) $L = 0{\cdot}25L + 6$

$0{\cdot}75L = 6$

$\frac{3}{4}L = 6$

$3L = 24$

$\underline{\underline{L = 8}}$

(10) a) $\int_0^2 x^3 - 4x^2 + 3x + 1 - (x^2 - 3x + 1)\, dx$

$= \int_0^2 (x^3 - 5x^2 + 6x)\, dx$

$= \left[\frac{1}{4}x^4 - \frac{5x^3}{3} + 3x^2 \right]_0^2$

$= \frac{1}{4}(2)^4 - \frac{5(2)^3}{3} + 3(2)^2 - 0$

$= 4 - \frac{40}{3} + 12$

$= 16 - \frac{40}{3}$

$= \frac{48}{3} - \frac{40}{3} = \underline{\underline{\frac{8}{3} \text{ units}^2}}$

b) $\int_0^2 1 - x - (x^2 - 3x + 1)\, dx$

$= \int_0^2 (-x^2 + 2x)\, dx$

$= \left[-\frac{x^3}{3} + x^2 \right]_0^2$

$= -\frac{2^3}{3} + 4 - 0$

$= -\frac{8}{3} + 4$

$= -\frac{8}{3} + \frac{12}{3}$

$= \underline{\underline{\frac{4}{3} \text{ units}^2}}$ $\therefore \frac{1}{2}\left(\frac{8}{3}\right) = \frac{4}{3}$ so <u>half shaded</u> area lies below the line.

(11) $3y - 2x = 4$

$3y = 2x + 4$

$y = \frac{2}{3}x + \frac{4}{3}$

$m = \frac{2}{3}$

$m_{AB} = \frac{2-a}{-7-5} = \frac{2-a}{-12}$

$\therefore \frac{2-a}{-12} = \frac{2}{3}$

$3(2-a) = -24$

$6 - 3a = -24$

$-3a = -30$

$\underline{\underline{a = 10}}$

(12) $\log_a 36 - \log_a 4 = \frac{1}{2}$

$\log_a 9 = \frac{1}{2}$

$9 = a^{1/2}$

$\therefore \underline{\underline{a = 81}}$

(13) $\int \frac{1}{(5-4x)^{1/2}}\,dx = \int (5-4x)^{-1/2} = \frac{(5-4x)^{1/2}}{\frac{1}{2}(-4)} + C$

$= -\frac{(5-4x)^{1/2}}{2} + C$

(14) a) $\sqrt{3}\sin x^\circ - \cos x^\circ = k\sin(x-a)^\circ$

$= k(\sin x\cos a - \cos x\sin a)^\circ$

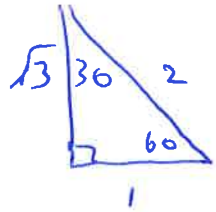
$= k\cos a^\circ \sin x^\circ - k\sin a^\circ \cos x^\circ$

$k\cos a^\circ = \sqrt{3}$

$k\sin a^\circ = 1$

$k = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

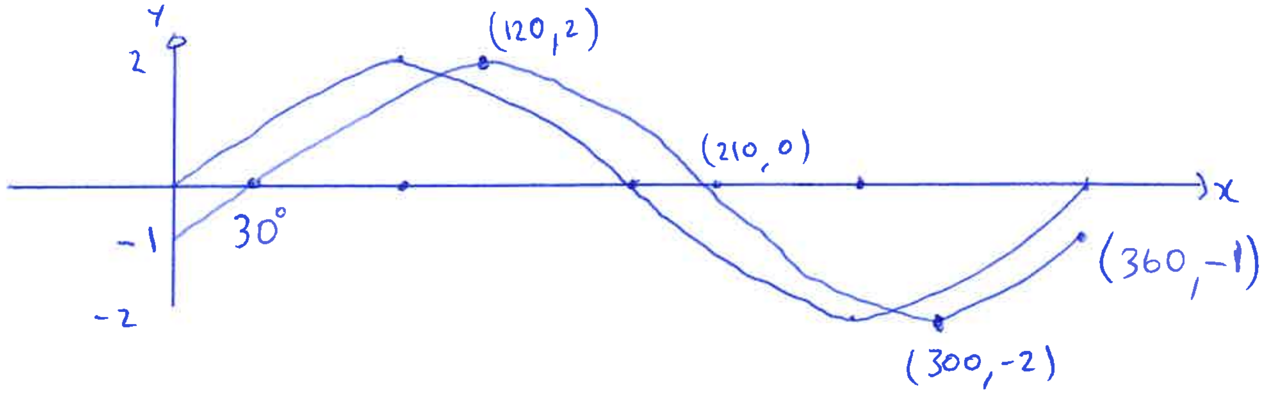
$\tan a = \dfrac{k\sin a}{k\cos a} = \dfrac{1}{\sqrt{3}}$



$a = 30°$

$\therefore \sqrt{3}\sin x° - \cos x° = \underline{\underline{2\sin(x-30)°}}$

b)



15 a) $h(x) = f(x-5) + 3$

$\therefore \underline{\underline{a = -5, b = 3}}$

b) $A_{\square} = lb = 2 \times 3 = 6$

So $\int_6^8 h(x)\,dx = 4 + 6 = \underline{\underline{10}}$

c) $h'(8) = \underline{\underline{-6}}$