Higher 2015 Paper 1 Solutions

(1) $u.v = 0$ for perpendicular vectors

$\therefore -24 + 2t + 6 = 0$

$2t - 18 = 0$

$2t = 18$

$t = 9$

(2) $y = 2x^3 + 3$ $\quad x = -2$

$\frac{dy}{dx} = 6x^2$

$y = 2(-2)^3 + 3$

$= -16 + 3$

$= -13$

when $x = -2$, $\frac{dy}{dx} = 6(-2)^2 = 24$ $\quad (-2, -13)$ (a, b)

$y - b = m(x - a)$

$y + 13 = 24(x + 2)$

$y + 13 = 24x + 48$

$y = 24x + 35$

(3)

-3	1	-3	-10	24
		-3	18	-24
	1	-6	8	0

no remainder means $(x+3)$ is a factor

$(x+3)(x^2 - 6x + 8)$

$= (x+3)(x-4)(x-2)$

(4) $y = 3\cos 4x + 1$

So $p = 3$, $q = 4$, $r = 1$

(5) a) $g(x) = 6 - 2x$ $\quad$ b) x

Let $y = 6 - 2x$

$2x = 6 - y$

$x = \frac{6 - y}{2}$

$\therefore g^{-1}(x) = \frac{6 - x}{2}$

(6) $\log_6 12 + \frac{1}{3}\log_6 27$

$= \log_6 12 + \log_6 27^{1/3}$

$= \log_6 12 + \log_6 3$

$= \log_6 36$

$= 2$

(7) $f(x) = x^{1/2}\left(3x - \frac{2}{x^1 x^{1/2}}\right)$

$f(x) = x^{1/2}(3x - 2x^{-3/2})$

$f(x) = 3x^{3/2} - 2x^{-1}$

$f'(x) = \frac{9}{2}x^{1/2} + 2x^{-2} = \frac{9}{2}x^{1/2} + \frac{2}{x^2}$

$f'(4) = \frac{9}{2}(4)^{1/2} + \frac{2}{4^2} = 9 + \frac{1}{8} = 9\frac{1}{8}$

14 cont.

$r = \sqrt{g^2+f^2-c}$

$6 = \sqrt{36+25-k}$

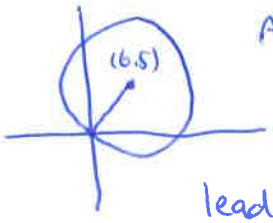
$36 = 61-k$

$k = 61-36$

$k = 25$

If $r = 5$, circle would only touch axes at 1 point

If $r = 6$, circle just touches y axis and cuts x axis in 2 places, meaning 3 places in total.



Alternative solution

$r = \sqrt{61}$

leads to $\sqrt{61} = \sqrt{61-k}$

$61 = 61-k$

$k = 0$

(15) $\frac{dT}{dt} = \frac{1}{25}t - k$

$T = \int \left(\frac{1}{25}t - k\right) dt$

$T = \frac{1}{25}\frac{t^2}{2} - kt + C$

$T = \frac{1}{50}t^2 - kt + C$

When $t = 0$, $T = 100$ $\Rightarrow$ $100 = \frac{1}{50}(0)^2 - k(0) + C$

$\Rightarrow 100 = C$

When $t = 10$, $T = 82$ $\Rightarrow$ $82 = \frac{1}{50}(10)^2 - 10k + 100$

$82 = 2 - 10k + 100$

$10k = 102 - 82$

$10k = 20$

$k = 2$

$\therefore T = \frac{1}{50}t^2 - 2t + 100$

(8) Area $= x(x-2) = x^2 - 2x$

So $x^2 - 2x < 15$

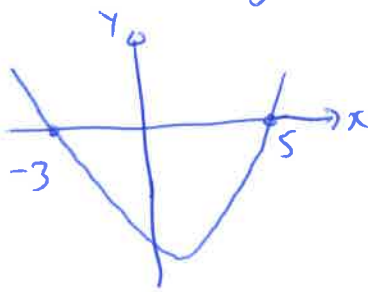
$x^2 - 2x - 15 < 0$

Sketch $y = x^2 - 2x - 15$

Set $y = 0 \Rightarrow x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ or $x = -3$



From graph

$\underline{-3 < x < 5}$

(9) $y = -\sqrt{3}x$

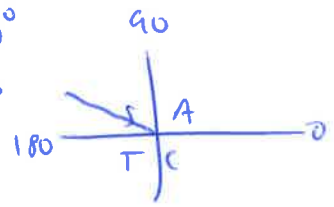
$m = -\sqrt{3}$

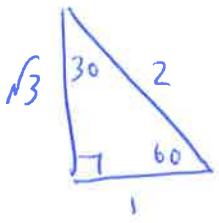
$m = \tan\theta$

$m = \tan 150°$

$= -\tan 30°$

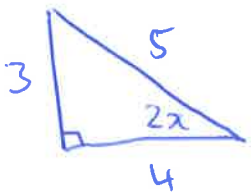
$= -\frac{1}{\sqrt{3}}$





$\underline{m_{AB} \neq m_{BC}}$ so points A, B and C are not collinear.

(10)



a) $\cos 2x = \frac{4}{5}$

b) $\cos 2x = 2\cos^2 x - 1$

$\Rightarrow 2\cos^2 x - 1 = \frac{4}{5}$

$2\cos^2 x = \frac{9}{5}$

$\cos^2 x = \frac{9}{10}$

$\cos x = \sqrt{\frac{9}{10}} = \underline{\frac{3}{\sqrt{10}}}$

(11) a) Centre $(-8, -2)$

$m_{radius} = \frac{-5 - -2}{-2 - -8} = \frac{-3}{6} = -\frac{1}{2}$

$\therefore m_\perp = 2$ $\quad$ (a, b) $(-2, -5)$

$y + 5 = 2(x + 2)$

$y + 5 = 2x + 4$

$\underline{y = 2x - 1}$

11b) equate: $-2x^2 + px + 1 - p = 2x - 1$

$-2x^2 + px - 2x + 2 - p = 0$

$-2x^2 + x(p-2) + 2 - p = 0$

11b cont $a = -2,\ b = p-2,\ c = 2-p$

$b^2 - 4ac = 0$ (for tangency)

$(p-2)^2 - 4(-2)(2-p) = 0$

$p^2 - 4p + 4 + 16 - 8p = 0$

$p^2 - 12p + 20 = 0$

$(p-10)(p-2) = 0$

$p = 10$ or $p = 2$

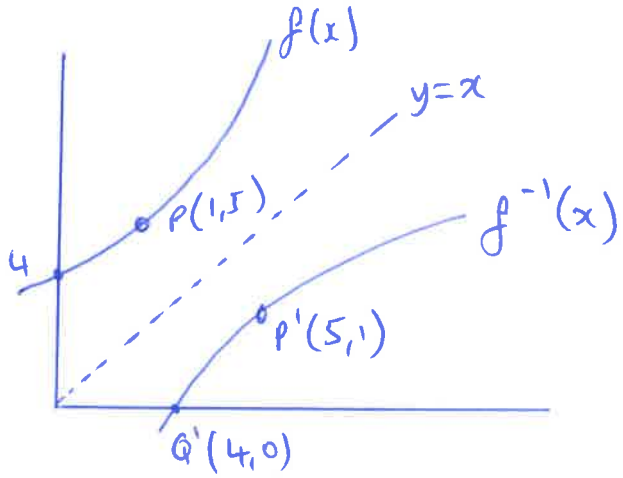
As $p > 3$, $\underline{p = 10}$

(12) $\frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \underline{-\frac{1}{2} \text{ units}^2}$

(13) a) $f(x) = 2^x + 3$

when $x = 1$, $f(x) = 2^1 + 3 = 5$ $\therefore$ $\underline{b = 5}$

bi)



c) $y = 4 - f(x+1)$

$(3,11) \rightarrow (2,11) \rightarrow (2,-11)$

$\rightarrow \underline{(2,-7)}$

(one unit to the left, reflect in x axis, move vertically up by 4)

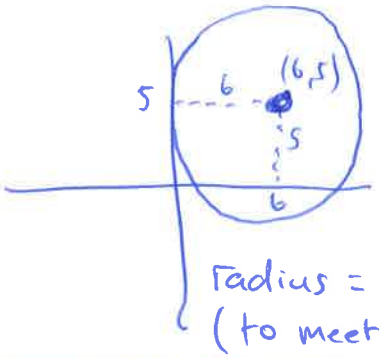
(14) $x^2 + y^2 - 12x - 10y + k = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -12$ $\quad 2f = -10$

$g = -6$ $\quad f = -5$

centre $(6, 5)$



radius = 6 units (to meet axes in 3 places)