Higher Maths 2013 paper 1

Q1-20 was originally multiple choice questions worth 2 marks each.

1. The functions f and g are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers.

   Find $g(f(x))$.

2. The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$.

   What is the gradient of the tangent to this curve at P?

3. Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

4. Draw the graph of $y = 4\cos 2x - 1$, for $0 \leq x \leq \pi$?

5. The line L passes through the point $(-2, -1)$ and is parallel to the line with equation $5x + 3y - 6 = 0$.

   What is the equation of L?

6. What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?

7. Find $\int x(3x + 2)\, dx$.

8. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$.

   Here are two statements about this sequence:

   (1) $u_0 = 9{\cdot}1$;

   (2) The sequence has a limit as $n \longrightarrow \infty$.

   Which of the following is true?

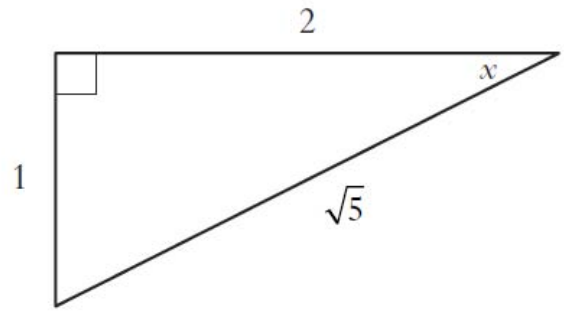
   A Neither statement is correct.

   B Only statement (1) is correct.

   C Only statement (2) is correct.

   D Both statements are correct.

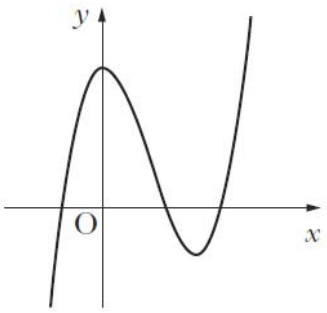
9. The diagram shows a right-angled triangle with sides and angles as marked.



Find the value of $\sin 2x$.

10. Simplify $\cos(270 - a)°$

11. The diagram shows a cubic curve with equation $y = f(x)$.



Draw a possible diagram to show $y = -f(x - k)$, where $k > 0$.

12. If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $|\mathbf{f} + \mathbf{g}|$.

13. A function f is defined on a suitable domain by $f(x) = \dfrac{x+2}{x^2 - 7x + 12}$.

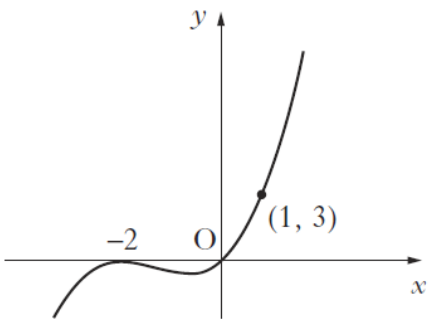
What value(s) of x cannot be in this domain?

14. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\mathbf{a}.\mathbf{b} = 5$, what is the value of $\mathbf{a}.(\mathbf{a} + \mathbf{b})$?

15. Solve $\tan\left(\dfrac{x}{2}\right) = -1$ for $0 \leq x < 2\pi$.

16. Find $\int (1 - 6x)^{-\frac{1}{2}} dx$ where $x < \dfrac{1}{6}$.

17. The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points $(-2, 0)$, $(0, 0)$ and $(1, 3)$.

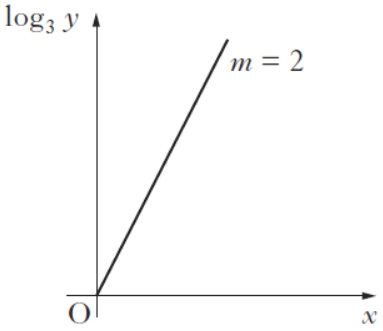


What are the values of a and k?

18. Given that $y = \sin(x^2 - 3)$, find $\dfrac{dy}{dx}$.

19. Solve $1 - 2x - 3x^2 > 0$, where x is a real number.

20. The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown.



Express y in terms of x.

21. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. **3**

22. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(*a*) Write down the centre and calculate the radius of C_1. **2**

(*b*) The point P(3, 2) lies on the circle C_1.

Find the equation of the tangent at P. **3**

(*c*) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1.

Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. **3**

(*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2. **4**

23. (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\sin(x - a)°$, where $k > 0$ and $0 \leq a < 360$.

Calculate the values of k and a. **4**

(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \leq x < 360$. **2**

24. (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

(ii) Find the ratio in which T divides AB. **4**

(*b*) The point C lies on the x-axis.

If TB and TC are perpendicular, find the coordinates of C. **5**

End of Question Paper