Higher 2012 Paper 1 solutions.

1. $u_{n+1} = 3u_n + 4$

$u_0 = 1$

$u_1 = 3(1) + 4 = 7$

$u_2 = 3(7) + 4 = \underline{\underline{25}}$

2. $y = x^3 - 6x + 1$

$\frac{dy}{dx} = 3x^2 - 6$

when $x = -2$, $\frac{dy}{dx} = 3(-2)^2 - 6 = \underline{\underline{6}}$

3. $x^2 - 6x + 14$

$= (x^2 - 6x + 9) - 9 + 14$

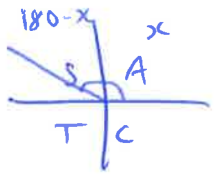
$= (x-3)^2 + 5$

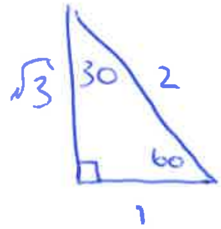
so $\underline{\underline{q = 5}}$

4. $\tan 150°$

$= -\tan 30°$

$= \underline{\underline{-\frac{1}{\sqrt{3}}}}$





5. $\cos 2a = \cos^2 a - \sin^2 a$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \underline{\underline{\frac{7}{25}}}$

6. $y = 3x^{-2} + 2x^{3/2}$

$\frac{dy}{dx} = -6x^{-3} + 3x^{1/2}$

$= \underline{\underline{-\frac{6}{x^3} + 3x^{1/2}}}$

7. $u.v = 0$ for perpendicular vectors

$\Rightarrow (-3 \times 1) + (1 \times t) + (2t \times -1) = 0$

$\Rightarrow -3 + t - 2t = 0$

$\Rightarrow -3 - t = 0$
$\quad +t \qquad +t$

$\underline{\underline{t = -3}}$

8. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$r = 2 \Rightarrow \frac{dV}{dr} = 4\pi(2)^2$

$= \underline{\underline{16\pi}}$

9 $y = \cos\left(x - \frac{\pi}{6}\right) - 1$

10 $\vec{RP} = -g - f + h$

11 $\int \left(\frac{1}{6x^2}\right) dx = \frac{1}{6}\int x^{-2}\,dx$

$= \frac{1}{6}\,\frac{x^{-1}}{-1} + c$

$= -\frac{1}{6}x^{-1} + c$

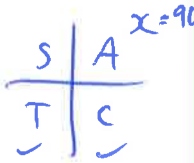
$= -\frac{1}{6x} + c$

12 $2 - 3\sin\left(x - \frac{\pi}{3}\right)$

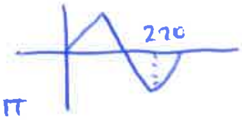
max value $= 5$

when $\sin\left(x - \frac{\pi}{3}\right) = -1$

$x - \frac{\pi}{3} = \frac{3\pi}{2}$



$x = \frac{3\pi}{2} + \frac{\pi}{3}$



$x = \frac{9\pi}{6} + \frac{2\pi}{6} = \frac{11\pi}{6}$

13 $y = k(x+1)(x+2)$

$(0,6)$ (x, y) $\quad 6 = k(1)(2)$

$2k = 6$

$k = 3$

So $y = 3(x+1)(x+2)$

14 $\int (2x-1)^{1/2}\,dx$

$= \frac{(2x-1)^{3/2}}{3/2 \times 2} + c = \frac{(2x-1)^{3/2}}{3} + c$

15 $\sqrt{3^2 + (-1)^2 + 0^2}$

$= \sqrt{10}$

$|u| = 1$ so $k = \frac{1}{\sqrt{10}}$

$\left(\frac{1}{\sqrt{10}} \times \sqrt{10}\right)$

16 $y = 3(\cos x)^4$

$\frac{dy}{dx} = 12(\cos x)^3 \times -\sin x$

$= -12\sin x\cos^3 x$

17 $a.(a+b) = 7$

$\Rightarrow a.a + a.b = 7$

$\Rightarrow 25 + a.b = 7$

$a.b = -18$

$a = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \quad |a| = \sqrt{3^2 + 4^2 + 0^2}$

$= 5$

$a.a = |a||a|\cos 0^\circ = 25$

18 Statement 1 is true

Statement 2 is false

So answer is $\underline{\underline{B}}$

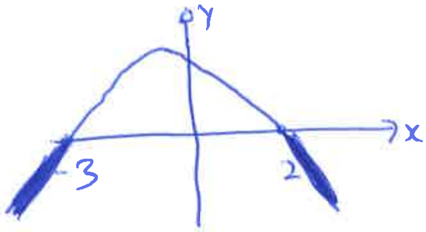
19a $6 - x - x^2 < 0$

Let $y = -x^2 - x + 6$

$y = 0 \Rightarrow -x^2 - x + 6 = 0$

$(-x+2)(x+3) = 0$

$x = 2$ or $x = -3$



So $6 - x - x^2 < 0$ for $\underline{\underline{x < -3 \text{ and } x > 2}}$

20
$$\frac{\log_b 9a^2}{\log_b 3a}$$

$$= \frac{\log_b (3a)^2}{\log_b 3a}$$

$$= \frac{2\log_b 3a}{\log_b 3a}$$

$$= \underline{\underline{2}}$$

Section B

21 ai)

4	1	-5	2	8
		4	-4	-8
	1	-1	-2	0

no remainder means $(x-4)$ is a factor.

ii) $(x-4)(x^2 - x - 2)$

$= \underline{\underline{(x-4)(x-2)(x+1)}}$

iii) $(x-4)(x-2)(x+1) = 0$

$\underline{\underline{x = 4 \text{ or } x = 2 \text{ or } x = -1}}$

b) $Q(2,0)$ $R(4,0)$ $P(-1,0)$

So area is $\int_0^2 (x^3 - 5x^2 + 2x + 8)\,dx$

$$= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + x^2 + 8x\right]_0^2$$

$$= 4 - \frac{40}{3} + 4 + 16 - 0$$

$$= 24 - \frac{40}{3} = \frac{72}{3} - \frac{40}{3} = \frac{32}{3} = \underline{\underline{10\tfrac{2}{3} \text{ units}^2}}$$

22 a) Let $\cos x - \sqrt{3}\sin x = k\cos(x+a)$

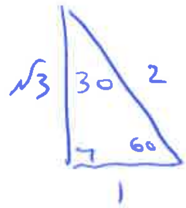
$$= k(\cos x \cos a - \sin x \sin a)$$

$$= k\cos a \cos x - k\sin a \sin x$$

$k\cos a = 1$

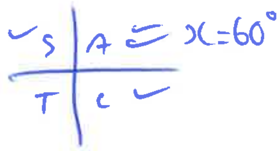
$k\sin a = \sqrt{3}$

$$\tan a = \frac{\sin a}{\cos a} = \frac{\sqrt{3}}{1}$$



$$k = \sqrt{1^2 + (\sqrt{3})^2} = \underline{\underline{2}}$$

$$\underline{\underline{a = \frac{\pi}{3}}}$$

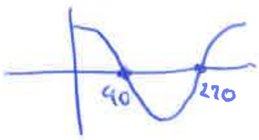


b) $y = 2\cos\left(x + \frac{\pi}{3}\right)$

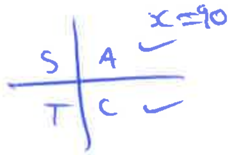
cuts y axis when $x = 0 \Rightarrow y = 2\cos\frac{\pi}{3} = 1$ $\underline{\underline{(0, 1)}}$

cuts x axis when $y = 0 \Rightarrow 2\cos\left(x + \frac{\pi}{3}\right) = 0$

$$\cos\left(x + \frac{\pi}{3}\right) = 0$$



$$x + \frac{\pi}{3} = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$



$$x = \frac{\pi}{2} - \frac{\pi}{3} \text{ or } \frac{3\pi}{2} - \frac{\pi}{3}$$

$$x = \frac{3\pi}{6} - \frac{2\pi}{6} \text{ or } \frac{9\pi}{6} - \frac{2\pi}{6}$$

$$\underline{\underline{x = \frac{\pi}{6} \text{ or } \frac{7\pi}{6}}}$$

23 a) $P(3,-3)$ $Q(-1,9)$

midpoint of PQ $\left(\frac{3+(-1)}{2}, \frac{-3+9}{2}\right) = (\overset{a}{1}, \overset{b}{3})$

$$m_{PQ} = \frac{9--3}{-1-3} = \frac{12}{-4} = -3$$

$\therefore m_{\perp} = \frac{1}{3}$ so

$$y-b = m(x-a)$$
$$y-3 = \tfrac{1}{3}(x-1)$$
$$3y-9 = x-1$$
$$3y = x+8$$
$$\underline{\underline{y = \tfrac{1}{3}x + \tfrac{8}{3}}}$$

b) $R(1,-2)$ $m=-3$

$$y+2 = -3(x-1)$$
$$y+2 = -3x+3$$
$$\underline{\underline{y = -3x+1}}$$

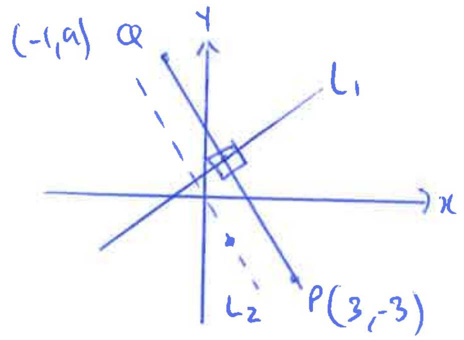
c) $\frac{1}{3}x + \frac{8}{3} = -3x+1$ $(\times 3)$

$$x+8 = -9x+3$$
$$10x = -5$$
$$x = \frac{-5}{10} = -\frac{1}{2}$$

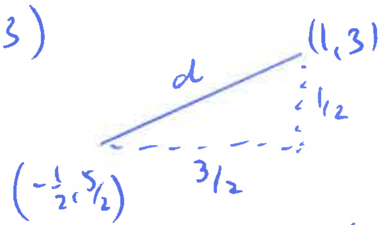
$$y = -3\left(-\tfrac{1}{2}\right)+1 = \frac{5}{2}$$

$$\underline{\underline{\left(-\tfrac{1}{2}, \tfrac{5}{2}\right)}}$$

d)



Shortest distance between PQ and l_2 is the distance between $\left(-\frac{1}{2}, \frac{5}{2}\right)$ and $(1,3)$



$$d = \sqrt{\left(\tfrac{3}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2} = \sqrt{\tfrac{9}{4} + \tfrac{1}{4}}$$

$$d = \sqrt{\tfrac{10}{4}} = \underline{\underline{\sqrt{\tfrac{5}{2}}}}$$

(or use distance formula)