

Higher Maths 2011 paper 1

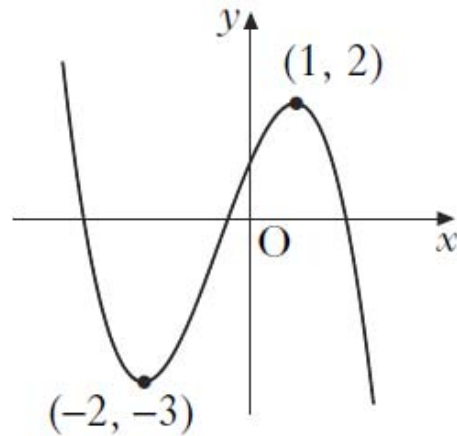
Q1-20 were originally multiple choice questions worth 2 marks each.

1. Given that $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, express $2\mathbf{p} - \mathbf{q} - \frac{1}{2}\mathbf{r}$ in component form.

2. A line l has equation $3y + 2x = 6$.

What is the gradient of any line parallel to l ?

3. The diagram shows the graph of $y = f(x)$.



Draw the graph of $y = f(x + 2) - 1$

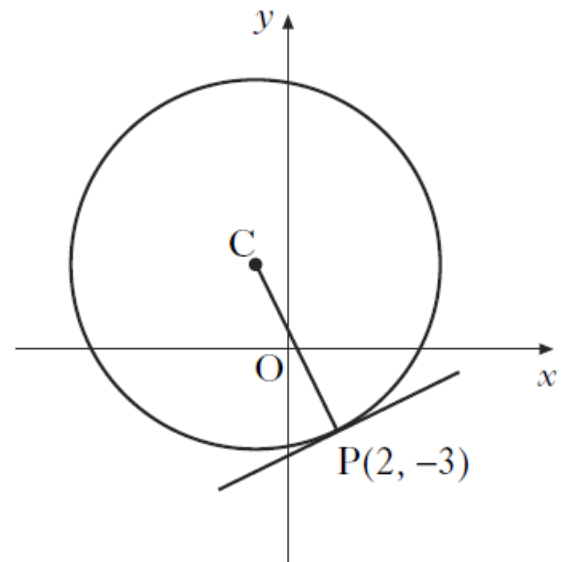
4. A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point $(2, 4)$.
What is the gradient of this tangent?

5. If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q ?

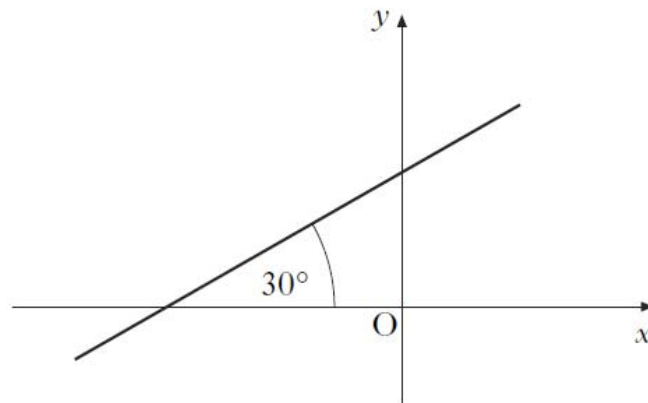
6. The point $P(2, -3)$ lies on the circle with centre C as shown.

The gradient of CP is -2 .

What is the equation of the tangent at P ?



7. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.
What is the remainder when $f(x)$ is divided by $(x - 1)$?
8. A line makes an angle of 30° with the positive direction of the x -axis as shown.



What is the gradient of the line?

9. The discriminant of a quadratic equation is 23.
Here are two statements about this quadratic equation:
- (1) the roots are real;
 - (2) the roots are rational.

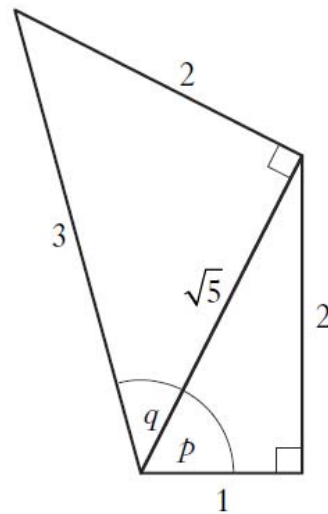
Which of the following is true?

- A Neither statement is correct.
- B Only statement (1) is correct.
- C Only statement (2) is correct.
- D Both statements are correct.

10. Solve $2 \cos x = \sqrt{3}$ for x , where $0 \leq x < 2\pi$.

11. Find $\int \left(4x^{\frac{1}{2}} + x^{-3} \right) dx$, where $x > 0$.

12. The diagram shows two right-angled triangles with sides and angles as given.



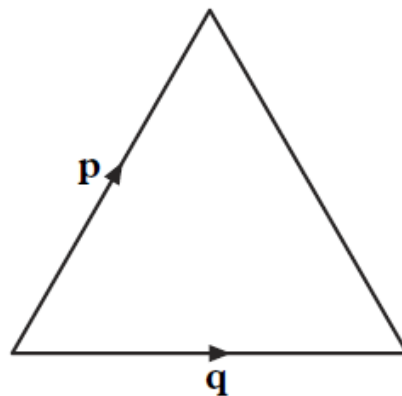
What is the value of $\sin(p + q)$?

13. Given that $f(x) = 4 \sin 3x$, find $f'(0)$.

14. An equilateral triangle of side 3 units is shown.

The vectors \mathbf{p} and \mathbf{q} are as represented in the diagram.

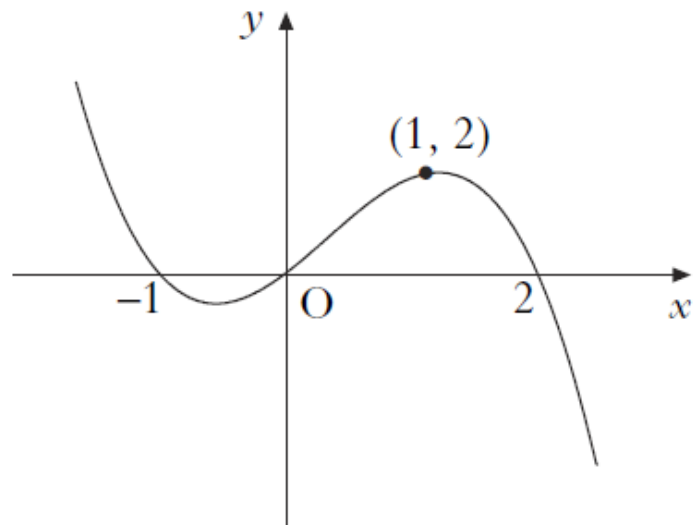
What is the value of $\mathbf{p} \cdot \mathbf{q}$?



15. Given that the points $S(-4, 5, 1)$, $T(-16, -4, 16)$ and $U(-24, -10, 26)$ are collinear, calculate the ratio in which T divides SU .

16. Find $\int \frac{1}{3x^4} dx$, where $x \neq 0$.

17. The diagram shows the graph of a cubic.



What is the equation of this cubic?

18. If $f(x) = (x - 3)(x + 5)$, for what values of x is the graph of $y = f(x)$ above the x -axis?

19. Draw the graph of $x = \log_3 y$

20. On a suitable domain, D , a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

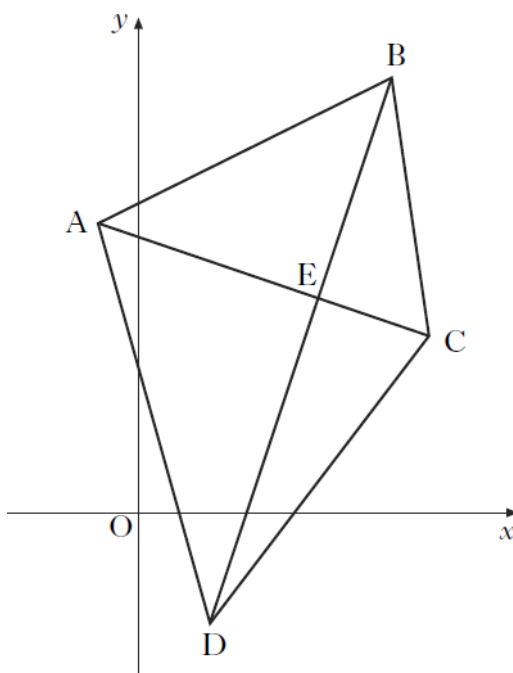
A $x \geq 0$ and $-1 \leq g(x) \leq 1$

B $x \geq 0$ and $0 \leq g(x) \leq 1$

C $x \geq 2$ and $-1 \leq g(x) \leq 1$

D $x \geq 2$ and $0 \leq g(x) \leq 1$

21. A quadrilateral has vertices $A(-1, 8)$, $B(7, 12)$, $C(8, 5)$ and $D(2, -3)$ as shown in the diagram.



- (a) Find the equation of diagonal BD . 2
- (b) The equation of diagonal AC is $x + 3y = 23$.
Find the coordinates of E , the point of intersection of the diagonals. 3
- (c) (i) Find the equation of the perpendicular bisector of AB .
(ii) Show that this line passes through E . 5
22. A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.
- (a) Find where the graph of $y = f(x)$ cuts:
(i) the x -axis;
(ii) the y -axis. 2
- (b) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. 8
- (c) On separate diagrams sketch the graphs of:
(i) $y = f(x)$;
(ii) $y = -f(x)$. 3
23. (a) Solve $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$ for $0 \leq x < 360$. 5
- (b) Hence solve $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$ for $0 \leq x < 360$. 2