Marks

Answer all the questions.

1. (a) Given $f(x) = \cos^2 x\, e^{\tan x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, obtain $f'(x)$ and evaluate $f'(\frac{\pi}{4})$. 3,1

(b) Differentiate $g(x) = \dfrac{\tan^{-1} 2x}{1+4x^2}$. 3

2. Obtain the binomial expansion of $(a^2 - 3)^4$. 3

3. A curve is defined by the equations

$$x = 5\cos\theta, \qquad y = 5\sin\theta, \qquad (0 \le \theta < 2\pi).$$

Use parametric differentiation to find $\dfrac{dy}{dx}$ in terms of θ. 2

Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{4}$. 3

4. Given $z = 1 + 2i$, express $z^2(z + 3)$ in the form $a + ib$. 2

Hence, or otherwise, verify that $1 + 2i$ is a root of the equation

$$z^3 + 3z^2 - 5z + 25 = 0.$$ 2

Obtain the other roots of this equation. 2

5. Express $\dfrac{1}{x^2 - x - 6}$ in partial fractions. 2

Evaluate $\displaystyle\int_0^1 \frac{1}{x^2 - x - 6}\, dx$. 4

6. Write down the 2×2 matrix M_1 associated with an anti-clockwise rotation of $\frac{\pi}{2}$ radians about the origin. 2

Write down the matrix M_2 associated with reflection in the x-axis. 1

Evaluate $M_2 M_1$ and describe geometrically the effect of the transformation represented by $M_2 M_1$. 2

7. Obtain the first three non-zero terms in the Maclaurin expansion of $f(x) = e^x \sin x$. 5

8. Use the Euclidean algorithm to show that $(231, 17) = 1$ where (a, b) denotes the highest common factor of a and b.

Hence find integers x and y such that $231x + 17y = 1$. 4

9. Use the substitution $x = (u - 1)^2$ to obtain $\displaystyle\int \frac{1}{(1+\sqrt{x})^3}\, dx$. 5

Marks

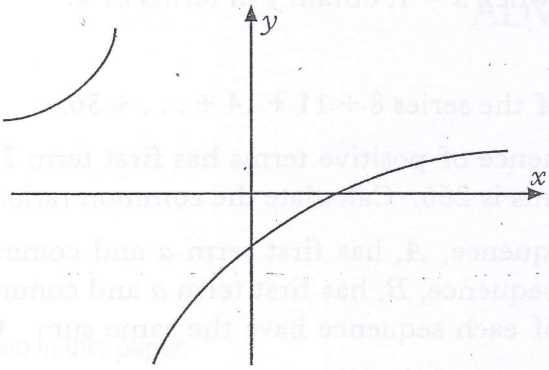
10. Determine whether the function $f(x) = x^4 \sin 2x$ is odd, even or neither.

Justify your answer. 3

11. A solid is formed by rotating the curve $y = e^{-2x}$ between $x = 0$ and $x = 1$ through $360°$ about the x-axis. Calculate the volume of the solid that is formed. 5

12. Prove by induction that $\frac{d^n}{dx^n}(xe^x) = (x + n)e^x$ for all integers $n \geq 1$. 5

13. The function f is defined by $f(x) = \frac{x-3}{x+2}$, $x \neq -2$, and the diagram shows part of its graph.



(a) Obtain algebraically the asymptotes of the graph of f. 3

(b) Prove that f has no stationary values. 2

(c) Does the graph of f have any points of inflexion? Justify your answer. 2

(d) Sketch the graph of the inverse function, f^{-1}. State the asymptotes and domain of f^{-1}. 3

14. (a) Find an equation of the plane π_1 containing the points $A(1, 0, 3)$, $B(0, 2, -1)$ and $C(1, 1, 0)$. 4

Calculate the size of the acute angle between π_1 and the plane π_2 with equation $x + y - z = 0$. 3

(b) Find the point of intersection of plane π_2 and the line

$$\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2}.$$ 3

[Turn over for Questions 15 and 16 on *Page four*

Marks

15. (a) A mathematical biologist believes that the differential equation $x\dfrac{dy}{dx} - 3y = x^4$ models a process. Find the general solution of the differential equation. 5

Given that $y = 2$ when $x = 1$, find the particular solution, expressing y in terms of x. 2

(b) The biologist subsequently decides that a better model is given by the equation $y\dfrac{dy}{dx} - 3x = x^4$.

Given that $y = 2$ when $x = 1$, obtain y in terms of x. 4

16. (a) Obtain the sum of the series $8 + 11 + 14 + \ldots + 56$. 2

(b) A geometric sequence of positive terms has first term 2, and the sum of the first three terms is 266. Calculate the common ratio. 3

(c) An arithmetic sequence, A, has first term a and common difference 2, and a geometric sequence, B, has first term a and common ratio 2. The first four terms of each sequence have the same sum. Obtain the value of a. 3

Obtain the smallest value of n such that the sum to n terms for sequence B is more than **twice** the sum to n terms for sequence A. 2

[END OF QUESTION PAPER]