Higher 2011 Paper 1 solutions

(Q1-20 were originally multiple choice)

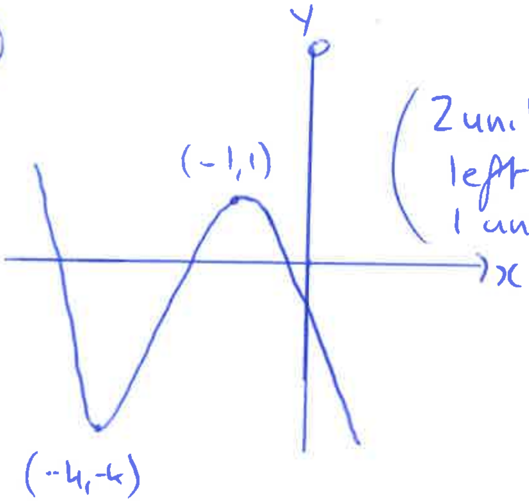
①
$$2p - q - \tfrac{1}{2}r = 2\begin{pmatrix}2\\5\\-7\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \tfrac{1}{2}\begin{pmatrix}-4\\2\\0\end{pmatrix} =$$

$$= \begin{pmatrix}4\\10\\-14\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \begin{pmatrix}-2\\1\\0\end{pmatrix} = \underline{\underline{\begin{pmatrix}5\\9\\-13\end{pmatrix}}}$$

②
$$3y + 2x = 6$$
$$3y = -2x + 6$$
$$y = -\tfrac{2}{3}x + 2$$

parallel $\underline{\underline{m = -\tfrac{2}{3}}}$

③



(2 units to the left then down 1 unit.)

④
$$y = x^3 - 2x$$
$$\frac{dy}{dx} = 3x^2 - 2$$

when $x = 2$, $\frac{dy}{dx} = m = 3(2)^2 - 2 = \underline{\underline{10}}$

⑤
$$x^2 - 8x + 7$$
$$= (x^2 - 8x + 16) - 16 + 7$$
$$= (x-4)^2 - 9$$

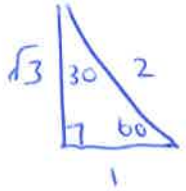
so $\underline{\underline{q = -9}}$

⑥
$$m_{\perp} = \tfrac{1}{2} \quad (\overset{a}{2}, \overset{b}{-3})$$
$$y - b = m(x - a)$$
$$y + 3 = \tfrac{1}{2}(x - 2)$$
$$y + 3 = \tfrac{1}{2}x - 1$$
$$\underline{\underline{y = \tfrac{1}{2}x - 4}}$$

⑦
$$\begin{array}{r|rrrr}
1 & 1 & -1 & 1 & 3 \\
 & & 1 & 0 & 1 \\
\hline
 & 1 & 0 & 1 & \boxed{4}
\end{array}$$

$\underline{\underline{\text{remainder} = 4}}$

⑧
$$m = \tan 30°$$
$$\underline{\underline{m = \frac{1}{\sqrt{3}}}}$$



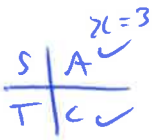
⑨ $b^2-4ac=23$ so roots are given by $x=\frac{-b\pm\sqrt{23}}{2a}$

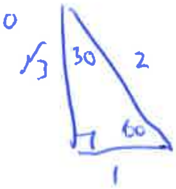
There are 2 real roots ∴ $\sqrt{23}$ is irrational

So only statement (1) is correct $\underline{\underline{(\text{option } B)}}$.

⑩ $2\cos x=\sqrt{3}$

$\cos x=\frac{\sqrt{3}}{2}$





$x=30^\circ$ or 330°

$x=\underline{\underline{\frac{\pi}{6} \text{ or } \frac{11\pi}{6}}}$

⑪ $\int(4x^{1/2}+x^{-3})\,dx$

$=4\frac{x^{3/2}}{3/2}+\frac{x^{-2}}{-2}+c$

$=\underline{\underline{\frac{8}{3}x^{3/2}-\frac{1}{2}x^{-2}+c}}$

⑫ $\sin(p+q)=\sin p\cos q+\cos p\sin q$

$=\frac{2}{\sqrt{5}}\times\frac{\sqrt{5}}{3}+\frac{1}{\sqrt{5}}\times\frac{2}{3}$

$=\underline{\underline{\frac{2}{3}+\frac{2}{3\sqrt{5}}}}$

⑬ $f(x)=4\sin 3x$

$f'(x)=4(3\cos 3x)$

$f'(x)=12\cos 3x$

$f'(0)=12\cos(0)\underline{\underline{=12}}$

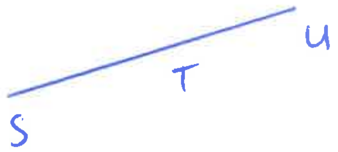
⑭ $p\cdot q=|p||q|\cos\theta$

$=3\times3\times\cos 60$

$=9\times\frac{1}{2}$

$=\underline{\underline{\frac{9}{2}}}$

⑮

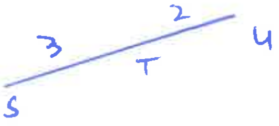


$\overrightarrow{ST}=\overset{t}{\begin{pmatrix}-16\\-4\\16\end{pmatrix}}-\overset{s}{\begin{pmatrix}-4\\5\\1\end{pmatrix}}=\begin{pmatrix}-12\\-9\\15\end{pmatrix}$

$\overrightarrow{TU}=u-t=\begin{pmatrix}-24\\-10\\26\end{pmatrix}-\begin{pmatrix}-16\\-4\\16\end{pmatrix}=\begin{pmatrix}-8\\-6\\10\end{pmatrix}$

$\overrightarrow{ST}=\frac{3}{2}\overrightarrow{TU}$

$\Rightarrow \frac{ST}{TU}=\frac{3}{2}$



ratio is $\underline{\underline{3:2}}$

⑯ $\int \frac{1}{3x^4}dx = \frac{1}{3}\int \frac{1}{x^4}dx = \frac{1}{3}\int x^{-4}\,dx$

$= \frac{1}{3}\,\frac{x^{-3}}{-3} + c$

$= -\frac{1}{9}x^{-3} + c$

$= -\frac{1}{9x^3} + c$

⑰ $y = k(x+1)(x-0)(x-2)$

$\overset{x\ y}{(1,2)} \Rightarrow 2 = k(2)(1)(-1)$

$2 = -2k$

$k = -1$

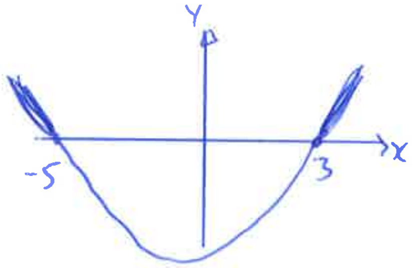
so $y = -1(x)(x+1)(x-2)$

$\Rightarrow y = -x(x+1)(x-2)$

⑱ Draw $y = (x-3)(x+5)$

$y = 0 \Rightarrow (x-3)(x+5) = 0$

$x = 3$ or $x = -5$



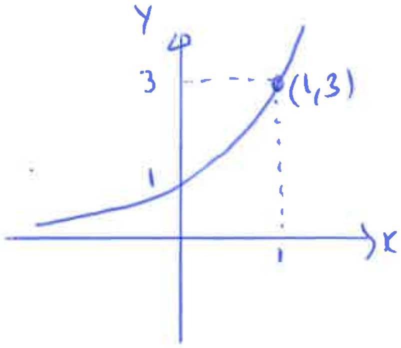
Above x axis when $x < -5$

and $x > 3$

⑲ $x = \log_3 y$

$\Rightarrow \log_3 y = x$

$y = 3^x$



⑳ $g(x) = \sin^2\sqrt{x-2}$

$x - 2 \geq 0$

$\Rightarrow x \geq 2$

For $g(x)$, $\sin^2$ means no negative values in the range so $0 \leq g(x) < 1$

Answer is D

㉑ a) equation of BD : $B(\overset{a}{7},\overset{b}{12})$ $D(2,-3)$

$$m_{BD} = \frac{-3-12}{2-7} = \frac{-15}{-5} = 3$$

$$y - b = m(x-a)$$
$$y - 12 = 3(x-7)$$
$$y - 12 = 3x - 21$$
$$\underline{\underline{y = 3x - 9}}$$

b) Solve $y = 3x - 9$ with $x + 3y = 23$ simultaneously

$\Rightarrow 3y = -x + 23$

$y = -\frac{1}{3}x + \frac{23}{3}$

$$3x - 9 = -\frac{1}{3}x + \frac{23}{3}$$
$$9x - 27 = -x + 23$$
$$10x = 50$$
$$x = 5$$

$y = 3(5) - 9 = 6$ $\underline{\underline{E(5,6)}}$

c i) $A(-1,8)$ $B(7,12)$

Midpoint of $AB = \left(\frac{-1+7}{2}, \frac{8+12}{2}\right) = (\overset{a}{3}, \overset{b}{10})$

$$m_{AB} = \frac{12-8}{7--1} = \frac{4}{8} = \frac{1}{2}$$

$\therefore m_{\perp} = -2$

$$y - b = m(x-a)$$
$$y - 10 = -2(x-3)$$
$$y - 10 = -2x + 6$$
$$\underline{\underline{y = -2x + 16}}$$

ii) when $x = 5$, $y = -2(5) + 16$
$= -10 + 16$
$= 6$

so $E(5,6)$ lies on this line.

(22) $f(x) = (x-2)(x^2+1)$

ai) cuts x axis when $y=0 \Rightarrow (x-2)(x^2+1)=0$

$x-2=0$ or $x^2+1=0$

$x=2$ or $x^2=-1$

$x=\sqrt{-1}$

no solutions

$\underline{\underline{(2,0) \text{ only}}}$

aii) cuts y axis when $x=0$

$y=(x-2)(x^2+1)$

$y=(-2)(1)$

$y=-2$

$\underline{\underline{(0,-2)}}$

b) $f(x) = x^3-2x^2+x-2$

$f'(x) = 3x^2-4x+1 = 0$ for S.P.

$(3x-1)(x-1)=0$

$3x-1=0$ or $x-1=0$

$x=\frac{1}{3}$ or $x=1$

When $x=1$, $y=1-2+1-2=-2$

$(1,-2)$

x	$0 \rightarrow$	$\frac{1}{3}$	$\frac{1}{2} \rightarrow$	1	$2 \rightarrow$
$f'(x)=(3x-1)(x-1)$	+	0	−	0	+
Slope	/	—	\	—	/

$x=\frac{1}{3}$, $y=\left(\frac{1}{3}\right)^3-2\left(\frac{1}{3}\right)^2+\frac{1}{3}-2$

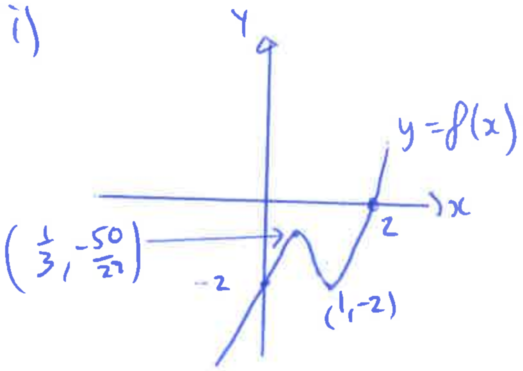
$=\frac{1}{27}-\frac{2}{9}+\frac{1}{3}-2$

$=-\frac{50}{27}$

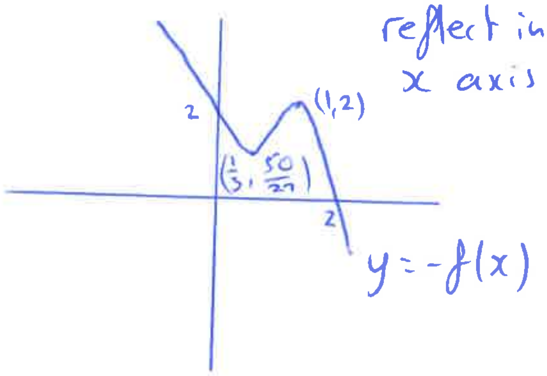
max T.P at $\left(\frac{1}{3}, -\frac{50}{27}\right)$

min T.P at $(1,-2)$

c) i)



ii)



reflect in x axis

(23) a) $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$

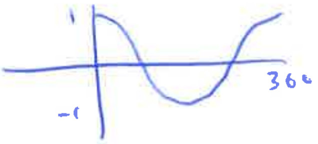
$2\cos^2 x^\circ - 1 \quad - 3\cos x^\circ + 2 = 0$

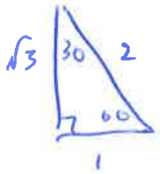
$2\cos^2 x \quad - 3\cos x \quad + 1 = 0$

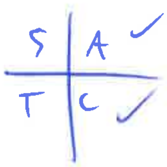
$(2\cos x^\circ - 1)(\cos x^\circ - 1) = 0$

$\cos x^\circ = \frac{1}{2}$ or $\cos x^\circ = 1$

$x = 60^\circ$ or 300° $\qquad$ $x = 0^\circ$ or 360°







For $0 \leq x < 360$, solutions are $\underline{\underline{0^\circ, 60^\circ \text{ and } 300^\circ}}$

b) $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

$2\cos^2 2x - 1 \quad - 3\cos 2x \quad + 2 = 0$

$2\cos^2 2x \quad - 3\cos 2x \quad + 1 = 0$

$(2\cos 2x \quad - 1)(\cos 2x - 1) = 0$

$\cos 2x = \frac{1}{2}$ or $\cos 2x = 1$

$2x = 60$ or 300 $\qquad$ $2x = 0$ or 360

$x = 30^\circ$ or 150° $\qquad$ $x = 0^\circ$ or 180°

$\cos 2x = 2\cos^2 x - 1$

$\cos 4x = 2\cos^2 2x - 1$

Same as part (a) except answers are halved.

Period $= 180^\circ$ so solutions are

$\underline{\underline{0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ \text{ and } 330^\circ}}$