Higher 2016 Paper 1 solutions

1

$y + 4x = 7$

$y = -4x + 7$

$m = -4 \qquad (\underset{a}{-2}, \underset{b}{3})$

$y - 3 = -4(x+2)$

$y - 3 = -4x - 8$

$\underline{\underline{y = -4x - 5}}$

2

$y = 12x^3 + 8x^{1/2}$

$\frac{dy}{dx} = 36x^2 + 4x^{-1/2}$

$= \underline{\underline{36x^2 + \frac{4}{x^{1/2}}}}$

3 a) $u_{n+1} = \frac{1}{3}u_n + 10$

$u_3 = 6$

$u_4 = \frac{1}{3}u_3 + 10$

$= \frac{1}{3}(6) + 10$

$= \underline{\underline{12}}$

b) $-1 < \frac{1}{3} < 1$ so sequence has a limit

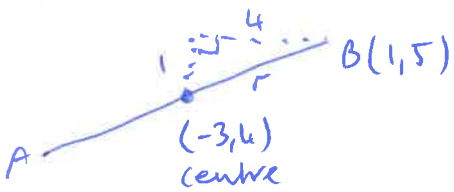
c) $L = \frac{1}{3}L + 10$

$\frac{2}{3}L = 10$

$2L = 30$

$L = \underline{\underline{15}}$

4 midpoint of AB $= \left(\frac{-7+1}{2}, \frac{3+5}{2}\right) = (-3, 4)$



$r^2 = 4^2 + 1^2$

$r^2 = 17$

$r = \sqrt{17}$

equation of circle

$(x-a)^2 + (y-b)^2 = r^2$

$\underline{\underline{(x+3)^2 + (y-4)^2 = 17}}$

5

$\int 8\cos(4x+1)\,dx$

$= 8\left(\frac{1}{4}\sin(4x+1)\right) + c$

$= \underline{\underline{2\sin(4x+1) + c}}$

6 a) Let $y = 3x + 5$

$3x + 5 = y$

$3x = y - 5$

$x = \frac{y-5}{3}$

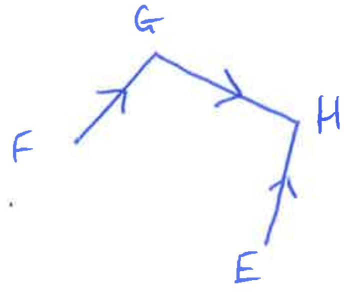
so $\underline{\underline{f^{-1}(x) = \frac{x-5}{3}}}$

change x to $f^{-1}(x)$

change y to x.

b) $g^{-1}(7) = \underline{\underline{2}}$

7



a) $\overrightarrow{FH} = \overrightarrow{FG} + \overrightarrow{GH}$

$= \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} = \underline{\underline{i + 3j - 4k}}$

b) $\overrightarrow{FE} = \overrightarrow{FG} + \overrightarrow{GH} + \overrightarrow{HE}$

$= \overrightarrow{FG} + \overrightarrow{GH} - \overrightarrow{EH}$

$= \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix} = \underline{\underline{-i - 5k}}$

8 $x^2 + y^2 + 2x - 4y - 5 = 0$, $y = 3x - 5$

$x^2 + (3x-5)^2 + 2x - 4(3x-5) - 5 = 0$

$x^2 + (3x-5)(3x-5) + 2x - 12x + 20 - 5 = 0$

$x^2 + 9x^2 - 15x - 15x + 25 + 2x - 12x + 20 - 5 = 0$

$10x^2 - 40x + 40 = 0$

$10(x^2 - 4x + 4) = 0$

$(x-2)(x-2) = 0$

$x = 2$

$y = 3(2) - 5 = 1$

$\underline{\underline{(2,1)}}$ only one point of contact means line is a tangent to the circle.

9 a) $f(x) = x^3 + 3x^2 - 24x$

$f'(x) = 3x^2 + 6x - 24 = 0$ for S.P

$3(x^2 + 2x - 8) = 0$

$(x+4)(x-2) = 0$

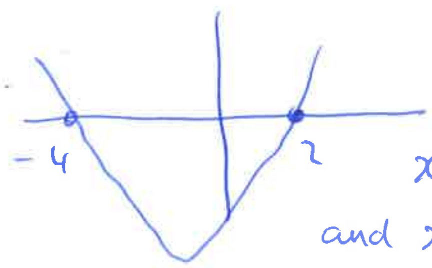
$\underline{\underline{x = -4 \text{ or } x = 2}}$

b) function increases when $f'(x) > 0$

ie $3x^2 + 6x - 24 > 0$

b) is only out of 2 marks so use this method

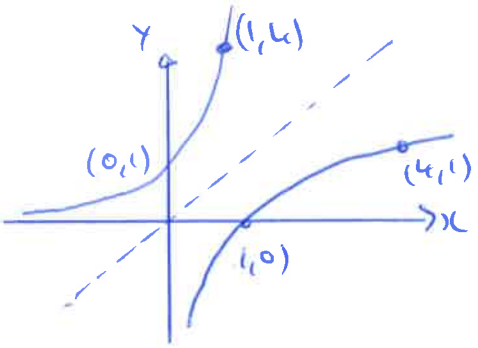
(You could use a nature table and draw the graph of $f(x)$)



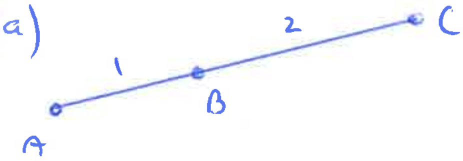
$x < -4$

and $\underline{\underline{x > 2}}$

10



11 a)



$$\frac{AB}{BC} = \frac{1}{2}$$

$$2AB = BC$$

$$2(b-a) = c-b$$

$$2b - 2a = c - b$$

$$3b = 2a + c$$

$$b = \tfrac{1}{3}(2a+c)$$

$$= \tfrac{1}{3}\left[2\begin{pmatrix}1\\3\\-2\end{pmatrix} + \begin{pmatrix}4\\-3\\4\end{pmatrix}\right]$$

$$= \tfrac{1}{3}\begin{pmatrix}6\\3\\0\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$\underline{\underline{B(2,1,0)}}$$

11b) $\vec{AC} = c - a$

$$= \begin{pmatrix}4\\-3\\4\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix} = \begin{pmatrix}3\\-6\\6\end{pmatrix}$$

$$|\vec{AC}| = \sqrt{3^2 + (-6)^2 + 6^2}$$

$$= \sqrt{9 + 36 + 36}$$

$$= \sqrt{81}$$

$$= 9$$

$k\,\vec{AC}$ is a unit vector so $\underline{\underline{k = \frac{1}{9}}}$

12 a) $f(x) = 2x^2 - 4x + 5 \qquad g(x) = 3 - x$

$$h(x) = f(g(x))$$

$$= f(3-x)$$

$$= 2(3-x)^2 - 4(3-x) + 5$$

$$= 2(9 - 6x + x^2) - 12 + 4x + 5$$

$$= 18 - 12x + 2x^2 - 12 + 4x + 5$$

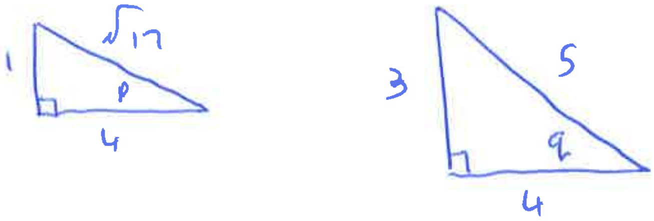
$= \underline{\underline{2x^2 - 8x + 11}}$ as required.

b) $2(x^2 - 4x) + 11$

$$= 2\left[(x^2 - 4x + 4) - 4\right] + 11$$

$$= 2\left[(x-2)^2 - 4\right] + 11 = 2(x-2)^2 - 8 + 11 = \underline{\underline{2(x-2)^2 + 3}}$$

13



$\cos(q-p) = \cos q \cos p + \sin q \sin p$

$= \frac{4}{5} \times \frac{4}{\sqrt{17}} + \frac{3}{5} \times \frac{1}{\sqrt{17}}$

rationalise the denominator

$= \frac{16}{5\sqrt{17}} + \frac{3}{5\sqrt{17}} = \frac{19}{5\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}} = \underline{\underline{\frac{19\sqrt{17}}{85}}}$

14 a) $\log_5 25 = \underline{\underline{2}}$

b) $\log_4 x + \log_4(x-6) = \log_5 25$

$\Rightarrow \quad \log_4 x(x-6) = 2$

$x(x-6) = 4^2$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ or $x = -2$

For $x > 6$, $\underline{\underline{x = 8}}$ is only solution

15 $f(x) = k(x-a)(x-b)^2$ roots are $x = -5 \Rightarrow (x+5) = 0$

$x = 4 \Rightarrow (x-4) = 0$

$y = k(x-4)(x+5)^2$

x y
(1, 9) $\quad 9 = k(-3)(6)^2$

$9 = k(-3)(36)$

$1 = k(-3)(4)$

$-12k = 1$

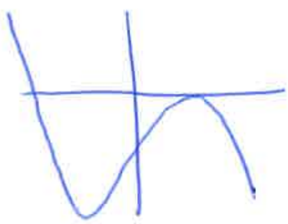
$\underline{\underline{k = -\frac{1}{12}}}$

$a = 4$

$\underline{\underline{b = -5}}$

b) $g(x) = f(x) - d$

move down 9 units



$\underline{\underline{d > 9}}$ (to leave only 1 root)