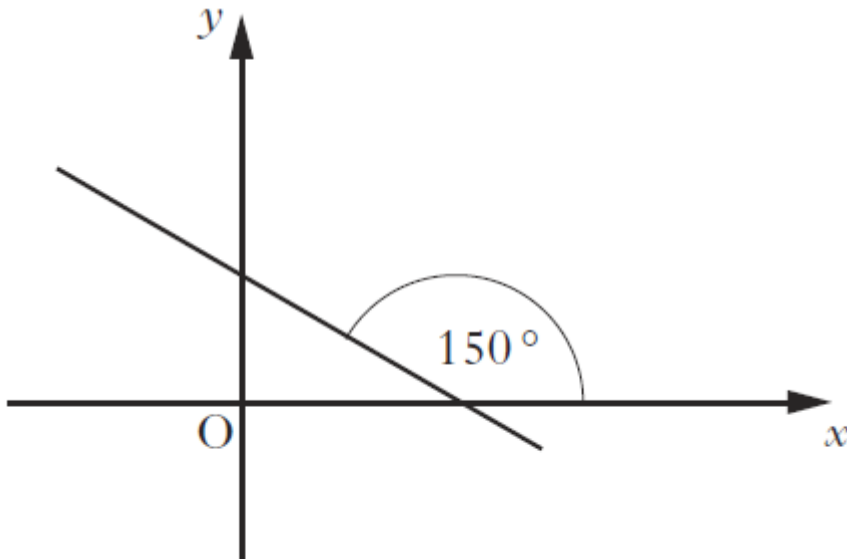


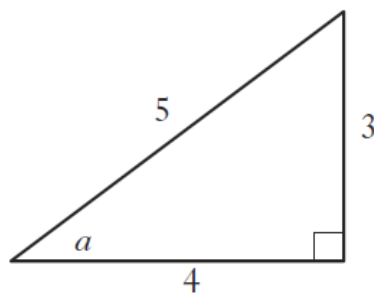
## Higher Maths 2012 paper 1

Q1-20 were originally multiple choice questions worth 2 marks each.

1. A sequence is defined by the recurrence relation  $u_{n+1} = 3u_n + 4$ , with  $u_0 = 1$ .  
Find the value of  $u_2$ .
2. What is the gradient of the tangent to the curve with equation  $y = x^3 - 6x + 1$  at the point where  $x = -2$ ?
3. If  $x^2 - 6x + 14$  is written in the form  $(x - p)^2 + q$ , what is the value of  $q$ ?
4. What is the gradient of this line



5. The diagram shows a right-angled triangle with sides and angles as marked.



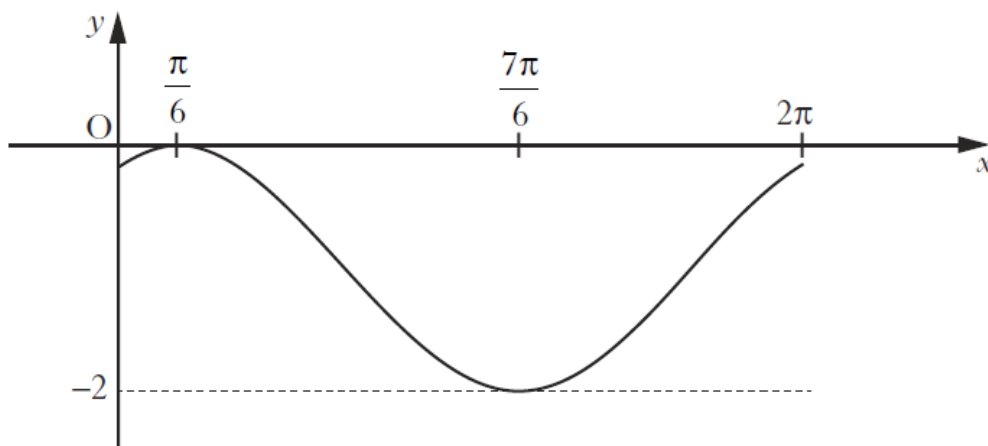
What is the value of  $\cos 2a$ ?

6. If  $y = 3x^{-2} + 2x^{\frac{3}{2}}$ ,  $x > 0$ , determine  $\frac{dy}{dx}$ .

7. If  $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$  are perpendicular, what is the value of  $t$ ?

8. The volume of a sphere is given by the formula  $V = \frac{4}{3}\pi r^3$ .  
What is the rate of change of  $V$  with respect to  $r$ , at  $r = 2$ ?

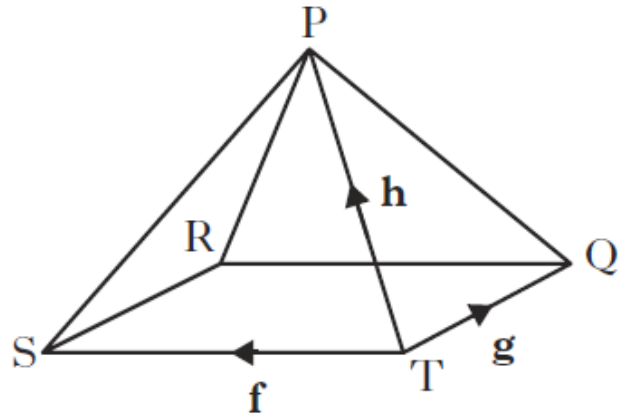
9. The diagram shows the curve with equation of the form  $y = \cos(x + a) + b$  for  $0 \leq x \leq 2\pi$ .



What is the equation of this curve?

10. The diagram shows a square-based pyramid P,QRST.

$\vec{TS}$ ,  $\vec{TQ}$  and  $\vec{TP}$  represent  $\mathbf{f}$ ,  $\mathbf{g}$  and  $\mathbf{h}$  respectively.



Express  $\vec{RP}$  in terms of  $\mathbf{f}$ ,  $\mathbf{g}$  and  $\mathbf{h}$ .

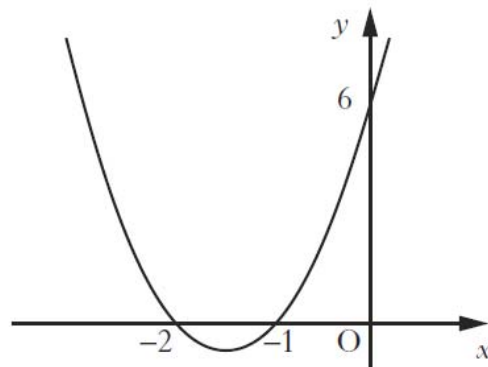
11. Find  $\int \left( \frac{1}{6x^2} \right) dx, x \neq 0.$

12. Find the maximum value of

$$2 - 3 \sin \left( x - \frac{\pi}{3} \right)$$

and the value of  $x$  where this occurs in the interval  $0 \leq x \leq 2\pi.$

13. A parabola intersects the axes at  $x = -2$ ,  $x = -1$  and  $y = 6$ , as shown in the diagram.



What is the equation of the parabola?

14. Find  $\int (2x-1)^{\frac{1}{2}} dx$  where  $x > \frac{1}{2}$ .

15. If  $\mathbf{u} = k \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ , where  $k > 0$  and  $\mathbf{u}$  is a unit vector, determine the value of  $k$ .

16. If  $y = 3\cos^4 x$ , find  $\frac{dy}{dx}$ .

17. Given that  $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$  and  $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 7$ , what is the value of  $\mathbf{a} \cdot \mathbf{b}$ ?

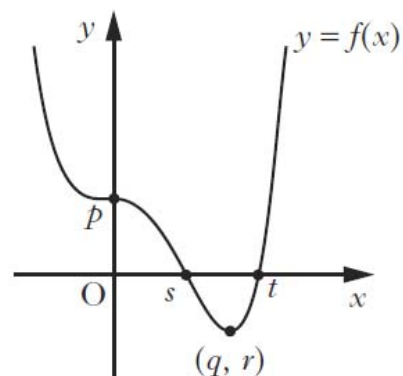
18. The graph of  $y = f(x)$  shown has stationary points at  $(0, p)$  and  $(q, r)$ .

Here are two statements about  $f(x)$ :

- (1)  $f(x) < 0$  for  $s < x < t$ ;
- (2)  $f'(x) < 0$  for  $x < q$ .

Which of the following is true?

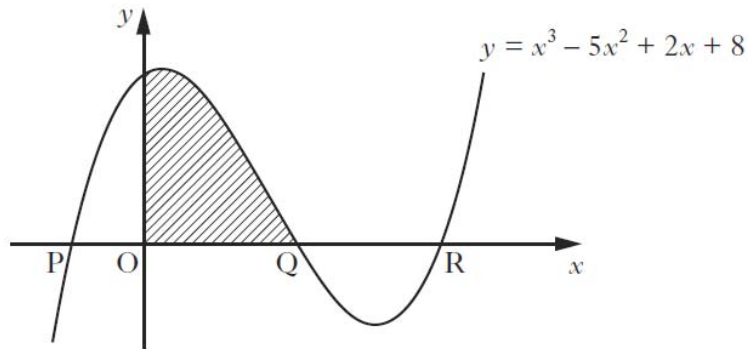
- A Neither statement is correct.
- B Only statement (1) is correct.
- C Only statement (2) is correct.
- D Both statements are correct.



19. Solve  $6 - x - x^2 < 0$ .

20. Simplify  $\frac{\log_b 9a^2}{\log_b 3a}$ , where  $a > 0$  and  $b > 0$ .

21. (a) (i) Show that  $(x - 4)$  is a factor of  $x^3 - 5x^2 + 2x + 8$ .  
(ii) Factorise  $x^3 - 5x^2 + 2x + 8$  fully.  
(iii) Solve  $x^3 - 5x^2 + 2x + 8 = 0$ . 6
- (b) The diagram shows the curve with equation  $y = x^3 - 5x^2 + 2x + 8$ .



The curve crosses the  $x$ -axis at P, Q and R.

Determine the shaded area. 6

22. (a) The expression  $\cos x - \sqrt{3} \sin x$  can be written in the form  $k \cos(x + a)$  where  $k > 0$  and  $0 \leq a < 2\pi$ .  
Calculate the values of  $k$  and  $a$ . 4
- (b) Find the points of intersection of the graph of  $y = \cos x - \sqrt{3} \sin x$  with the  $x$  and  $y$  axes, in the interval  $0 \leq x \leq 2\pi$ . 3
23. (a) Find the equation of  $\ell_1$ , the perpendicular bisector of the line joining P(3, -3) to Q(-1, 9). 4
- (b) Find the equation of  $\ell_2$  which is parallel to PQ and passes through R(1, -2). 2
- (c) Find the point of intersection of  $\ell_1$  and  $\ell_2$ . 3
- (d) Hence find the shortest distance between PQ and  $\ell_2$ . 2

[END OF SECTION B]

[END OF QUESTION PAPER]

