X100/12/03

MATHEMATICS
HIGHER
Paper 2

Read carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

LI

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

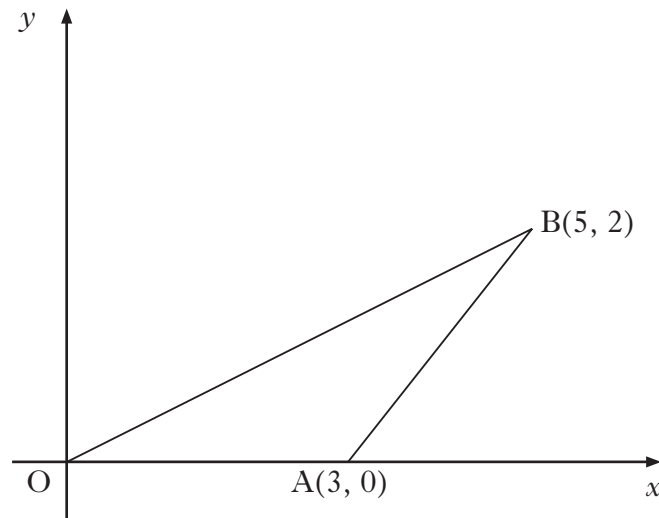**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

| $f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + c$
$\cos ax$	$\dfrac{1}{a}\sin ax + c$

1. A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.

   (*a*) Obtain the equation of the perpendicular bisector of AB. **4**

   (*b*) The median from A has equation $y + 2x = 6$.

      Find T, the point of intersection of this median and the perpendicular bisector of AB. **2**

   (*c*) Calculate the angle that AT makes with the positive direction of the *x*-axis. **2**

2. A curve has equation $y = x^4 - 2x^3 + 5$.

   Find the equation of the tangent to this curve at the point where $x = 2$. **4**

3. Functions f and g are defined on suitable domains by

$$f(x) = x(x - 1) + q \ \text{ and } \ g(x) = x + 3.$$

   (*a*) Find an expression for $f(g(x))$. **2**

   (*b*) Hence, find the value of q such that the equation $f(g(x)) = 0$
      has equal roots. **4**

[Turn over

4. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.
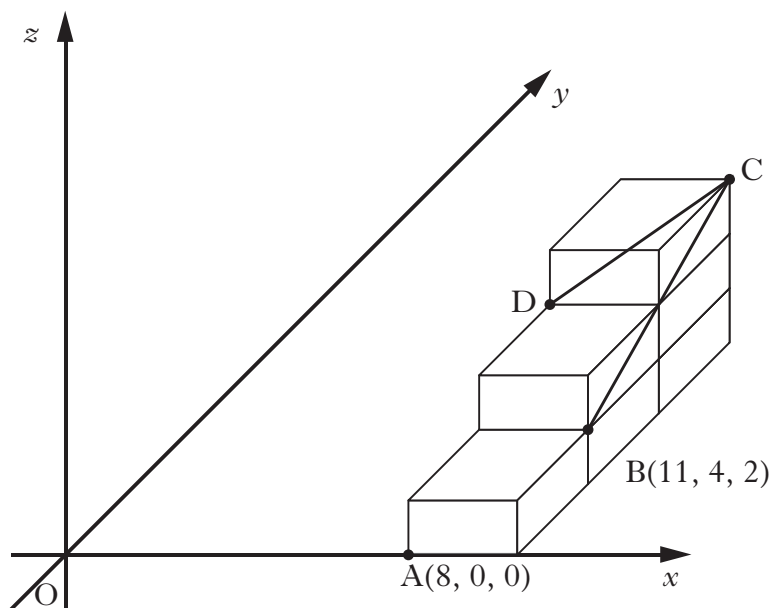
A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D.   **2**

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$.   **2**

(*c*) Find the size of the angle BCD.   **5**

5. Given that $\int_{4}^{t} (3x+4)^{-\frac{1}{2}} \, dx = 2$, find the value of *t*.   **5**

6. Solve the equation

$$\sin x - 2 \cos 2x = 1 \qquad \text{for } 0 \le x < 2\pi.$$ **5**

7. Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,\text{m}^2$ of land.
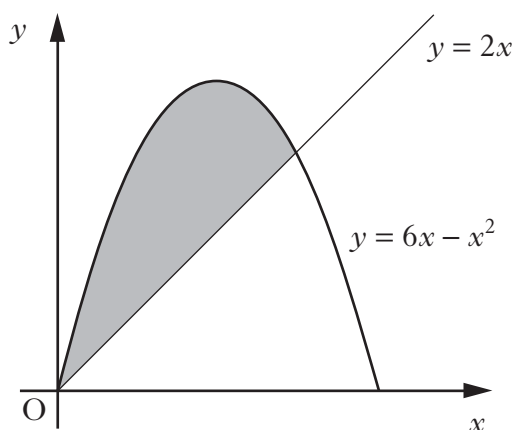

Diagram 1

(*a*) Calculate the area of land being developed.　　　　　　　　　**5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.
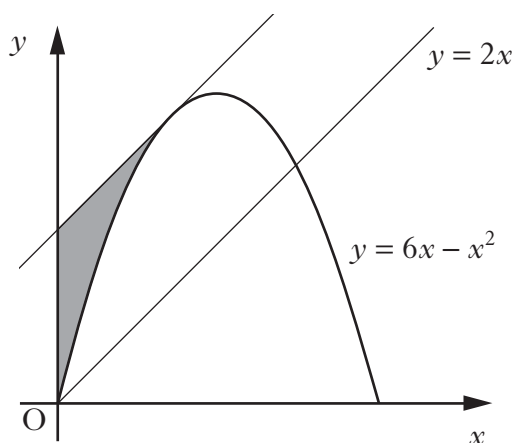

Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park.　　　　　　　　　**5**

[Turn over

8. Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p. **5**

9. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object,

t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(*c*) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

[*END OF QUESTION PAPER*]