

X100/12/03

NATIONAL
QUALIFICATIONS
2013

WEDNESDAY, 22 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

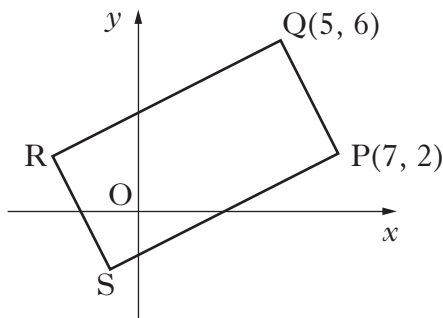
1. The first three terms of a sequence are 4, 7 and 16.
The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of m and c .

4

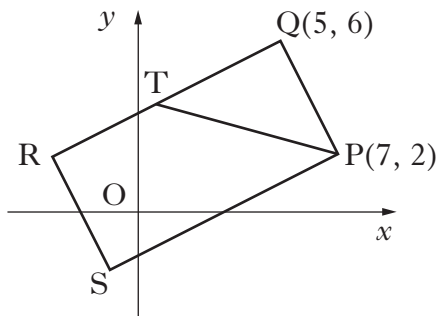
2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



- (a) Find the equation of QR.

3

- (b) The line from P with the equation $x + 3y = 13$ intersects QR at T.



Find the coordinates of T.

3

- (c) Given that T is the midpoint of QR, find the coordinates of R and S.

3

[Turn over

3. (a) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.

4

- (b) Show that the curve with equation

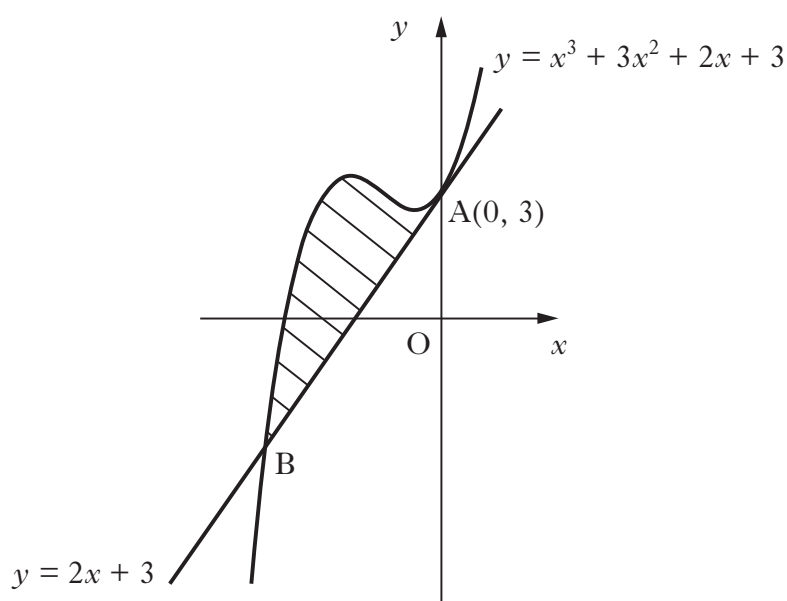
$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x -coordinate and determine the nature of this point.

5

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at $A(0, 3)$, as shown in the diagram.



The line meets the curve again at B .

Show that B is the point $(-3, -3)$ and find the area enclosed by the line and the curve.

6

5. Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$

4

6. Given that $\int_0^a 5\sin 3x \, dx = \frac{10}{3}$, $0 \leq a < \pi$,

calculate the value of a .

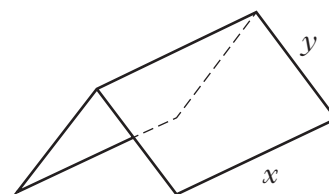
5

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2 .

- (a) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

3

- (b) These rods cost $\pounds 8.25$ per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

- (i) Find the value of x for which L is a minimum.
- (ii) Calculate the minimum cost of a frame.

7

8. Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \quad \text{for } 0 \leq x < 2\pi$$

6

[Turn over for Question 9 on Page six

9. The concentration of the pesticide, X_{pesto} , in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
 - P_t is the concentration at time t ;
 - t is the time, in days, after the application of the pesticide.
- (a) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.
If the half-life of X_{pesto} is 25 days, find the value of k to 2 significant figures. **4**
- (b) Eighty days after the initial application, what is the percentage decrease in concentration of X_{pesto} ? **3**

[END OF QUESTION PAPER]