Higher 2013 Paper 2 Solutions

(1)

$4, \; 7, \; 16$

$u_1 \quad u_2 \quad u_3$

$u_{n+1} = m u_n + c$

$u_2 = m u_1 + c \Rightarrow 7 = 4m + c \quad — (1)$

$u_3 = m u_2 + c \Rightarrow 16 = 7m + c \quad — (2)$

Subtract $\quad -9 = -3m$

$\underline{\underline{m = 3}}$

Put $m = 3$ into (1)

$4m + c = 7$

$12 + c = 7$

$\quad -12 \qquad -12$

$\underline{\underline{c = -5}}$

(2) $m_{PQ} = \dfrac{6-2}{5-7} = \dfrac{4}{-2} = -2$

$\therefore \; m_{QR} = \dfrac{1}{2} \qquad Q(\overset{a}{5}, \overset{b}{6})$

$y - 6 = \frac{1}{2}(x - 5)$

$2y - 12 = 1(x - 5)$

$2y = x + 7$

$\underline{\underline{y = \frac{1}{2}x + \frac{7}{2}}}$

b) $x + 3y = 13 \quad — (1)$

$-x + 2y = 7 \quad — (2)$

Add $\quad 5y = 20$

$y = 4$

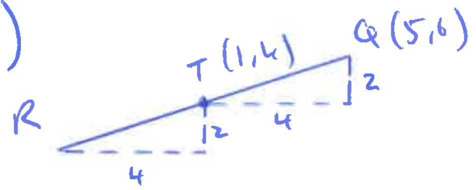
Put $y = 4$ into (1)

$x + 12 = 13$

$x = 1$

$\underline{\underline{T(1, 4)}}$

c)



$R(-3, 2)$

$\underline{\underline{S(-1, -2)}}$ S is connected to R the same as P is connected to Q

ie 2 along, 4 down.

3 a)

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 1 & -5 \\ & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & \boxed{0} \end{array}$$

$(x-1)(x^2+4x+5)$

$= (x-1)(x^2+4x+5)$

$b^2-4ac = 4^2-4(1)(5)$
$= 16-20$
$= -4$ (no roots so quadratic doesn't factorise)

b) $y = x^4+4x^3+2x^2-20x+3$

$\frac{dy}{dx} = 4x^3+12x^2+4x-20 = 0$ for stationary points

$4(x^3+3x^2+x-5) = 0$

$4(x-1)(x^2+4x+5) = 0$ (from part 'a')

$x-1=0$

$x=1$ is only solution.

x	$0 \longrightarrow$	1	$\longrightarrow 2$
$\frac{dy}{dx} = (x-1)(x^2+4x+5)$	$-$	0	$+$
Slope	$\backslash$	$\_$	$/$

Minimum Turning Point when $x=1$

4 $x^3+3x^2+2x+3 = 2x+3$

$x^3+3x^2 = 0$

$x^2(x+3) = 0$

$x^2=0$ or $x+3=0$

$x=0$ or $x=-3$

when $x=-3$, $y = 2(-3)+3$
$= -6+3$
$= -3$

so B is $(-3,-3)$

3

Area is $\int_{-3}^{0} [\overset{\text{top}}{x^3 + 3x^2 + 2x + 3} - \overset{\text{bottom}}{(2x+3)}]\,dx$

$= \int_{-3}^{0} (x^3 + 3x^2)\,dx = \left[\frac{1}{4}x^4 + x^3\right]_{-3}^{0}$

$= 0 - \left(\frac{1}{4}(-3)^4 + (-3)^3\right)$

$= -\left(\frac{81}{4} - 27\right)$

$= -\left(\frac{81}{4} - \frac{108}{4}\right) = -\left(-\frac{27}{4}\right) = 6\frac{3}{4}$ units^2

5 $\log_5(3-2x) + \log_5(2+x) = 1$

$\log_5(3-2x)(2+x) = \log_5 5$ $\qquad 1 = \log_5 5$

$(3-2x)(2+x) = 5$

$6 - x - 2x^2 - 5 = 0$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

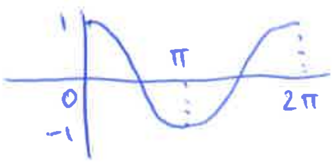
$x = \frac{1}{2}$ or $x = -1$

6 $\int_0^a 5\sin 3x\,dx = \frac{10}{3}$

$\Rightarrow \left[-\frac{5}{3}\cos 3x\right]_0^a = \frac{10}{3} \Rightarrow -\frac{5}{3}\cos 3a - \left(-\frac{5}{3}\cos(3\times 0)\right) = \frac{10}{3}$

$\Rightarrow -\frac{5}{3}\cos 3a + \frac{5}{3} = \frac{10}{3}$

$\Rightarrow -\frac{5}{3}\cos 3a = \frac{5}{3}$



$\cos 3a = -1$

$3a = \pi$

$a = \frac{\pi}{3}$

Period $= \frac{2\pi}{3}$.

7 a) $L = 3x + 4y$

$L = 3x + 4\left(\frac{12}{x}\right)$

$L = 3x + \frac{48}{x}$ as required

$A = 2xy$

$24 = 2xy$

$2xy = 24$

$y = \frac{24}{2x} = \frac{12}{x}$

b) i) $L = 3x + 48x^{-1}$

$\cancel{\frac{dy}{dx}} = 3 - 48x^{-2} = 0$ for stationary points

$3 - \frac{48}{x^2} = 0$

$3x^2 - 48 = 0$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

x	$3 \rightarrow$	4	$\rightarrow 5$
$L'(x) = 3x^2 - 48$	$-$	0	$+$
Slope	$\backslash$	$\_\_$	$/$

$x = 4$ gives minimum length of rods.

ii) $L = 3(4) + \frac{48}{4}$

$= 12 + 12$

$= 24$ metres

cost $= 24 \times 8.25 = $ £198

8 $\sin 2x = 2\cos^2 x$

$2\sin x \cos x - 2\cos^2 x = 0$

$2\cos x(\sin x - \cos x) = 0$

$2\cos x = 0$ or $\sin x - \cos x = 0$

$\cos x = 0$ or $\sin x = \cos x$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \pi/4$

S	A ✓
T ✓	C

$\pi + x$

$x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

Solutions are $\underline{\underline{\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4} \text{ and } \frac{3\pi}{2}.}}$

9 a) $P_t = P_0 e^{-kt}$

Let $P_0 = 100$ so $P_t = 50$ (make up values for P_0 and P_t)

So $50 = 100 e^{-k(25)}$

$100 e^{-25k} = 50$

$e^{-25k} = 0.5$

$\log_e e^{-25k} = \log_e 0.5$ take logs of both sides

$-25k \log_e e = \log_e 0.5$

$-25k = \log_e 0.5$ $(\log_e = \ln)$

$k = \dfrac{\log_e 0.5}{-25} = \underline{\underline{0.028}}$

b) $P_t = 100 e^{-0.028t}$

$t = 80 \Rightarrow P_t = 100 e^{-0.028(80)}$

$= 10.88$

After 80 days, concentration decreases from 100 to 10.88

So percentage decrease $= \dfrac{89.12}{100} \times 100\% = \underline{\underline{89.12\%}}$