Higher 2014 Paper 1 Solutions (Q1-20 were multiple choice)

(1) $u_{n+1} = \frac{1}{3}u_n + 1$

$u_2 = 15$

$u_3 = \frac{1}{3}(15) + 1 = 6$

$u_4 = \frac{1}{3}(6) + 1 = \underline{3}$

(2) $m_{CT} = \frac{2-(-1)}{1-3} = \frac{3}{-2}$

$\therefore m_{tan} = \frac{2}{3}$

(3) $\log_4 12 - \log_4 x = \log_4 6$

$\Rightarrow \log_4 \frac{12}{x} = \log_4 6$

$\frac{12}{x} = 6$

$x = \frac{12}{6} = 2$

(4) $3\sin x - 4\cos x = k\cos(x-a)$

$= k(\cos x \cos a + \sin x \sin a)$

$= k\cos a \cos x + k \sin a \sin x$

$k\cos a = -4$

$k \sin a = 3$

(5) $\int (2x+9)^5 dx$

$= \frac{(2x+9)^6}{6 \times 2} + C$

$= \frac{1}{12}(2x+9)^6 + C$

(6) $2\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

(7) $\sin 2a = 2\sin a \cos a$

$= 2 \times \frac{3}{\sqrt{34}} \times \frac{5}{\sqrt{34}}$

$= \frac{30}{34} = \frac{15}{17}$

(8) $\frac{1}{2}(4-9x^4)^{-1/2} \times -36x^3$ chain rule

$= -18x^3(4-9x^4)^{-1/2}$

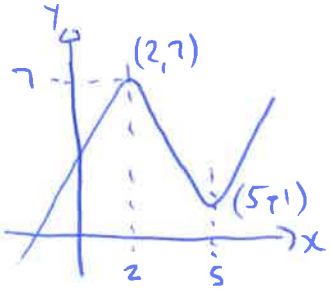
$= -\frac{18x^3}{(4-9x^4)^{1/2}}$

(9) $5 \times 2 = \underline{10}$

(10) $-1 < (k-2) < 1$

$\Rightarrow 1 < k < 3$

(11)



(12) $x^2+6x-16 \neq 0$

$(x+8)(x-2) \neq 0$

$x+8 \neq 0$ or $x-2 \neq 0$

$x \neq -8$ or $x \neq 2$

(13) $\sin\frac{\pi}{3} - \cos\frac{5\pi}{4}$

$= \sin 60 - \cos 225$

$= \frac{\sqrt{3}}{2} - \left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$

$\cos 225$

$= -\cos 45^\circ$

(14) $u.v = 0$ for perp vectors

$\Rightarrow -6+2k+5k = 0$

$7k = 6$

$k = \frac{6}{7}$

(15) $y = k(x+1)(x-2)(x-2)$

$(0,-8)$ $\quad -8 = k(1)(-2)(-2)$

$x \quad y$

$4k = -8$

$k = -2$

$\therefore y = -2(x+1)(x-2)(x-2)$

(16) $a.(a+2b)$

$= a.a + a.2b$

$= a.a + 2a.b$

$= 1 + 2\left(\frac{2}{3}\right)$

$= 1 + \frac{4}{3}$

$= 1\frac{4}{3} = \frac{7}{3} = 2\frac{1}{3}$

(17) $3x^2+12x+17$

$= 3(x^2+4x)+17$

$= 3[(x^2+4x+4)-4]+17$

$= 3[(x+2)^2-4]+17$

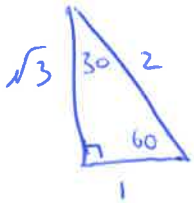
$= 3(x+2)^2+5$

(18) $1-2\sin^2 15^\circ$

$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

$\cos 2A = 1-2\sin^2 A$



(19) $\overrightarrow{SW} = -u-2v$

(20) $2 - \log_5 \frac{1}{25}$

$= 2 - \log_5 5^{-2}$

$= 2 + 2\log_5 5$

$= 2+2$

$= 4$

(21) a) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2 = 0$ for S.P

$\Rightarrow 3x(2-x) = 0$

$3x = 0$ or $2 - x = 0$

$x = 0$ or $x = 2$

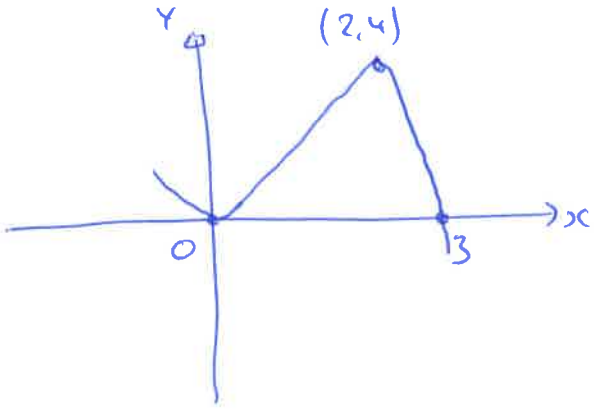
x	$-1 \rightarrow$	0	$\rightarrow 1 \rightarrow$	2	$\rightarrow 3 \rightarrow$
$\frac{dy}{dx}$	$-$	0	$+$	0	$-$
Slope	\	—	/	—	\
		Min T.P at $(0,0)$		Max T.P at $(2,4)$	

When $x = 0$, $y = 0$ $(0,0)$

When $x = 2$, $y = 3(2)^2 - 2^3$

$= 12 - 8$

$= 4$ $(2,4)$



b) When $x = 0$, $y = 0$ $(0,0)$

When $y = 0$, $3x^2 - x^3 = 0$

$x^2(3-x) = 0$

$x^2 = 0$ or $3 - x = 0$

$x = 0$ or $x = 3$

$(0,0)$ $(3,0)$

(22)

-1	6	7	a	b
		-6	-1	$-a+1$
	6	1	$a-1$	$-a+b+1$

$\Rightarrow -a + b + 1 = 0$

$-a + b = -1$ — (1)

2	6	7	a	b
		12	38	$2a+76$
	6	19	$a+38$	$2a+b+76$

$\Rightarrow 2a + b + 76 = 72$

$2a + b = -4$ — (2)

(1) $\times 2$ $-2a + 2b = -2$ — (3)

Add $2a + b = -4$ — (2)

$3b = -6$

$b = -2$

Put $b = -2$ into (2)

$2a - 2 = -4$

$2a = -2$

$a = -1$

22b) $(6x^2+x-2)(x+1)$

$= \underline{(3x+2)(2x-1)(x+1)}$

23 a) $x^2+y^2+2x-4y-15=0$, $y=3x-5$

$x^2+(3x-5)^2+2x-4(3x-5)-15=0$

$x^2+9x^2-30x+25+2x-12x+20-15=0$

$10x^2-40x+30=0$

$10(x^2-4x+3)=0$

$10(x-3)(x-1)=0$

$x=3$ or $x=1$

When $x=3$, $y=3(3)-5=4$ $(3,4)$ P

When $x=1$, $y=3(1)-5=-2$ $\underline{(1,-2)\ Q}$

b) Comparing with $x^2+y^2+2gx+2fy+c=0$

$2g=2$ $2f=-4$

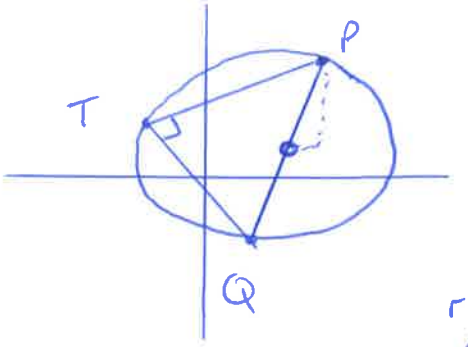
$g=1$ $f=-2$

centre $(-g,-f)=\underline{(-1,2)T}$

$M_{PT}=\frac{4-2}{3-(-1)}=\frac{2}{4}=\frac{1}{2}$ $M_{QT}=\frac{-2-2}{1-(-1)}=\frac{-4}{2}=-2$

$M_{PT}\times M_{QT}=\frac{1}{2}\times -2=-1$ $\therefore$ PT and QT are perpendicular.

c)

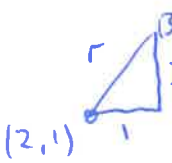


Angle in the $\Delta = 90°$ (from part b)

So PQ is the diameter.

Midpoint of PQ is $\left(\frac{3+1}{2}, \frac{4+(-2)}{2}\right)$

$=(2,1)$



$r=\sqrt{3^2+1^2}$

$r=\sqrt{10}$

equation is $\underline{(x-2)^2+(y-1)^2=10}$

(24) Two ways to do this question

$m = \frac{5-2}{6-0} = \frac{3}{6} = \frac{1}{2}$

$c = 2$

$\log_a \boxed{3} = \frac{1}{2}$

$\log_a \boxed{81} = 2$

$y = mx + c$

$y = \frac{1}{2}x + 2$

$\log_a y = x\left(\frac{1}{2}\right) + 2$

$\log_a y = x \log_a 3 + \log_a 81$

$\log_a y = \log_a 3^x + \log_a 81$

$\log_a y = \log_a 81(3^x)$

$y = 81(3^x)$

so $k = 81$, $a = 3$

or $y = ka^x$

$\log_a y = \log_a ka^x$

$\log_a y = \log_a k + \log_a a^x$

$\log_a y = x \log_a a + \log_a k$

$Y = mx + c$

$m = \log_a a$

$\log_a a = \frac{1}{2}$

$a = a^{1/2} = 3$

$\log_a k = 2$

$k = a^2$

$k = 81$