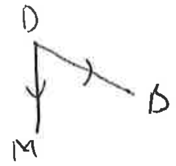
2011 Higher Paper 2 – Answers

1\. a) $B(4,4,0)$ $\qquad$ $M(2,0,0)$

b) $\overrightarrow{DB} = b - d = \begin{pmatrix}4\\4\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix} = \begin{pmatrix}2\\2\\-6\end{pmatrix}$ $\qquad$ $|\overrightarrow{DB}| = \sqrt{4+4+36} = \sqrt{44}$

$\overrightarrow{DM} = m - d = \begin{pmatrix}\cancel{0}\,2\\0\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix} = \begin{pmatrix}0\\-2\\-6\end{pmatrix}$ $\qquad$ $|\overrightarrow{DM}| = \sqrt{0+4+36} = \sqrt{40}$

c) 

$$\cos\theta = \frac{\overrightarrow{DB}.\overrightarrow{DM}}{|DB||DM|} = \frac{0-4+36}{\sqrt{44}\sqrt{40}} = \frac{32}{\sqrt{1760}}$$

$$\cos\theta = 0{\cdot}762$$

$$\theta = \underline{\underline{40{\cdot}3^\circ}}$$

2\. a) $g(f(x))$

$= g(x^3-1)$

$= 3(x^3-1)+1$

$= \underline{\underline{3x^3-2}}$

b) $g(f(x)) + x\,h(x)$

$= 3x^3-2 + x(4x-5)$

$= 3x^3-2 + 4x^2-5x$

$= \underline{\underline{3x^3+4x^2-5x-2}}$

c i)

$$\begin{array}{c|cccc} 1 & 3 & 4 & -5 & -2 \\ & & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & \boxed{0} \end{array}$$

no remainder means $x-1$ is a factor

ii) $(x-1)(3x^2+7x+2)$

$= \underline{\underline{(x-1)(3x+1)(x+2)}}$

d) $(x-1)(3x+1)(x+2)=0$

$x-1=0$ or $3x+1=0$ or $x+2=0$

$\underline{\underline{x=1}}$ or $\underline{\underline{x=-\frac{1}{3}}}$ or $\underline{\underline{x=-2}}$.

3 a) $u_{n+1} = -\frac{1}{2}u_n$

$u_0 = -16$

$u_1 = -\frac{1}{2}(-16) = 8$

$u_2 = -\frac{1}{2}(8) = -4$

b) $v_{n+1} = pv_n + q$

$v_1 = 4$

$v_2 = p(4) + q = 5 \Rightarrow 4p + q = 5$

$v_3 = p(5) + q = 7 \Rightarrow 5p + q = 7$

$\underline{\underline{p = 2}}$

$\therefore 8 + q = 5$

$\underline{\underline{q = -3}}$

check

$v_4 = 2v_3 - 3$

$= 14 - 3$

$= \underline{\underline{11}}$

c) i sequence in a as

$-1 < -\frac{1}{2} < 1$

$u_{n+1} = -\frac{1}{2}u_n$

$L = -\frac{1}{2}L$

$\frac{3}{2}L = 0$

$L = 0$

ii) $v_{n+1} = 2v_n - 3$

no limit as 2 does not lie between -1 and 1

4. $$\int_{-2}^{0} x^3 - x^2 - 4x + 4 - (2x+4) + \int_{0}^{3} 2x + 4 - (x^3 - x^2 - 4x + 4)$$

$$= \int_{-2}^{0} x^3 - x^2 - 6x + \int_{0}^{3} -x^3 + x^2 + 6x$$

$$= \left[\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - 3x^2 \right]_{-2}^{0} + \left[-\tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 + 3x^2 \right]_{0}^{3}$$

$$= 0 - \left(4 + \frac{8}{3} - 12 \right) + \left(-\frac{81}{4} + 9 + 27 \right)$$

$$= -\left(-8 + \frac{8}{3} \right) + \left(36 - \frac{81}{4} \right)$$

$$= -\left(-\frac{24}{3} + \frac{8}{3} \right) + \left(\frac{144}{4} - \frac{81}{4} \right)$$

$$= \frac{16}{3} + \frac{63}{4}$$

$$= \frac{253}{12} = \underline{\underline{21\tfrac{1}{12} \text{ units}}}$$

5. $$m = \frac{7-5}{4-0} = \frac{2}{4} = \frac{1}{2}$$

$$y = mx + c$$
$$y = \tfrac{1}{2}x + 5$$
$$\log_2 y = \tfrac{1}{2}\log_2 x + 5$$
$$\log_2 y = \log_2 x^{1/2} + \log_2 32$$
$$\log_2 y = \log_2 32x^{1/2}$$
$$\underline{\underline{y = 32x^{1/2}}} \qquad \underline{\underline{k = 32}}, \ \underline{\underline{n = \tfrac{1}{2}}}.$$

$$\log_2 \square = 5$$
$$\square = 2^5 = 32$$

6a) $3\sin x - 5\cos x = R\sin(x + a)$

$= R(\sin x \cos a + \cos x \sin a)$

$= R\cos a \sin x + R\sin a \cos x$

$R\cos a = 3$

$R\sin a = -5$

$\tan a = -\frac{5}{3}$

S | A ✓

✓ T | C ✓

$k = \sqrt{3^2 + (-5)^2}$

$= \underline{\underline{\sqrt{34}}}$

$a = \underline{\underline{5.25}}$ radians

(5.3 to 1 d.p.)

$\therefore\ 3\sin x - 5\cos x = \sqrt{34}\sin(x + 5.3)$

b) $\int_0^t (3\cos x + 5\sin x)\,dx = 3$

$\Rightarrow [3\sin x - 5\cos x]_0^t = 3$

$\Rightarrow 3\sin t - 5\cos t - (3\sin 0 - 5\cos 0) = 3$

$\Rightarrow 3\sin t - 5\cos t + 5 = 3$

$\Rightarrow 3\sin t - 5\cos t = -2$

$\Rightarrow \sqrt{34}\sin(t + 5.3) = -2$

$\Rightarrow \sin(t + 5.3) = \frac{-2}{\sqrt{34}}$

$t = 0.35$ (Ignore negative and shift $\sin 2/\sqrt{34}$

S | A

T ✓ | C ✓

$\pi + t$ $2\pi - t$

$t + 5.3 = \pi + 0.35$ or $2\pi - 0.35$

$t + 5.3 = 3.5$ or 5.9

$t = -1.8$ or 0.6

For $0 \le t \le 2$, $\underline{\underline{t = 0.6}}$

7. $(x+1)^2+(y-1)^2=121$

centre $(-1,1)$ radius $=\sqrt{121}=11$

$x^2+y^2-4x+6y+p=0$

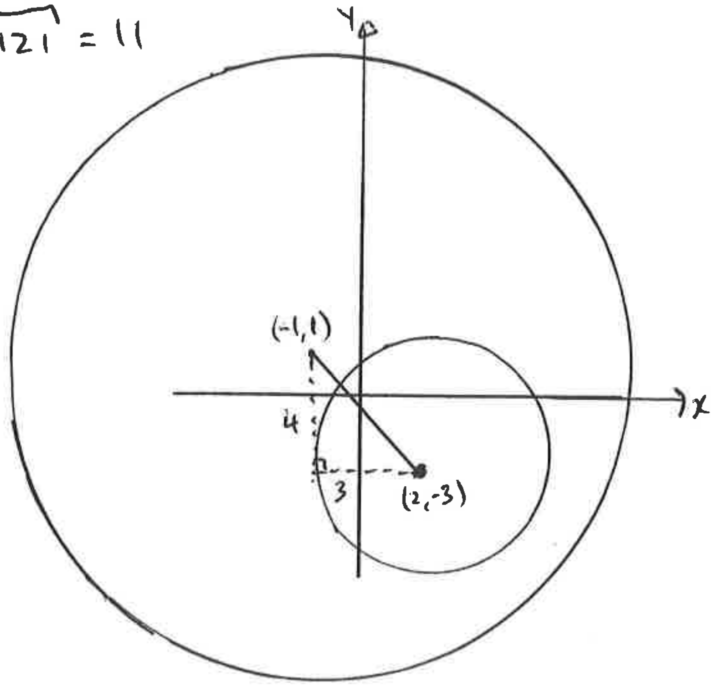
$2g=-4$ $2f=6$

$g=-2$ $f=3$

centre $(-g,-f)=(2,-3)$

Distance between centres:

$\sqrt{4^2+3^2}=5$



The radius of C_2 must be less than 6 $(11-5)$ if it has no points of contact with C_1

radius of C_2: $\sqrt{g^2+f^2-c}$

$=\sqrt{4+9-p}$

$=\sqrt{13-p}$

So $\sqrt{13-p}<6$

$13-p<36$

$-p<23$

Change all signs

$\underline{\underline{p>-23}}$