Higher 2013 Paper 1 Solutions

(Questions 1-20 were multiple choice)

1 $g(f(x)) = g(x^2+1) = 3(x^2+1) - 4 = \underline{\underline{3x^2 - 1}}$

2 $y = x^2 - 4x + 7$

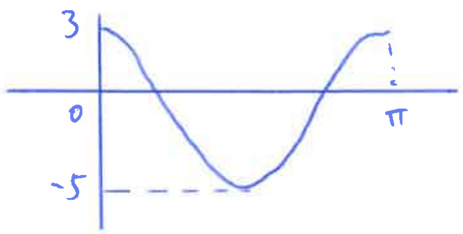
$\frac{dy}{dx} = 2x - 4$

when $x = 5$, $\frac{dy}{dx} = 2(5) - 4$
$= 10 - 4$
$= \underline{\underline{6}}$

3 $2x^2 + 4x + 5 = 0$

$b^2 - 4ac$
$= (4)^2 - 4(2)(5)$
$= 16 - 40$
$= \underline{\underline{-24}}$

4 $y = 4\cos 2x - 1$



5 $5x + 3y - 6 = 0$

$3y = -5x + 6$

$y = -\frac{5}{3}x + 2$

$m = -\frac{5}{3}$ $\quad (\overset{a}{-2}, \overset{b}{-1})$

$y - b = m(x - a)$

$y + 1 = -\frac{5}{3}(x + 2)$

$3y + 3 = -5(x + 2)$

$3y + 3 = -5x - 10$

$\underline{\underline{5x + 3y + 13 = 0}}$ or $y = -\frac{5}{3}x - \frac{13}{3}$

6

$$\begin{array}{c|cccc} 2 & 1 & 3 & -5 & -6 \\ & & 2 & 10 & 10 \\ \hline & 1 & 5 & 5 & \boxed{4} \end{array}$$

remainder $= \underline{\underline{4}}$

7 $\int (3x^2 + 2x)\,dx$

$= \left[\frac{3x^3}{3} + \frac{2x^2}{2} + c \right]$

$= \underline{\underline{x^3 + x^2 + c}}$

8 $u_{n+1} = 0 \cdot 1 u_n + 8$

$u_1 = 11$

$u_1 = 0 \cdot 1 u_0 + 8$

$11 = 0.1 u_0 + 8$

$0 \cdot 1 u_0 = 3$

$u_0 = 30$

Statement 1 is false
Statement 2 is true.

9 $\sin 2x = 2\sin x\cos x$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$

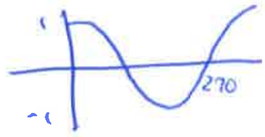
$= \underline{\underline{\frac{4}{5}}}$

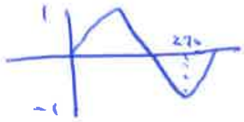
10 $\cos(270-a)^\circ$

$= \cos 270 \cos a + \sin 270 \sin a$

$= 0 + (-1)\sin a$

$= \underline{\underline{-\sin a^\circ}}$





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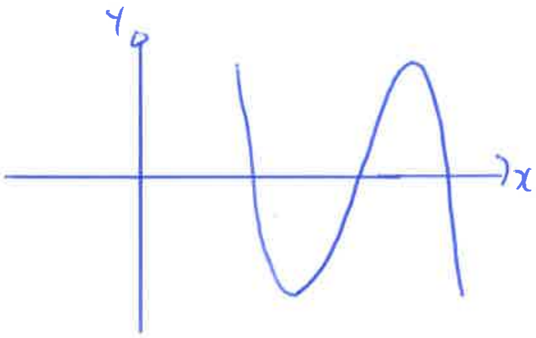


$-f(x)$ reflects in x axis

$f(x-k)$ moves graph to the right by k units

12 $f+g = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 5 \end{pmatrix}$

$|f+g| = \sqrt{5^2+4^2+5^2} = \underline{\underline{\sqrt{66}}}$

13 $x^2 - 7x + 12 \neq 0$

$(x-4)(x-3) \neq 0$

$\underline{\underline{x \neq 4 \text{ or } x \neq 3}}$

14 $a.(a+b)$

$= a.a + a.b$

$= 9 + 5$

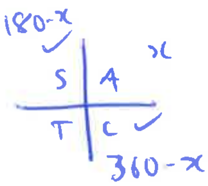
$= \underline{\underline{14}}$

$a.a = |a||a|\cos 0^\circ$

$= 3 \times 3 \times 1$

$= 9$

15 $\tan\left(\frac{x}{2}\right) = -1$



$\frac{x}{2} = 135^\circ$ or 315°

$x = 270^\circ$ or 630°

For $0 \le x \le 2\pi$, $x = 270^\circ = \frac{270}{180}\pi = \underline{\underline{\frac{3\pi}{2}}}$

16

$$\int (1-6x)^{-1/2}\,dx$$

$$= \frac{(1-6x)^{1/2}}{\frac{1}{2}x-6} + c$$

$$= -\frac{1}{3}(1-6x)^{1/2} + c$$

17

$$y = kx(x+a)^2$$

$$y = k(x-0)(x+2)^2$$

$\overset{x\ y}{(1,3)}$ $\quad 3 = k(1)(3)^2$

$$3 = 9k$$

$$k = \frac{3}{9} = \frac{1}{3}, \quad a = 2$$

18

$$y = \sin(x^2-3)$$

$$\frac{dy}{dx} = \cos(x^2-3) \times 2x$$

$$= 2x\cos(x^2-3)$$

19

$$1-2x-3x^2 > 0$$

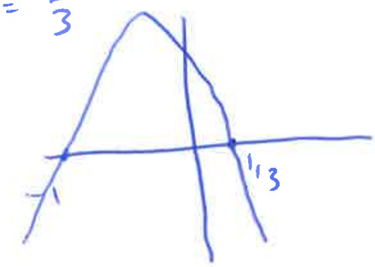
Sketch $y = 1-2x-3x^2$

$$\therefore 1-2x-3x^2 = 0$$

$$(1-3x)(1+x) = 0$$

$3x = 1$ or $x = -1$

$$x = \frac{1}{3}$$



So for $1-2x-3x^2 > 0$

$-1 < x < \frac{1}{3}$ (above the x axis.)

20

$$y = mx + c$$

$$\log_3 y = 2x$$

$$\log_3 y = x\log_3 9$$

$$\log_3 y = \log_3 9^x$$

$$y = 9^x$$

21

$$2x^2 + 12x + 1$$

$$= 2(x^2+6x) + 1$$

$$= 2[(x^2+6x+9) - 9] + 1$$

$$= 2[(x+3)^2 - 9] + 1$$

$$= 2(x+3)^2 - 17$$

22 a) $x^2+y^2+2x+4y-27=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=2 \qquad 2f=4 \qquad c=-27$

$g=1 \qquad f=2$

centre $(-g,-f)=\underline{\underline{(-1,-2)}}$

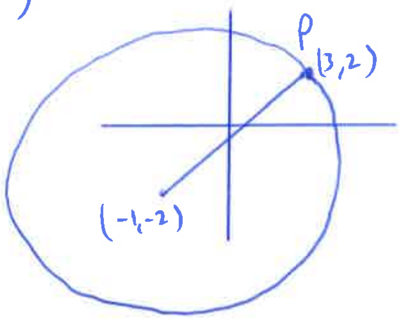
$r=\sqrt{g^2+f^2-c}$

$r=\sqrt{1+4+27}$

$=\sqrt{32}=\sqrt{16\times 2}$

$=\underline{\underline{4\sqrt{2}}}$

b)



$M_{radius}=\dfrac{2--2}{3--1}=\dfrac{4}{4}=1$

$\therefore M_{\perp}=-1 \qquad P(\overset{a}{3},\overset{b}{2})$

$y-b=m(x-a)$

$y-2=-1(x-3)$

$y-2=-x+3$

$\underline{\underline{y=-x+5}}$

c) $(\overset{a}{10},\overset{b}{-1})$

$r=2\sqrt{2}$ (half of $4\sqrt{2}$)

$(x-a)^2+(y-b)^2=r^2$

$(x-10)^2+(y+1)^2=(2\sqrt{2})^2$

$x^2-20x+100+y^2+2y+1=8$

$\underline{\underline{x^2+y^2-20x+2y+93=0}}$

d) $y=-x+5$

$x^2+y^2-20x+2y+93=0$

$x^2+(-x+5)^2-20x+2(-x+5)+93=0$

$x^2+x^2-10x+25-20x-2x+10+93=0$

$2x^2-32x+128=0$

$2(x^2-16x+64)=0$

$(x-8)(x-8)=0$

$x=8$

$y=-x+5$

$=-8+5$

$=-3$

$(8,-3)$ only one point of contact means line is a tangent.

⑤

23a) $\sqrt{3}\sin x^\circ - \cos x^\circ = k\sin(x-a)^\circ$

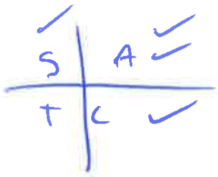
$$= k(\sin x\cos a - \cos x\sin a)$$

$$= k\cos a\sin x - k\sin a\cos x$$

$k\cos a^\circ = \sqrt{3}$

$k\sin a^\circ = 1$

$\tan a^\circ = \dfrac{k\sin a^\circ}{k\cos a^\circ} = \dfrac{1}{\sqrt{3}}$

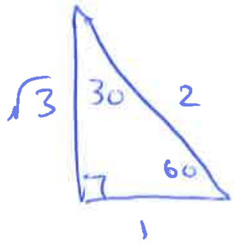


$k = \sqrt{(\sqrt{3})^2 + 1^2}$

$k = \sqrt{4}$

$\underline{\underline{k = 2}}$

$\underline{\underline{a = 30^\circ}}$



b) $4 + 5\cos x^\circ - 5\sqrt{3}\sin x^\circ$

$= 4 + [5(\cos x - \sqrt{3}\sin x)]$

$= 4 + [-5(-\cos x^\circ + \sqrt{3}\sin x^\circ)]$

$= 4 - 5(\sqrt{3}\sin x^\circ - \cos x^\circ)$

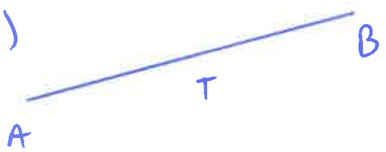
$= 4 - 5(2\sin(x-30)^\circ)$

$= 4 - 10\sin(x-30)^\circ$

max value $= 4 - -10 = \underline{\underline{14}}$

(min value $= 4 - 10 = -6$)

24) a) i)

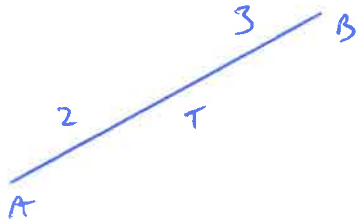


$AT = t - a = \begin{pmatrix}3\\2\\5\end{pmatrix} - \begin{pmatrix}-7\\-8\\1\end{pmatrix} = \begin{pmatrix}10\\10\\4\end{pmatrix}$

$\overrightarrow{TB} = b - t = \begin{pmatrix}18\\17\\11\end{pmatrix} - \begin{pmatrix}3\\2\\5\end{pmatrix} = \begin{pmatrix}15\\15\\6\end{pmatrix}$

$\therefore \overrightarrow{AT} = \frac{2}{3}\overrightarrow{TB}$ so $\overrightarrow{AT}$ is parallel to $\overrightarrow{TB}$. T is a common point so A, T and B are collinear.

24aii) $\overrightarrow{AT} = \frac{2}{3}\overrightarrow{TB}$



$\Rightarrow \frac{AT}{TB} = \frac{2}{3}$

So T divides AB in the ratio $\underline{\underline{2:3}}$

b) $\overrightarrow{TB} = \begin{pmatrix} 15 \\ 15 \\ 6 \end{pmatrix}$ $\quad \overrightarrow{TC} = c - t = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} x-3 \\ -2 \\ -5 \end{pmatrix}$

$TB \cdot TC = 0$ for perpendicular vectors

$15(x-3) + 15(-2) + 6(-5) = 0$

$15x - 45 - 30 - 30 = 0$

$15x - 105 = 0$

$15x = 105$

$x = 7$

So c is $\underline{\underline{(7, 0, 0)}}$