H

National Qualifications

X847/76/11

Mathematics
Paper 1 (Non-calculator)

Duration — 1 hour 15 minutes

2022 PAPER 1 – WORKED SOLUTIONS

Total marks — 55

HWallace

1. Determine the equation of the line perpendicular to $5x+2y=7$, passing through $(-1,6)$. **3**

$5x + 2y = 7$

$2y = -5x + 7$

$y = -\frac{5}{2}x + \frac{7}{2}$

$m = -5/2$ ✓

$m_{\perp} = \frac{2}{5}$ ✓

since $m \times m_{\perp} = -1$

$-\frac{5}{2} \times \frac{2}{5} = -1$

$m_{\perp} = \frac{2}{5}$ and point $(-1, 6)$
a, b

$y - b = m(x - a)$

$y - 6 = \frac{2}{5}(x + 1)$

$5y - 30 = 2x + 2$

$5y = 2x + 32.$ ✓

2. Evaluate $2\log_3 6 - \log_3 4$. **3**

$2\log_3 6 - \log_3 4$

$= \log_3 6^2 - \log_3 4$ ✓

$= \log_3 \left(\frac{36}{4}\right)$ ✓

$= \log_3 9$

$= \log_3 3^2$

$= 2\log_3 3$

$= 2.$ ✓

3. A function, h, is defined by $h(x) = 4 + \frac{1}{3}x$, where $x \in \mathbb{R}$.

Find the inverse function, $h^{-1}(x)$. **3**

let $y = 4 + \frac{1}{3}x$

$y - 4 = \frac{1}{3}x$ ✓

$3(y - 4) = x$ ✓

↳ $h^{-1}(x) = 3(x - 4)$. ✓

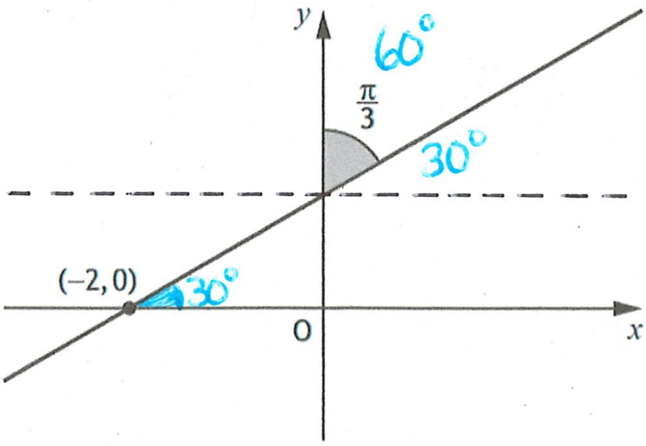
4. Differentiate $y = \sqrt{x^3} - 2x^{-1}$, where $x > 0$. **3**

Prepare $y = x^{3/2}$ ✓ $- 2x^{-1}$

Diff: $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$ ✓ $+ 2x^{-2}$ ✓

$\frac{dy}{dx} = \frac{3}{2}\sqrt{x} + \frac{2}{x^2}$.

5. A line makes an angle of $\frac{\pi}{3}$ radians with the y-axis, and passes through the point $(-2, 0)$ as shown below.



Determine the equation of the line. **3**

$m = \tan 30°$

$m = \frac{1}{\sqrt{3}}$

$y - b = m(x - a)$

$y - 0 = \frac{1}{\sqrt{3}}(x + 2)$

$\sqrt{3}y = x + 2$

6. Evaluate $\int_{-5}^{2} (10 - 3x)^{-\frac{1}{2}}\,dx$. **4**

$= \left[\frac{(10-3x)^{1/2}}{1/2 \times (-3)} \right]_{-5}^{2} = \left[-\frac{2}{3}\sqrt{10-3x} \right]_{-5}^{2}$

$= \left(-\frac{2}{3}\sqrt{10-3(2)}\right) - \left(-\frac{2}{3}\sqrt{10-3(-5)}\right)$

$= -\frac{2}{3}\sqrt{4} + \frac{2}{3}\sqrt{25}$

$= -\frac{2}{3}(2) + \frac{2}{3}(5)$

$= -\frac{4}{3} + \frac{10}{3}$

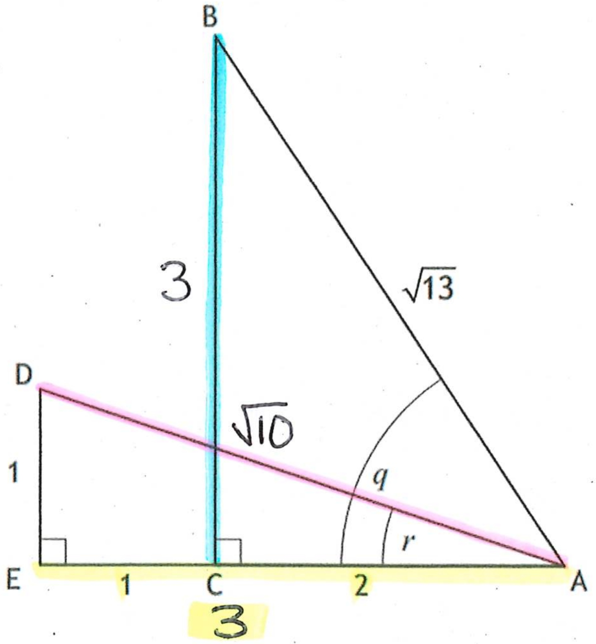
$= 2$

7. Triangles ABC and ADE are both right angled.

Angle BAC $= q$ and angle DAE $= r$ as shown in the diagram.

$AD = \sqrt{1^2 + 3^2}$

$AD = \sqrt{10}$



$BC = \sqrt{(\sqrt{13})^2 - 2^2}$

$BC = \sqrt{9}$

$BC = 3.$

(a) Determine the value of:

(i) $\sin r$ 1

(ii) $\sin q$. 1

(b) Hence determine the value of $\sin(q - r)$. 3

$\cos r = \dfrac{3}{\sqrt{10}}$

$\cos q = \dfrac{2}{\sqrt{13}}$

a) $\sin r = \dfrac{1}{\sqrt{10}}$ ✓ and $\sin q = \dfrac{3}{\sqrt{13}}$. ✓

b) $\sin(q - r) = \sin q \cos r - \cos q \sin r$ ✓

$= \left(\dfrac{3}{\sqrt{13}}\right)\left(\dfrac{3}{\sqrt{10}}\right) - \left(\dfrac{2}{\sqrt{13}}\right)\left(\dfrac{1}{\sqrt{10}}\right)$ ✓

$= \dfrac{9}{\sqrt{130}} - \dfrac{2}{\sqrt{130}}$

$\sin(q - r) = \dfrac{7}{\sqrt{130}}$ ✓

8. Solve $\log_6 x + \log_6 (x+5) = 2$, where $x>0$. **4**

$$\log_6 x + \log_6 (x+5) = 2\log_6 6$$

$$\log_6 x(x+5) = \log_6 6^2$$

$$x(x+5) = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x+9)(x-4) = 0$$

$x + 9 = 0$ $\qquad$ $x - 4 = 0$

$x = -9$ $\qquad$ $x = 4$

but $x > 0$

$\therefore$ no solution

$\rightarrow$ $x = 4$ only.

9. Solve the equation $\cos 2x° = 5\cos x° - 3$ for $0 \le x < 360$. **5**

$$2\cos^2 x° - 1 = 5\cos x° - 3$$

$$2\cos^2 x° - 5\cos x° + 2 = 0$$

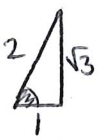
let $2c^2 - 5c + 2 = 0$

$(2c - 1)(c - 2) = 0$

$$\therefore (2\cos x° - 1)(\cos x° - 2) = 0$$

$2\cos x° - 1 = 0$ $\qquad$ $\cos x° - 2 \neq 0$

$\cos x° = \frac{1}{2}$ $\qquad$ S | A, T | C $\qquad$ $\cos x° = 2$



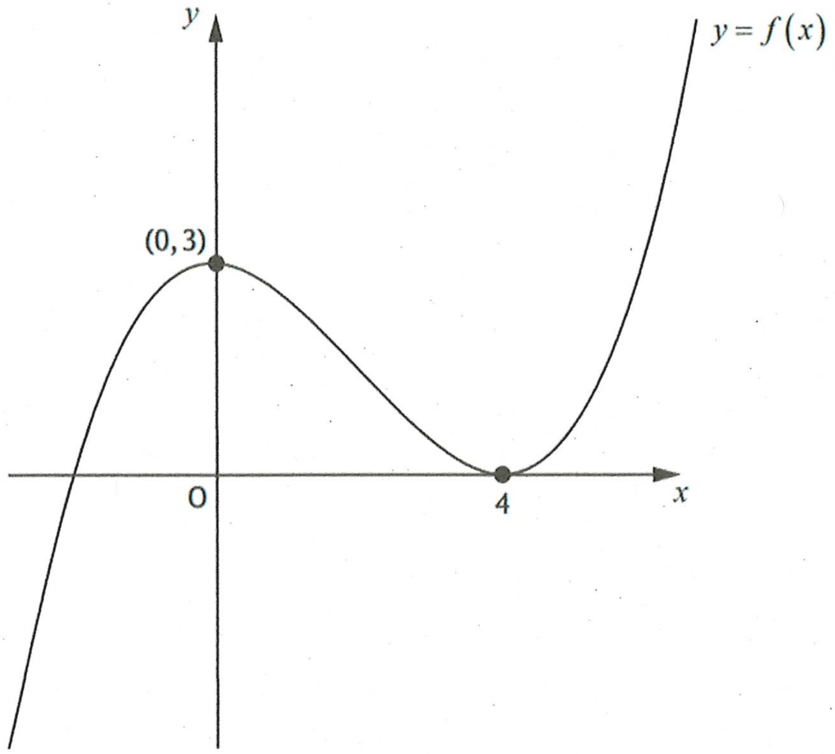
$ra = 60°$ $\qquad$ no solutions.

$x = 60, 360 - 60$

$x° = 60°, 300°$

10. The diagram shows the graph of a cubic function with equation $y = f(x)$.

The curve has stationary points at $(0, 3)$ and $(4, 0)$.



(a) Sketch the graph of $y = 2f(x) + 1$. **3**

Use the diagram provided in the answer booklet.

(b) State the coordinates of the stationary points on the graph of $y = f\left(\frac{1}{2}x\right)$. **1**

a) $y = 2f(x) + 1$

double y-values

up 1

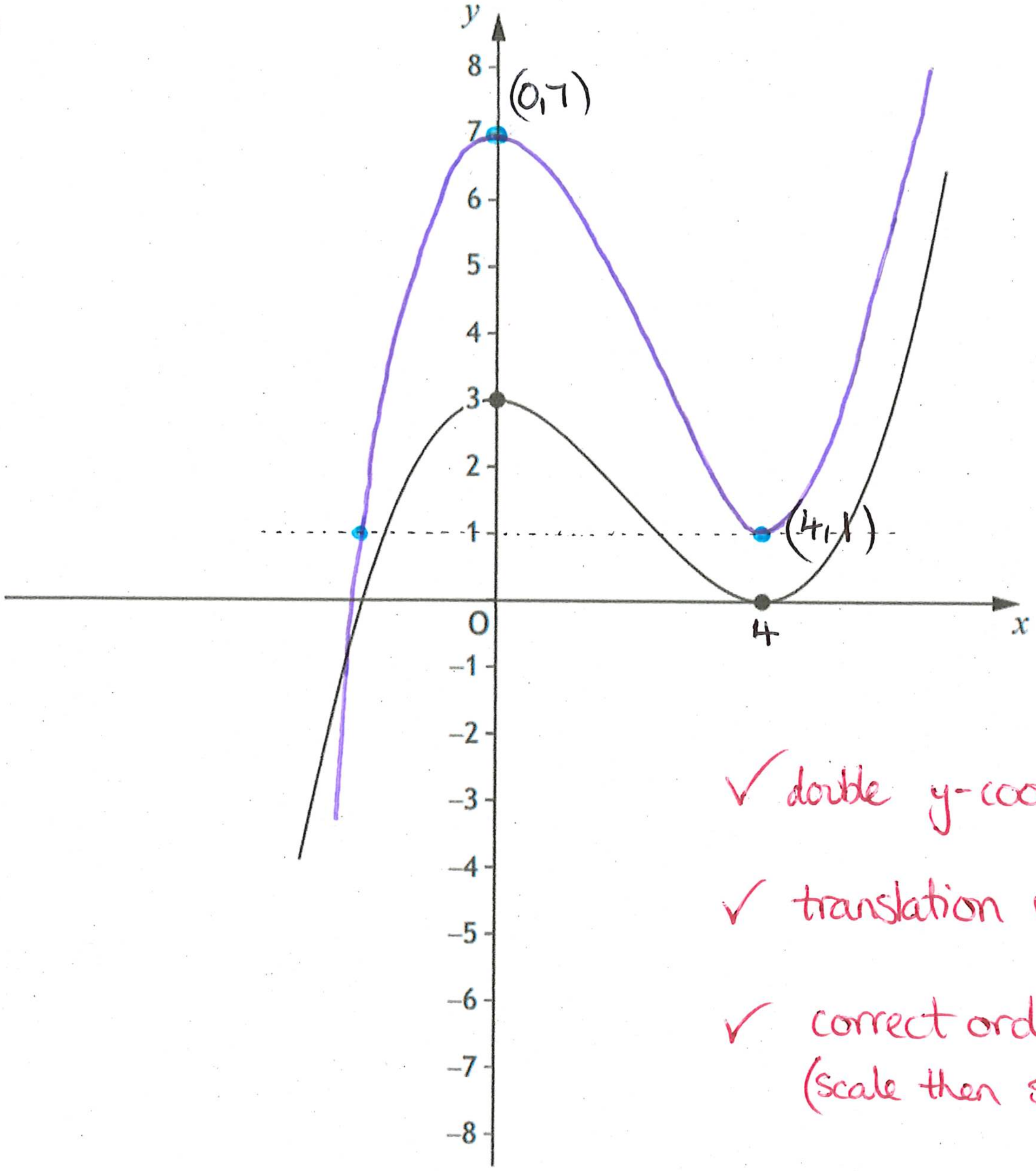
$f(x) \rightarrow 2f(x) \rightarrow 2f(x) + 1$

$(0, 3) \rightarrow (0, 6) \rightarrow (0, 7)$

$(4, 0) \rightarrow (4, 0) \rightarrow (4, 1)$

Diagram provided in Answer Booklet:

10a)



✓ double y-coords

✓ translation up 1.

✓ correct order (scale then slide)

b) $y = f\left(\frac{1}{2}x\right)$

double x-coords.

Stationary Points at (0,3) and (8,0). ✓

11. Express $2x^2+12x+23$ in the form $p(x+q)^2+r$. **3**

$$2\left[x^2+6x\right]+23$$

$$2\left[(x+3)^2-9\right]+23.$$

$$2(x+3)^2-18+23$$

$$\underline{2(x+3)^2+5}$$

12. Given that $f(x)=4\sin\left(3x-\frac{\pi}{3}\right)$, evaluate $f'\left(\frac{\pi}{6}\right)$. **3**

$$f'(x)=4\times 3\cos\left(3x-\frac{\pi}{3}\right)$$

$$f'(x)=12\cos\left(3x-\frac{\pi}{3}\right)$$

$$f'\left(\frac{\pi}{6}\right)=12\cos\left(\frac{3\pi}{6}-\frac{\pi}{3}\right)$$

$$=12\cos\frac{\pi}{6}$$

$$=12\cos 30^\circ$$

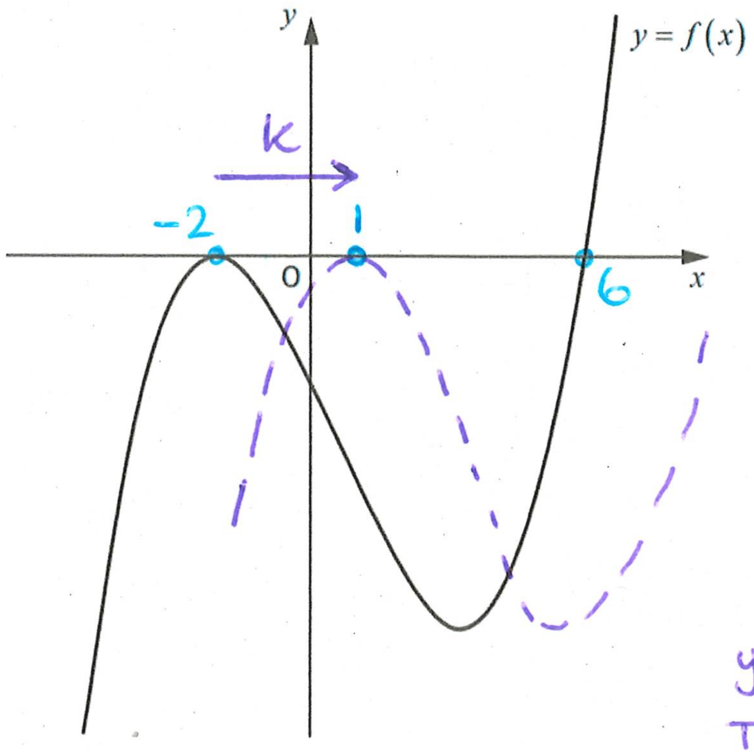
$$=12\left(\frac{\sqrt{3}}{2}\right)$$

$$\underline{f'\left(\frac{\pi}{6}\right)=6\sqrt{3}}$$

13. (a) (i) Show that $(x+2)$ is a factor of $f(x)=x^3-2x^2-20x-24$. **2**

(ii) Hence, or otherwise, solve $f(x)=0$. **3**

The diagram shows the graph of $y=f(x)$.



b) $\underline{k=3}$

$y=f(x-k)$

Translation to the right by k.

(b) The graph of $y=f(x-k)$, $k>0$ has a stationary point at $(1,0)$.

State the value of k. **1**

a) i)

-2	1	-2	-20	-24
	↓	-2	8	24
	1	-4	-12	$\boxed{0}$

Since remainder is zero, $x=-2$ is a root and $(x+2)$ is a factor.

ii) $f(x) = x^3-2x^2-20x-24$

$(x+2)(x^2-4x-12)=0$

$(x+2)(x-6)(x+2)=0$

$(x+2)^2(x-6)=0$

↓ ↓

$x+2=0$ $\quad$ $x-6=0$

$x=-2$ $\quad$ $x=6$

↳ $\underline{x=-2,\ 6.}$

14. C_1 is the circle with equation $(x-7)^2+(y+5)^2=100$.

(a) (i) State the centre and radius of C_1. **2**

(ii) Hence, or otherwise, show that the point $P(-2,7)$ lies outside C_1. **2**

C_2 is a circle with centre P and radius r.

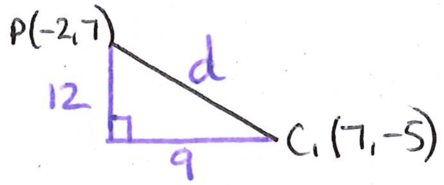
(b) Determine the value(s) of r for which circles C_1 and C_2 have exactly one point of intersection. **2**

a) $(x-7)^2+(y+5)^2=100$

$(x-a)^2+(y-b)^2=r^2$

i) Centre $C_1(7,-5)$ ✓ radius $r_1=10$ ✓

ii) Distance C_1 to P



$d^2=12^2+9^2$

$d^2=225$

$d=15$ ✓

Since $d>r_1$, $15>10$, then point P lies outside the circle ✓

b) For one point of intersection,

then $d=r_1+r$ or $r=d+r_1$

$15=10+r$ $\quad$ $r=15+10$

$r=5$ ✓ or $r=25$ ✓