Higher 2015 Paper 2 Solutions

1 a) $m_{AB} = \frac{-5-7}{-1-(-5)} = \frac{-12}{4} = -3$

$m_{\perp} = \frac{1}{3}$ $\quad C(\overset{a}{13},\overset{b}{3})$

$y - b = m(x-a)$

$y - 3 = \frac{1}{3}(x-13)$

$3y - 9 = x - 13$

$x - 3y = 4$ as required.

b) midpoint of AC

$= \left(\frac{-5+13}{2}, \frac{7+3}{2}\right)$

$= (\overset{a}{4},\overset{b}{5})$ $\quad B(-1,-5)$

$m_{median} = \frac{-5-5}{-1-4} = \frac{-10}{-5} = 2$

$y - 5 = 2(x-4)$

$y - 5 = 2x - 8$

$y = 2x - 3$

c) $x - 3y = 4$

$\Rightarrow x - 3(2x-3) = 4$

$x - 6x + 9 = 4$

$-5x = -5$

$x = 1$

$y = 2(1) - 3$

$= -1$ $\quad (1,-1)$

2 $f(x) = 10 + x \quad g(x) = (1+x)(3-x)+2$

$= 5 + 2x - x^2$

a) $f(g(x))$

$= f(5+2x-x^2)$

$= 10 + 5 + 2x - x^2$

$= 15 + 2x - x^2$

b) $-x^2 + 2x + 15$

$= -(x^2 - 2x) + 15$

$= -[(x^2 - 2x + 1) - 1] + 15$

$= -[(x-1)^2 - 1] + 15$

$= -(x-1)^2 + 16$

2c) $f(g(x)) \neq 0$

So $-x^2 + 2x + 15 \neq 0$

$(-x+5)(x+3) \neq 0$

$-x + 5 \neq 0$ or $x + 3 \neq 0$

$x \neq 5$ or $x \neq -3$

3 a) $t_{n+1} = \frac{3}{4} t_n + 13$

$t_1 = 13$

$t_2 = \frac{3}{4}(13) + 13 = 22\frac{3}{4}$ feet

b) $f_{n+1} = \frac{1}{3} f_n + 32$

$L = \frac{1}{3} L + 32$

$\frac{2}{3} L = 32$

$2L = 96$

$L = 48$

Frog will not escape as $48 < 50$

$t_{n+1} = \frac{3}{4} t_n + 13$

$L = \frac{3}{4} L + 13$

$\frac{1}{4} L = 13$

$L = 52$

Toad will escape as $52 > 50$

4 a) $\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$ (times every term by 4)

$x^2 - 2x + 12 = x^2 - 6x + 20$

$4x = 8$

$x = 2$

b) $\int_0^2 \left[\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \left(\frac{3}{8}x^2 - \frac{9}{4}x + 3\right)\right] dx$

$= \int_0^2 \left(-\frac{1}{8}x^2 + \frac{7}{4}x\right) dx$

$= \left[-\frac{1}{24}x^3 + \frac{7}{8}x^2\right]_0^2 = -\frac{1}{24}(2)^3 + \frac{7}{8}(2)^2 - 0$

$= -\frac{1}{3} + \frac{7}{2} = \frac{-2}{6} + \frac{21}{6} = \frac{19}{6}$

Shape is symmetrical so total Area $= 2 \times \frac{19}{6} = \frac{19}{3}$ units^2

5 a) Circle C_1 : $x^2+y^2+6x+10y+9=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=6 \qquad 2f=10$

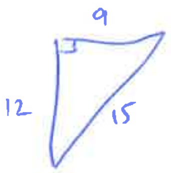
$g=3 \qquad f=5$

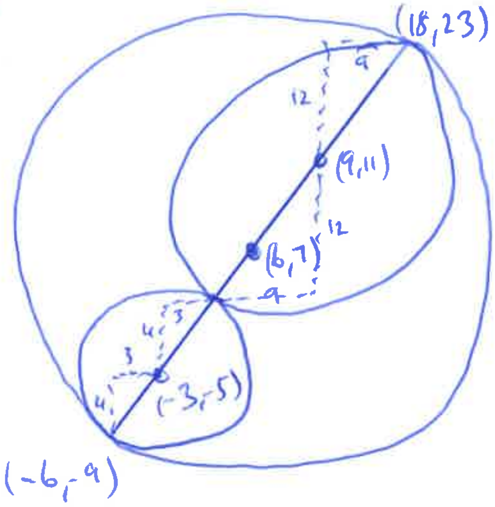
centre $(-3,-5)$

$r=\sqrt{g^2+f^2-c}$

$r=\sqrt{9+25-9}$

$r=5$



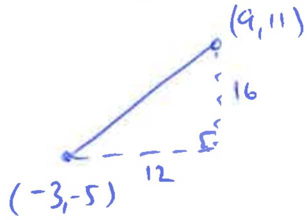


Distance between centres is

$\sqrt{12^2+16^2}$

$=\sqrt{400}$

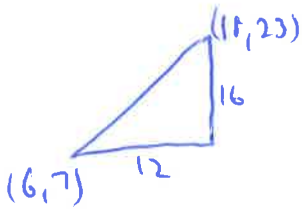
$=20$



So radius of C_2 is $\underline{\underline{15}}$

b) centre of C_3 is $\left(\frac{-6+18}{2}, \frac{-9+23}{2}\right) = (6,7)$

radius of C_3 is



$\sqrt{12^2+16^2}$

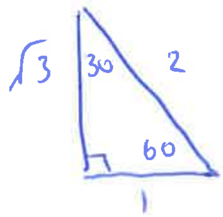
$=20$

equation of C_3 is $\underline{\underline{(x-6)^2+(y-7)^2=400}}$

6 a) $p\cdot(q+r) = p\cdot q + p\cdot r$

$= \frac{9}{2} + 0$

$= \underline{\underline{4\frac{1}{2}}}$



$p\cdot q = |p||q|\cos\theta$

$= 3\times 3\times \cos 60°$

$= \frac{9}{2}$

$p\cdot r = |p||r|\cos\theta$

$= 3\times|r|\times\cos 90°$

$= 0$

6b) $\overrightarrow{EC} = \underline{\underline{-q + p + r}}$

c) $\overrightarrow{AE} \cdot \overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$

$\Rightarrow q \cdot (p + r - q) = 9\sqrt{3} - \frac{9}{2}$

$\Rightarrow p \cdot q + q \cdot r - q \cdot q = 9\sqrt{3} - \frac{9}{2}$

$\Rightarrow \frac{9}{2} + 3\frac{\sqrt{3}}{2}|r| - 9 = 9\sqrt{3} - \frac{9}{2}$

$\Rightarrow 3\frac{\sqrt{3}}{2}|r| - \frac{9}{2} = 9\sqrt{3} - \frac{9}{2}$

$\Rightarrow 3\frac{\sqrt{3}}{2}|r| = 9\sqrt{3}$

$\Rightarrow 3\sqrt{3}|r| = 18\sqrt{3}$

$\Rightarrow |r| = \frac{18\sqrt{3}}{3\sqrt{3}}$

$\Rightarrow \underline{\underline{|r| = 6}}$

$q \cdot r = |q||r|\cos\theta$
$= 3 \times |r| \times \cos 30$
$= 3\frac{\sqrt{3}}{2}|r|$

$q \cdot q = |q||q|\cos\theta$
$= 3 \times 3 \times \cos 0°$
$= 9$

7 a) $\int (3\cos 2x + 1)\,dx = \underline{\underline{\frac{3}{2}\sin 2x + x + c}}$

b) $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$

LHS : $3\cos 2x + 1$

$= 3(\cos^2 x - \sin^2 x) + 1$

$= 3\cos^2 x - 3\sin^2 x + 1$

$= 3\cos^2 x - 3\sin^2 x + (\sin^2 x + \cos^2 x)$

$= 4\cos^2 x - 2\sin^2 x$

= RHS as required

$\sin^2 x + \cos^2 x = 1$

7c) $\int (\sin^2 x - 2\cos^2 x)\,dx$

$= \int (-2\cos^2 x + \sin^2 x)\,dx$

$= \int -\frac{1}{2}(4\cos^2 x - 2\sin^2 x)\,dx$

$= \int -\frac{1}{2}(3\cos 2x + 1)\,dx$

$= -\frac{1}{2}\left[\frac{3}{2}\sin 2x + x + c\right]$

$= \underline{\underline{-\frac{3}{4}\sin 2x - \frac{1}{2}x - \frac{1}{2}c}}$

8 ai) $x = 20 \Rightarrow T(20) = 5\sqrt{36+20^2} + 4(20-20)$

$= 5\sqrt{436}$

$= 104.4$ tenths

$= \underline{\underline{10.44 \text{ seconds.}}}$

aii) $x = 0 \Rightarrow T(0) = 5\sqrt{36} + 4(20)$

$= 30 + 80$

$= 110$ tenths

$= \underline{\underline{11 \text{ seconds.}}}$

b) $T(x) = 5(36 + x^2)^{1/2} + 80 - 4x$

$T'(x) = \frac{5}{2}(36 + x^2)^{-1/2} \times 2x - 4 = 0$ for S.P.

$\Rightarrow \dfrac{5x}{(36+x^2)^{1/2}} - 4 = 0$

$\Rightarrow 5x - 4(36 + x^2)^{1/2} = 0$

$\Rightarrow 5x = 4(36 + x^2)^{1/2}$ square both sides

$$25x^2 = 16(36 + x^2)$$
$$25x^2 = 576 + 16x^2$$
$$9x^2 = 576$$
$$x^2 = 64$$
$$\underline{\underline{x = 8}} \quad \text{(Positive value only)}$$

x	$\rightarrow$	8	$\rightarrow$
$T'(x)$	$-$	0	$+$
Slope	\	—	/

$x = 8$ gives minimum time taken.

$$\therefore \quad T(8) = 5\sqrt{36 + 8^2} + 4(20 - 8)$$
$$= 50 + 48$$
$$= 98 \text{ tenths}$$
$$= \underline{\underline{9.8 \text{ seconds.}}}$$

9

$$36\sin(1.5t) - 15\cos(1.5t) = k\sin(1.5t - a)$$
$$= k(\sin 1.5t \cos a - \cos 1.5t \sin a)$$
$$= k\cos a \sin 1.5t - k\sin a \cos 1.5t$$

$$k\cos a = 36$$
$$k\sin a = 15$$
$$k = \sqrt{36^2 + 15^2}$$
$$k = 39$$

$$\tan a = \frac{15}{36}$$

S ✓	A ✓
T	C ✓

$$a = 0.395 \text{ radians.}$$

So $36\sin(1.5t) - 15\cos(1.5t) = \underline{\underline{39\sin(1.5t - 0.395)}}$

$$h = 36\sin(1.5t) - 15\cos(1.5t) + 65$$

$h = 100 \quad \Rightarrow \quad 39\sin(1.5t - 0.395) + 65 = 100$

$$\sin(1.5t - 0.395) = \frac{35}{39}$$

$\pi - x$ S	A x
T	C

$$1.5t - 0.395 = 1.11 \text{ or } 2.03$$
$$1.5t = 1.505 \text{ or } 2.42$$
$$\underline{\underline{t = 1}} \text{ or } \underline{\underline{1.61}} \text{ seconds.}$$