Higher 2014 Paper 2 Solutions

1 a) $M_{AB} = \dfrac{2-0}{5-3} = \dfrac{2}{2} = 1$

$m_{\perp} = -1$ midpoint of $AB = \left(\dfrac{3+5}{2}, \dfrac{2+0}{2}\right) = (\overset{a}{4}, \overset{b}{1})$

$y - b = m(x - a)$

$y - 1 = -1(x - 4)$

$y - 1 = -x + 4$

$\underline{\underline{y = -x + 5}}$

b) $y = -2x + 6$ with $y = -x + 5$

$-x + 5 = -2x + 6$

$\quad +2x - 5 \quad\quad +2x - 5$

$x = 1$

$y = -2(1) + 6 = 4$

$\underline{\underline{T(1,4)}}$

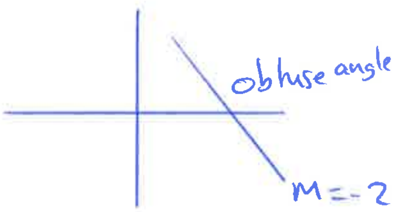
c) $M_{AT} = \dfrac{4-0}{1-3} = \dfrac{4}{-2} = -2$

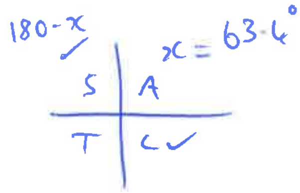
$m = \tan\theta$

$\tan\theta = -2$

$\theta = 180 - 63.4$

$= \underline{\underline{116.57°}}$





2 $y = x^4 - 2x^3 + 5$ when $x = 2$, $y = 2^4 - 2(2)^3 + 5$

$= 16 - 16 + 5$

$= 5$

$(\underset{a}{2}, \underset{b}{5})$

$\dfrac{dy}{dx} = 4x^3 - 6x^2$

when $x = 2$, $\dfrac{dy}{dx} = 4(2)^3 - 6(2)^2$

$= 32 - 24$

$m = 8$

$y - b = m(x - a)$

$y - 5 = 8(x - 2)$

$y - 5 = 8x - 16$

$\underline{\underline{y = 8x - 11}}$

3 $f(x) = x(x-1) + q \quad , \quad g(x) = x + 3$

a) $f(g(x))$

$= f(x+3)$

$= (x+3)(x+3-1) + q$

$= (x+3)(x+2) + q$

$= \underline{\underline{x^2 + 5x + 6 + q}}$

b) $x^2 + 5x + 6 + q = 0$

$b^2 - 4ac = 0$ for equal roots

$5^2 - 4(1)(6+q) = 0$

$25 - 4(6+q) = 0$

$25 - 24 - 4q = 0$

$1 - 4q = 0$

$4q = 1$

$\underline{\underline{q = \frac{1}{4}}}$

4 a) $C(11, 12, 6)$

$D(8, 8, 4)$

b) $\vec{CB} = b - c = \begin{pmatrix} 11 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 11 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$

$|CB| = \sqrt{(-8)^2 + (-4)^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80}$

$\vec{CD} = d - c = \begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 11 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$

$|\vec{CD}| = \sqrt{(-3)^2 + (-4)^2 + (-2)^2}$

$= \sqrt{9 + 16 + 4}$

$= \sqrt{29}$

c) $a \cdot b = |a||b| \cos\theta$

$\cos\theta = \dfrac{a \cdot b}{|a||b|} = \dfrac{\vec{CB} \cdot \vec{CD}}{|\vec{CB}||\vec{CD}|} = \dfrac{(0 \times -3) + (-8 \times -4) + (-4 \times -2)}{\sqrt{80}\,\sqrt{29}}$

$\cos\theta = \dfrac{40}{\sqrt{80}\sqrt{29}} = 0.83$

$\angle BCD = \theta = \underline{\underline{33.9^\circ}}$

5 $\displaystyle\int_4^t (3x+4)^{-1/2}\,dx = 2$

$\Rightarrow \left[\dfrac{(3x+4)^{1/2}}{\frac{1}{2} \times 3} \right]_4^t = 2$

$$\Rightarrow \left[\frac{2}{3}(3x+4)^{1/2}\right]_4^t = 2$$

$$\Rightarrow \frac{2}{3}(3t+4)^{1/2} - \frac{2}{3}(16)^{1/2} = 2$$

$$\Rightarrow \frac{2}{3}(3t+4)^{1/2} - \frac{8}{3} = 2$$

$$2 + \frac{8}{3} = \frac{6}{3} + \frac{8}{3} = \frac{14}{3}$$

$$\Rightarrow \frac{2}{3}(3t+4)^{1/2} = \frac{14}{3}$$

$$\Rightarrow 2(3t+4)^{1/2} = 14$$

$$\Rightarrow (3t+4)^{1/2} = 7$$ square both sides

$$3t+4 = 49$$

$$3t = 45$$

$$\underline{\underline{t = 15}}$$

6 $\sin x - 2\cos 2x = 1$

$$\Rightarrow \sin x - 2(1-2\sin^2 x) - 1 = 0$$ sub for $\cos 2x$

$$\Rightarrow \sin x - 2 + 4\sin^2 x - 1 = 0$$

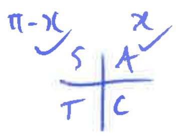
$$\Rightarrow 4\sin^2 x + \sin x - 3 = 0$$

$$\Rightarrow (4\sin x - 3)(\sin x + 1) = 0$$

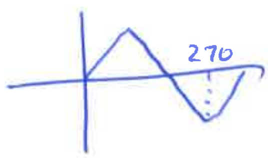
$$\sin x = \frac{3}{4} \quad \text{or} \quad \sin x = -1$$

$x = 0.84$

or 2.29



$x = \frac{3\pi}{2}$



$\underline{\underline{\text{Solutions are } 0.84,\ 2.29 \text{ and } \frac{3\pi}{2} \text{ radians}}}$

7 a) Graphs intersect when $2x = 6x - x^2$

$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \text{ or } x = 4$$

Area is $\int_0^4 \underbrace{(6x - x^2)}_{\text{top}} - \underbrace{(2x)}_{\text{bottom}} \, dx$

$$= \int_0^4 (4x - x^2)\,dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= 2(4)^2 - \frac{4^3}{3} - 0$$

$$= 32 - \frac{64}{3}$$

$$= \underline{\underline{10\tfrac{2}{3} \text{ units}^2}}$$

Area of land $= 10\frac{2}{3} \times 300$

$$= \underline{\underline{3200\,m^2}}$$

b) $y = 6x - x^2$

$$\frac{dy}{dx} = 6 - 2x = 2$$
$$-2x = -4$$
$$x = 2$$

when $x = 2$, $y = 6(2) - 2^2$
$= 12 - 4$
$= 8$

$(\overset{a}{2}, \overset{b}{8})$ is where tangent meets parabola.

$$y - b = m(x - a)$$
$$y - 8 = 2(x - 2)$$
$$y - 8 = 2x - 4$$

$y = 2x + 4$ is equation of road (tangent)

Shaded area is $\int_0^2 (2x + 4) - (6x - x^2)\,dx$

$$= \int_0^2 (x^2 - 4x + 4)\,dx$$

$$= \left[\tfrac{1}{3}x^3 - \frac{4x^2}{2} + 4x\right]_0^2 = \left[\tfrac{1}{3}x^3 - 2x^2 + 4x\right]_0^2$$

$$= \frac{1}{3}(2)^3 - 2(2)^2 + 4(2) - 0$$

$$= \frac{8}{3} - 8 + 8$$

$$= \frac{8}{3} = 2\tfrac{2}{3} \text{ units}^2$$

$\therefore$ area of car park $= 2\frac{2}{3} \times 300 = \underline{\underline{800m^2}}$

8 $x^2 + y^2 - 2px - 4py + 3p + 2 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2p \qquad 2f = -4p \qquad c = 3p + 2$

$g = -p \qquad f = -2p$

For circle to exist $r > 0$

$\Rightarrow \sqrt{g^2 + f^2 - c} > 0$

$= \sqrt{(-p)^2 + (-2p)^2 - 3p - 2} > 0$

$\Rightarrow p^2 + 4p^2 - 3p - 2 > 0$

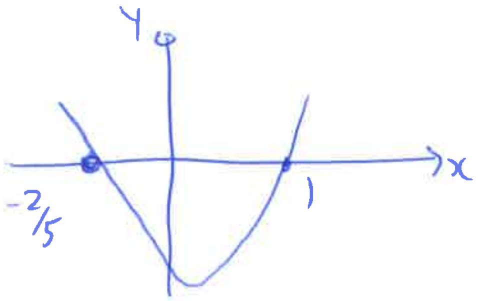
$\Rightarrow 5p^2 - 3p - 2 > 0$ quadratic Inequation can only be solved by sketching a graph.

Sketch $y = 5x^2 - 3x - 2$

$5x^2 - 3x - 2 = 0$

$(5x + 2)(x - 1) = 0$

$x = -\frac{2}{5}$ or $x = 1$



For $5p^2 - 3p - 2 > 0$

$\underline{\underline{p < -\frac{2}{5} \text{ or } p > 1}}$

9 a) $V(t) = 8\cos\left(2t - \frac{\pi}{2}\right)$

$a(t) = V'(t) = -8\sin\left(2t - \frac{\pi}{2}\right) \times 2$ Chair Rule

$= \underline{\underline{-16\sin\left(2t - \frac{\pi}{2}\right)}}$

b) $a(t) = -16\sin\left(2t - \frac{\pi}{2}\right)$

when $t = 10$, $a(t) = -16\sin\left(20 - \frac{\pi}{2}\right)$

$= -16\sin(18.429)$

$= 6.52$

$\underline{\underline{a(t) > 0 \text{ so velocity is increasing at } t = 10}}$

c) $s(t) = \int v(t) = \int 8\cos\left(2t - \frac{\pi}{2}\right)$

$s(t) = 8\left[\frac{1}{2}\sin\left(2t - \frac{\pi}{2}\right)\right] + c$

$s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + c$

$4 = 4\sin\left(0 - \frac{\pi}{2}\right) + c$

$4 = -4 + c$

$c = 8$

so $\underline{\underline{s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8}}$