H

National
Qualifications
2018

X747/76/11

Mathematics
Paper 1 (Non-Calculator)

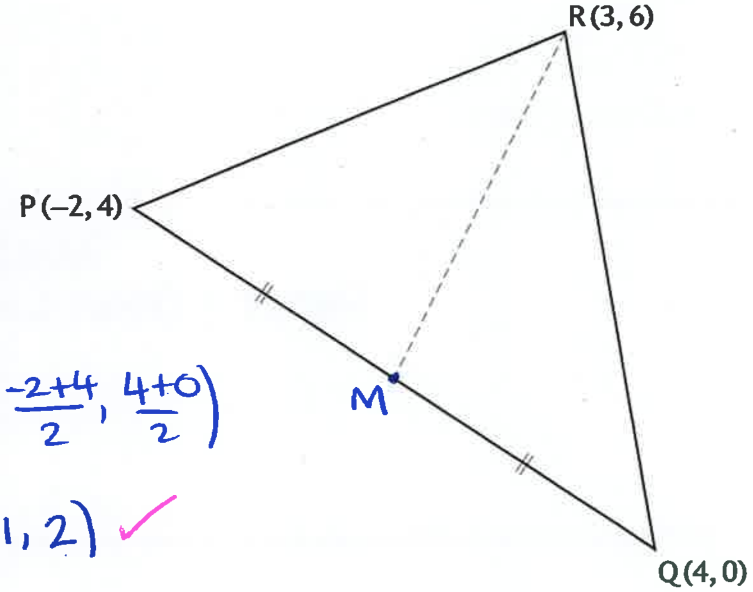
THURSDAY, 3 MAY
9:00 AM – 10:10 AM

60 marks

2018 PAPER 1 – WORKED SOLUTIONS

HWallace

1. PQR is a triangle with vertices P(–2, 4), Q(4, 0) and R(3, 6).



Midpoint $\left(\frac{-2+4}{2}, \frac{4+0}{2}\right)$

$M(1, 2)$ ✓

Find the equation of the median through R. **3**

Gradient RM

$m_{RM} = \frac{6-2}{3-1} = \frac{4}{2} = 2$ ✓

Equation of Median

$y - b = m(x - a)$

$y - 6 = 2(x - 3)$

$y - 6 = 2x - 6$

$y = 2x$ ✓

2. A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by

$$g(x) = \frac{1}{5}x - 4.$$

Find the inverse function, $g^{-1}(x)$. **3**

Let $y = \frac{1}{5}x - 4$

$y + 4 = \frac{1}{5}x$ ✓

$5(y+4) = x$ ✓

↳ $g^{-1}(x) = 5(x+4)$ ✓ or $g^{-1}(x) = 5x + 20$

3. Given $h(x) = 3\cos 2x$, find the value of $h'\left(\frac{\pi}{6}\right)$. **3**

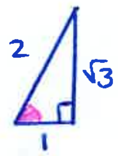
$h'(x) = 3 \times (-2\sin 2x)$ ✓✓

$h'(x) = -6\sin 2x$

$h'\left(\frac{\pi}{6}\right) = -6\sin 2\left(\frac{\pi}{6}\right)$

$\frac{\pi}{3} = \frac{180}{3} = 60^\circ$

$h'\left(\frac{\pi}{6}\right) = -6\sin\frac{\pi}{3}$

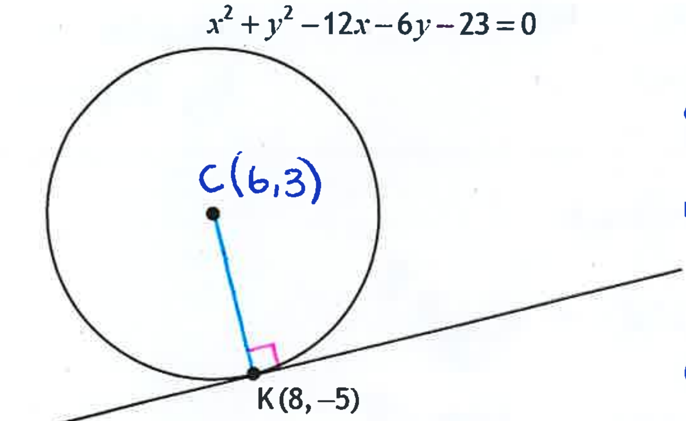


$h'\left(\frac{\pi}{6}\right) = -6\left(\frac{\sqrt{3}}{2}\right)$

$\sin 60 = \frac{\sqrt{3}}{2}$

$h'\left(\frac{\pi}{6}\right) = -3\sqrt{3}$ ✓

4. The point K (8, –5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.



Gradient of radius

$m_{rad} = \frac{3-(-5)}{6-8} = \frac{8}{-2} = -4$ ✓

Gradient of tangent

$m_{tan} = \frac{1}{4}$ ✓

Find the equation of the tangent to the circle at K. **4**

$x^2 + y^2 - 12x - 6y - 23 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -12 \qquad 2f = -6$

$g = -6 \qquad f = -3 \qquad$ Centre $(6, 3)$ ✓

Equation of tangent $\quad y - b = m(x - a)$

$y + 5 = \frac{1}{4}(x - 8)$

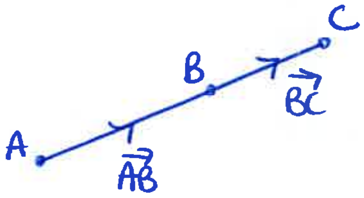
$y + 5 = \frac{1}{4}x - 2$

$y = \frac{1}{4}x - 7$ ✓

5. A$(-3, 4, -7)$, B$(5, t, 5)$ and C$(7, 9, 8)$ are collinear.

(a) State the ratio in which B divides AC. **1**

(b) State the value of t. **1**



a) $\vec{AB} = b - a$

$= \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ -7 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 8 \\ t-4 \\ 12 \end{pmatrix}$

$\vec{BC} = c - b$

$= \begin{pmatrix} 7 \\ 9 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 2 \\ 9-t \\ 3 \end{pmatrix}$

$\vec{AB} = 4\vec{BC}$

B divides AC in the ratio 4 : 1 ✓

b)
$$t - 4 = 4(9 - t)$$
$$t - 4 = 36 - 4t$$
$$5t = 40$$
$$t = 8$$ ✓

This question can also be answered by inspection.

6. Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$. **3**

$= \log_5 250 - \log_5 8^{1/3}$ ✓

$= \log_5 250 - \log_5 2$

$= \log_5 \left(\frac{250}{2}\right)$ ✓

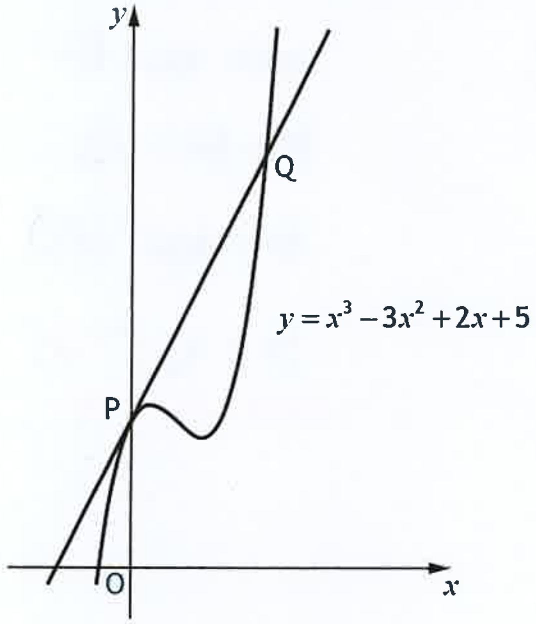
$= \log_5 125$

$= \log_5 5^3$

$= 3\log_5 5$

$= 3$ ✓

7. The curve with equation $y = x^3 - 3x^2 + 2x + 5$ is shown on the diagram.



(a) Write down the coordinates of P, the point where the curve crosses the y-axis . **1**

(b) Determine the equation of the tangent to the curve at P. **3**

(c) Find the coordinates of Q, the point where this tangent meets the curve again. **4**

a) y-intercept at $x = 0$

$y = 0^3 - 3(0)^2 + 2(0) + 5$

$y = 5$.

↳ $P(0, 5)$ ✓

b) $y = x^3 - 3x^2 + 2x + 5$

$\frac{dy}{dx} = 3x^2 - 6x + 2$ ✓

$m = 3(0)^2 - 6(0) + 2$

$m = 2$ ✓

Equation of tangent

$y = 2x + 5$ ✓

c) Let $y = y$

$x^3 - 3x^2 + 2x + 5 = 2x + 5$

$x^3 - 3x^2 = 0$ ✓

$x^2(x - 3) = 0$ ✓

↓ ↓

$x^2 = 0$ $\quad$ $x - 3 = 0$

$x = 0$ $\quad$ $x = 3$ ✓

Point P $\quad$ Point Q.

At Q, $x = 3$

$y = 2(3) + 5$

$y = 11$

Point $Q(3, 11)$ ✓

8. A line has equation $y-\sqrt{3}\,x+5=0$.

Determine the angle this line makes with the positive direction of the x-axis. **2**

$y-\sqrt{3}x+5=0$

$y=\sqrt{3}x-5$

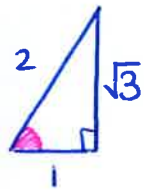
$\therefore\ m=\sqrt{3}$ ✓

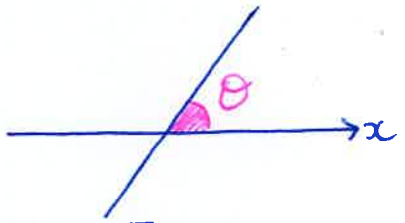
$m=\tan\theta$

$\tan\theta=\sqrt{3}$

$\theta=\tan^{-1}(\sqrt{3})$

$\theta=60^\circ$ ✓

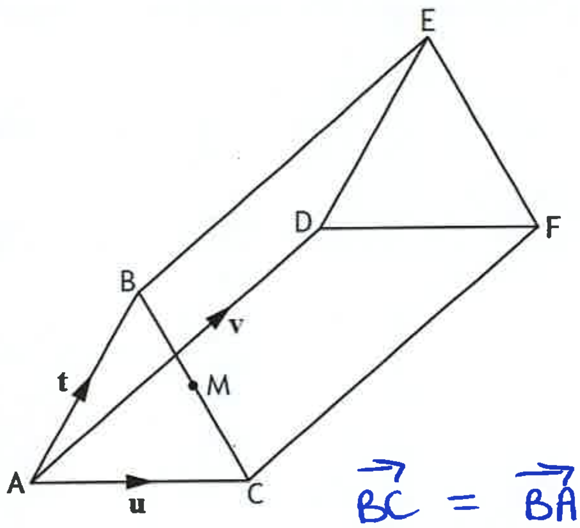




$m=\sqrt{3}$

9. The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB}=\mathbf{t}$, $\overrightarrow{AC}=\mathbf{u}$ and $\overrightarrow{AD}=\mathbf{v}$.



(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. **1**

$\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}$

$\overrightarrow{BC}=-t+u$

$\overrightarrow{BC}=u-t$ ✓

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$. **2**

$\overrightarrow{MD}=\overrightarrow{MC}+\overrightarrow{CA}+\overrightarrow{AD}$ ✓

$\overrightarrow{MD}=\frac{1}{2}(u-t)-u+v$

$\overrightarrow{MD}=\frac{1}{2}u-\frac{1}{2}t-u+v$

$\overrightarrow{MD}=v-\frac{1}{2}u-\frac{1}{2}t$ ✓

10. Given that

- $\frac{dy}{dx} = 6x^2 - 3x + 4$, and
- $y = 14$ when $x = 2$,

express y in terms of x. **4**

$$y = \int \frac{dy}{dx} \cdot dx$$

$$y = \int 6x^2 - 3x + 4 . \ dx$$

$$y = \frac{6x^3}{3} - \frac{3x^2}{2} + 4x + c$$

$$y = 2x^3 - \frac{3x^2}{2} + 4x + c \checkmark$$

At $x = 2$, $y = 14$

$$14 = 2(2)^3 - \frac{3(2)^2}{2} + 4(2) + c \checkmark$$

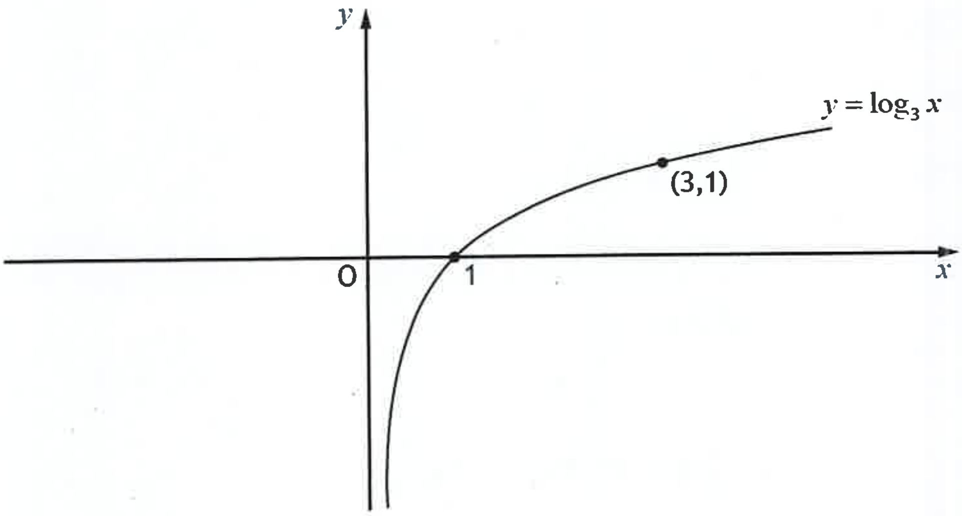
$$14 = 16 - 6 + 8 + c$$

$$14 = 18 + c$$

$$c = -4$$

$$\hookrightarrow \ y = 2x^3 - \frac{3x^2}{2} + 4x - 4 \checkmark$$

11. The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. **2**

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves. **3**

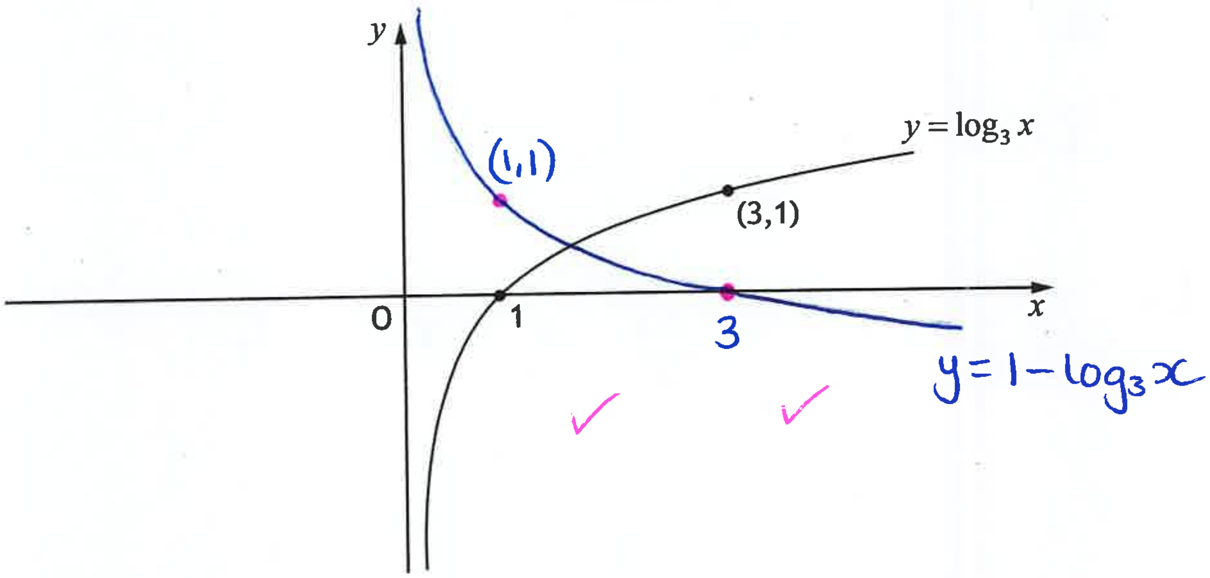
a) if $y = 1 - \log_3 x$

then $y = -\log_3 x + 1$

reflect on x-axis — up 1

$(1, 0)$ $(1, 0)$ $(1, 1)$

$(3, 1)$ $(3, -1)$ $(3, 0)$



11) b) For point of intersection, let $y = y$

$$\log_3 x = 1 - \log_3 x$$

$$2\log_3 x = 1$$

$$\log_3 x^2 = \log_3 3$$

$$x^2 = 3$$

$$x = \sqrt{3} \quad (x > 0)$$

12. Vectors **a** and **b** are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. **1**

(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$. **3**

a) $$2a + b = 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$2a + b = \begin{pmatrix} 6 \\ -3 \\ p+4 \end{pmatrix}$$

b) If $|2a + b| = 7$

then $\sqrt{(6)^2 + (-3)^2 + (p+4)^2} = 7$

$$36 + 9 + p^2 + 8p + 16 = 49$$

$$p^2 + 8p + 12 = 0$$

$$(p + 6)(p + 2) = 0$$

$$\downarrow \qquad \downarrow$$

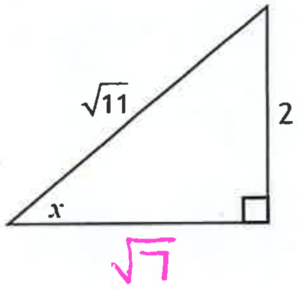
$$p = -6 \text{ or } p = -2$$

13. The right-angled triangle in the diagram is such that $\sin x = \dfrac{2}{\sqrt{11}}$ and $0 < x < \dfrac{\pi}{4}$.

$a^2 = (\sqrt{11})^2 - 2^2$

$a^2 = 7$

$a = \sqrt{7}$



$\cos x = \dfrac{\sqrt{7}}{\sqrt{11}}$

(a) Find the exact value of:

(i) $\sin 2x$ **3**

(ii) $\cos 2x$. **1**

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$. **3**

a) i) $\sin 2x = 2\sin x \cos x$

$= 2\left(\dfrac{2}{\sqrt{11}}\right)\left(\dfrac{\sqrt{7}}{\sqrt{11}}\right)$

$\sin 2x = \dfrac{4\sqrt{7}}{11}$

ii) $\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\dfrac{\sqrt{7}}{\sqrt{11}}\right)^2 - \left(\dfrac{2}{\sqrt{11}}\right)^2$

$= \dfrac{7}{11} - \dfrac{4}{11}$

$\cos 2x = \dfrac{3}{11}$

b) $\sin 3x = \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= \left(\dfrac{4\sqrt{7}}{11}\right)\left(\dfrac{\sqrt{7}}{\sqrt{11}}\right) + \left(\dfrac{3}{11}\right)\left(\dfrac{2}{\sqrt{11}}\right)$

$= \dfrac{28}{11\sqrt{11}} + \dfrac{6}{11\sqrt{11}}$

$\sin 3x = \dfrac{34}{11\sqrt{11}}$

14. Evaluate $\displaystyle\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}}\,dx$. **5**

Prepare!

$$= \int_{-4}^{9} (2x+9)^{-2/3} \cdot dx$$

$$= \left[\frac{(2x+9)^{1/3}}{1/3 \times 2} \right]_{-4}^{9}$$

$$= \left[\frac{3}{2} \sqrt[3]{2x+9} \right]_{-4}^{9}$$

$$= \left(\frac{3}{2} \sqrt[3]{2(9)+9} \right) - \left(\frac{3}{2} \sqrt[3]{2(-4)+9} \right)$$

$$= \frac{3}{2} \sqrt[3]{27} - \frac{3}{2} \sqrt[3]{1}$$

$$= \frac{3}{2}(3) - \frac{3}{2}(1)$$

$$= \frac{9}{2} - \frac{3}{2}$$

$$= \frac{6}{2}$$

$$= 3.$$

15. A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$
- $x=2$ is a repeated root of $f(x)$
- $f'(-2)=0$
- $f'(x)>0$ where the graph with equation $y=f(x)$ crosses the y-axis

Sketch a possible graph of $y=f(x)$ on the diagram in your answer booklet. **4**

If $(x+4)$ is a factor, then $x=-4$ is a root!
We also have another repeated root at $x=2$, and a stationary Point. Roots: $(2,0)$ and $(-4,0)$

At $f'(-2)=0$, then there will be a stationary point at $x=-2$.

If $f'(0)>0$, then the gradient of the curve is positive when $f(x)$ crosses the y-axis.