Higher 2016 Paper 2 Solutions

1 ai) $\left(\frac{-6+10}{2}, \frac{2+6}{2}\right) = M(2,4)$ $\qquad P(\overset{a}{0}, \overset{b}{-4})$

ii) $m_{PM} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-4}{0-2} = \frac{-8}{-2} = 4$

$y-b = m(x-a)$

$y+4 = 4(x-0)$

$\underline{\underline{y = 4x-4}}$

b) $m_{PR} = \frac{6--4}{10-0} = \frac{10}{10} = 1$ $\qquad R(10,6)\ P(0,-4)$

$\therefore m_{\perp} = -1$ through $M(\overset{a}{2},\overset{b}{4})$

$y-4 = -1(x-2)$

$y-4 = -x+2$

$\underline{\underline{y = -x+6}}$

c) midpoint of PR $= \left(\frac{0+10}{2}, \frac{-4+6}{2}\right) = (5,1)$

$y = -x+6$

when $x=5$, $y=-5+6=1$ so point $(5,1)$ lies on the line.
ie the line passes through the midpoint of PR.

2 $x^2-2x+3-p=0$

$a=1$
$b=-2$
$c=3-p$

$b^2-4ac < 0$ for no real roots

$(-2)^2-4(3-p) < 0$

$4-12+4p < 0$

$4p-8 < 0$

$4p < 8$

$\underline{\underline{p < 2}}$

3 a i)

$$\begin{array}{r|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & 11 & -14 \\ \hline & 2 & -11 & 14 & \boxed{0} \end{array}$$

no remainder means $(x+1)$ is a factor.

ii) $(x+1)(2x^2-11x+14)=0$

$(x+1)(2x-7)(x-2)=0$

$\underline{\underline{x=-1 \text{ or } x=\frac{7}{2} \text{ or } x=2}}$

b) i) $A(-1,0)$ $B(2,0)$ $\left[C\left(\frac{7}{2},0\right)\right]$

ii) $\displaystyle\int_{-1}^{2}(2x^3-9x^2+3x+14)\,dx$

$= \left[\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x\right]_{-1}^{2} = \left[\frac{x^4}{2}-3x^3+\frac{3}{2}x^2+14x\right]_{-1}^{2}$

$= 8-24+6+28-\left(\frac{1}{2}+3+\frac{3}{2}-14\right)$

$= 18-(-9)$

$= \underline{\underline{27 \text{ units}^2}}$

4 a) $(x+5)^2+(y-6)^2=9$

$(x-a)^2+(y-b)^2=r^2$

centre $\underline{\underline{(-5,6)}}$

$\underline{\underline{r=3}}$

$x^2+y^2-6x-16=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=-6$ $2f=0$

$g=-3$ $f=0$

centre $(-g,-f)=\underline{\underline{(3,0)}}$

$r=\sqrt{g^2+f^2-c}=\sqrt{9--16}=\underline{\underline{5}}$

b) $r_1+r_2=3+5=8$

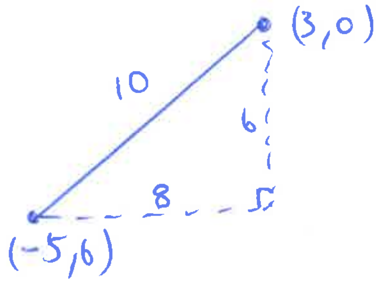
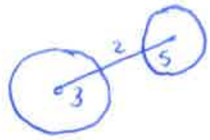


Diagram is only a sketch

The distance between the centres is 10 units. The sum of the radii is 8 units. The circles do not touch (2 units apart)



5 a) $\vec{AB} = b - a = \begin{pmatrix} -10 \\ 18 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$

$\vec{AC} = c - a = \begin{pmatrix} -4 \\ -6 \\ 21 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$

b) $a.b = |a||b| \cos\theta$

$$\cos\theta = \frac{a.b}{|a||b|} = \frac{\vec{AB}.\vec{AC}}{|\vec{AB}||\vec{AC}|} = \frac{16 - 128 + 32}{\sqrt{324}\sqrt{324}} = -0.247$$

$\Rightarrow \theta = \underline{\underline{104.3^\circ}}$

6 $B(t) = 200e^{0.107t}$

a) when $t = 0$, $B(0) = 200e^{-0.107(0)} = \underline{\underline{200}}$

b) $400 = 200e^{0.107t}$

$200e^{0.107t} = 400$

$e^{0.107t} = 2$

$\log_e e^{0.107t} = \log_e 2$

$0.107t \log_e e = \log_e 2$

$t = \dfrac{\log_e 2}{0.107} = \underline{\underline{6.48 \text{ hours}}}$

7 a) $L = 9x + 8y$

$L = 9x + 8\left(\dfrac{18}{x}\right)$

$\underline{\underline{L = 9x + \dfrac{144}{x}}}$ as required.

Area $= 3x(2y) = 6xy$

$6xy = 108$

$y = \dfrac{108}{6x} = \underline{\underline{\dfrac{18}{x}}}$

7b) $L(x) = 9x + 144x^{-1}$

$L'(x) = 9 - 144x^{-2} = 0$ for stationary points

$$\Rightarrow 9 - \frac{144}{x^2} = 0$$

$$9x^2 - 144 = 0$$

$$9x^2 = 144$$

$$x^2 = 16$$

$$x = \pm 4$$

Positive value

x	$2 \to$	4	$\to 6$
$f'(x)$	$-$	0	$+$
Slope	\	—	/

$x = 4$ gives minimum length of fencing.

8) a) $5\cos x - 2\sin x = k\cos(x+a)$

$$= k(\cos x\cos a - \sin x \sin a)$$

$$= k\cos a \cos x - k\sin a \sin x$$

$k\cos a = 5$

$k\sin a = 2$

$$k = \sqrt{5^2+2^2} = \sqrt{29}$$

$$\tan a = \frac{k\sin a}{k\cos a} = \frac{2}{5}$$

S ✓	A ✓
T	C ✓

$$a = 21.8^\circ = \frac{21.8\pi}{180} = 0.38 \text{ radians}$$

So $5\cos x - 2\sin x = \sqrt{29}\cos(x + 0.38)$

b) $5\cos x - 2\sin x + 10 = 12$

$$\sqrt{29}\cos(x+0.38) = 2$$

$$\cos(x+0.38) = \frac{2}{\sqrt{29}}$$

S	A ✓ x
T	C ✓

$2\pi - x$

$360 - x$

$$x + 0.38 = 1.19 \text{ or } 5.09$$

$$x = 0.81 \text{ or } 4.71 \text{ radians}$$

(9) $f'(x) = \dfrac{2x+1}{x^{1/2}} = \dfrac{2x}{x^{1/2}} + \dfrac{1}{x^{1/2}} = 2x^{1/2} + x^{-1/2}$

$f(x) = \int (2x^{1/2} + x^{-1/2})\,dx = 2\dfrac{x^{3/2}}{3/2} + \dfrac{x^{1/2}}{1/2} + C$

$f(x) = \frac{4}{3}x^{3/2} + 2x^{1/2} + C$

$f(9) = 40$

$x = 9 \quad f(x) = 40$

$40 = \frac{4}{3}(9)^{3/2} + 2(9)^{1/2} + C$

$40 = 36 + 6 + C$

$C + 42 = 40$

$C = -2$

$\therefore \quad \underline{\underline{f(x) = \frac{4}{3}x^{3/2} + 2x^{1/2} - 2}}$

(10) a) $y = (x^2+7)^{1/2}$

$\dfrac{dy}{dx} = \frac{1}{2}(x^2+7)^{-1/2} \times 2x = x(x^2+7)^{-1/2} = \underline{\underline{\dfrac{x}{(x^2+7)^{1/2}}}}$

b) $4\int \dfrac{x}{(x^2+7)^{1/2}}\,dx = 4\left[(x^2+7)^{1/2}\right] + C = \underline{\underline{4(x^2+7)^{1/2} + C}}$

(11) a) $\sin 2x \tan x = 1 - \cos 2x$

LHS : $\sin 2x \tan x = 2\sin x \cos x \dfrac{\sin x}{\cos x} = 2\sin^2 x$

RHS : $1 - \cos 2x = 1 - (1 - 2\sin^2 x) = 2\sin^2 x$

LHS = RHS as required.

b) $f(x) = \sin 2x \tan x = 1 - \cos 2x$ (from part 'a')

$f'(x) = 0 - -2\sin 2x = \underline{\underline{2\sin 2x}}$