

FOR OFFICIAL USE



National
Qualifications
2023

Mark

X847/75/01

Mathematics Paper 1 (Non-calculator)

WEDNESDAY, 3 MAY

9:00 AM – 10:00 AM



* X 8 4 7 7 5 0 1 *

Fill in these boxes and read what is printed below.

Full name of centre

WORKED SOLUTIONS

Town

th

Forename(s)

Surname

Number of seat

Date of birth

Day

Month

Year

Scottish candidate number

Total marks — 40

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



* X 8 4 7 7 5 0 1 0 1 *

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle $A = \frac{1}{2}ab \sin C$

Volume of a sphere $V = \frac{4}{3}\pi r^3$

Volume of a cone $V = \frac{1}{3}\pi r^2 h$

Volume of a pyramid $V = \frac{1}{3}Ah$

Standard deviation $s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$

or $s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n - 1}}$, where n is the sample size.



* X 8 4 7 7 5 0 1 0 2 *

Total marks — 40
Attempt ALL questions

1. Evaluate $2\frac{1}{6} \div \frac{8}{9}$.

Give your answer in its simplest form.

2

$$\frac{13}{6} \div \frac{8}{9}$$

$$\frac{13}{\cancel{6}^2} \times \frac{9^3}{8} \checkmark$$

$$\underline{\underline{\frac{39}{16} = 2\frac{7}{16} \checkmark}}$$

2. Expand and simplify $(x+7)^2 + 6(x^2 - 10)$.

3

$$(x+7)(x+7) + 6(x^2 - 10)$$

$$x^2 + 14x + 49 \checkmark + 6x^2 - 60 \checkmark$$

$$\underline{\underline{7x^2 + 14x - 11 \checkmark}}$$

[Turn over



3. Solve, algebraically, the system of equations

$$2x + 3y = 8 \quad (\times 2)$$

$$5x + 2y = -2 \quad (\times 3)$$

$$\begin{array}{r} 4x + 6y = 16 \\ \ominus 15x + 6y = -6 \end{array} \quad \checkmark$$

$$11x = -22$$

$$x = -2 \quad \checkmark$$

$$\hookrightarrow 2x + 3y = 8$$

$$2(-2) + 3y = 8$$

$$-4 + 3y = 8$$

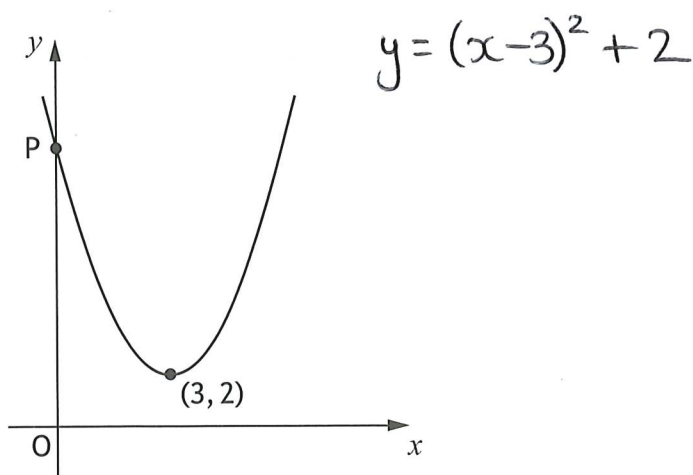
$$3y = 12$$

$$y = 4.$$

Solution $x = -2, y = 4.$ \checkmark



4. The graph below shows part of a parabola of the form $y = (x+a)^2 + b$.



(a) (i) State the value of a .

1

$$\underline{\underline{a = -3}} \quad \checkmark$$

(ii) State the value of b .

1

$$\underline{\underline{b = 2}} \quad \checkmark$$

(b) P is the point $(0, c)$.

Find the value of c .

1

$$y = (x - 3)^2 + 2$$

let $x = 0$ and $y = c$

$$c = (0 - 3)^2 + 2$$

$$c = (-3)^2 + 2$$

$$c = 9 + 2$$

$$\underline{\underline{c = 11}} \quad \checkmark$$

[Turn over



5. Determine the nature of the roots of the function $f(x) = 4x^2 + 6x - 1$.

Consider roots at $4x^2 + 6x - 1 = 0$
 $ax^2 + bx + c = 0$

$$a = 4$$

$$b = 6$$

$$c = -1$$

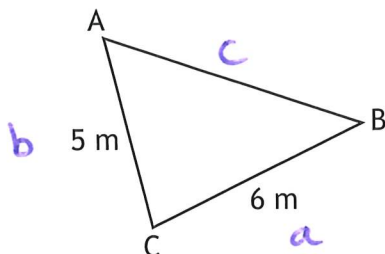
$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (6)^2 - 4(4)(-1) \\ &= 36 + 16 \\ &= 52 \checkmark \end{aligned}$$

Since $b^2 - 4ac > 0$, there
are 2 real and distinct roots. \checkmark



6. In triangle ABC:

- AC = 5 metres
- BC = 6 metres
- $\cos C = \frac{1}{5}$.



Calculate the length of AB.

3

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 6^2 + 5^2 - 2(6)(5) \left(\frac{1}{5}\right) \checkmark$$

$$c^2 = 36 + 25 - 12$$

$$c^2 = 49 \checkmark$$

$$c = \sqrt{49}$$

$$c = 7$$

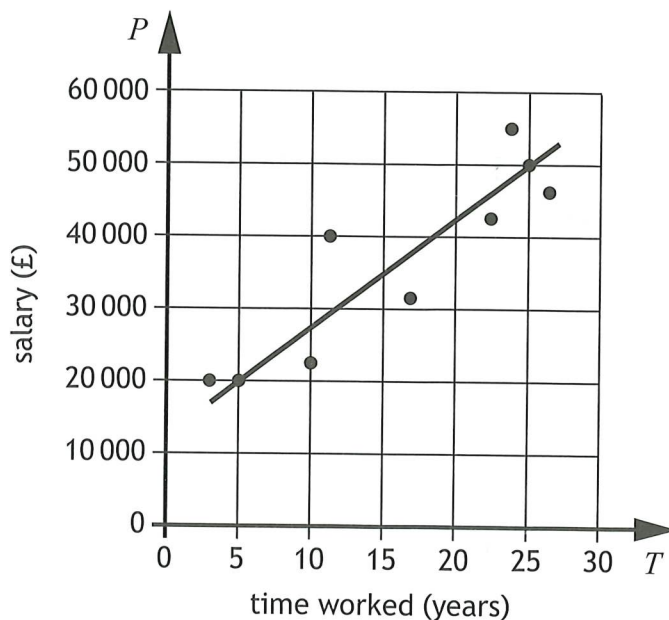
Length AB = 7 m. \checkmark

[Turn over



7. A business recorded the salaries of a sample of its employees and the length of time they have worked for the business.

The scattergraph shows the relationship between their salary, P pounds, and the length of time, T years, they have worked.



A line of the best fit has been drawn.

- (a) Find the equation of the line of best fit in terms of P and T .

Give the equation in its simplest form.

3

Points $(5, 20000)$ and $(25, 50000)$

$$m = \frac{50000 - 20000}{25 - 5}$$

$$y - b = m(x - a)$$

$$m = \frac{30000}{20}$$

$$y - 20000 = 1500(x - 5) \checkmark$$

$$m = 1500 \checkmark$$

$$y - 20000 = 1500x - 7500$$

$$y = 1500x + 12500$$

$$\hookrightarrow \underline{\underline{P = 1500T + 12500}} \checkmark$$



7. (continued)

- (b) Use your equation from part (a) to estimate the salary of an employee who has worked for the business for 8 years.

1

$$\text{Let } T = 8$$

$$P = 1500(8) + 12500$$

$$P = 12000 + 12500$$

$$P = 24500$$

Salary £24,500 ✓

8. Express $\frac{12}{\sqrt{15}}$ with a rational denominator.

Give your answer in its simplest form.

2

$$\frac{12}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} = \frac{12\sqrt{15}}{15} = \frac{4\sqrt{15}}{5}$$

[Turn over



9. A magazine company conducted a survey of the ages of its readers. A sample of ten readers' ages, in years, are shown below.

33 55 38 47 36 41 42 41 35 31
 ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

(a) Calculate the median and interquartile range of the ages of readers for this sample.

3

order 31 33 35 36 38 | 41 41 42 47 55

$Q_1 = 35$

Median = 39.5 ✓

$Q_2 = 39.5$

$IQR = 42 - 35$

$Q_3 = 42$ ✓

IQR = 7 ✓

A newspaper company also conducted a survey of the ages of its readers.

The median age of a sample of its readers was 41 years and the interquartile range was 9 years.

(b) Make two valid comments comparing the ages of the readers of the magazine and the ages of the readers of the newspaper.

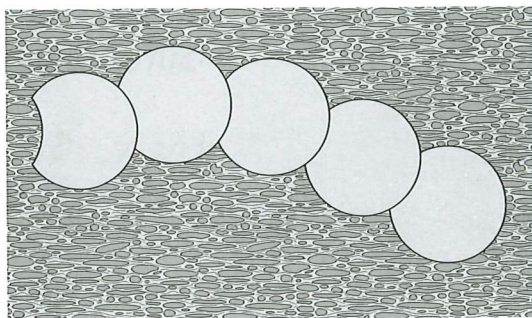
2

Summary	median	IQR
Magazine	39.5	7
Newspaper	41	9

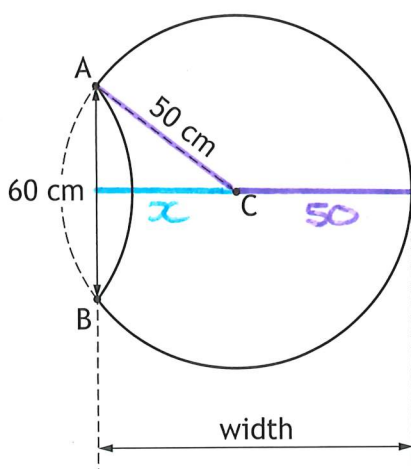
- On average, the ages of newspaper readers is higher than magazine readers. ✓
- The ages of magazine readers are more consistent than newspaper readers. ✓



10. Alan buys some identical paving slabs to make a path.
Each slab is part of a circle.



The diagram below shows a single slab.

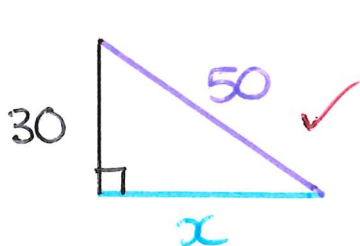


The circle, centre C, has a radius of 50 centimetres.

Length AB is 60 centimetres.

Calculate the width of the paving slab.

4



$$x^2 = 50^2 - 30^2 \checkmark$$

$$x^2 = 2500 - 900$$

$$x^2 = 1600$$

$$x = 40 \text{ cm} \checkmark$$

$$\text{width} = 40 + 50$$

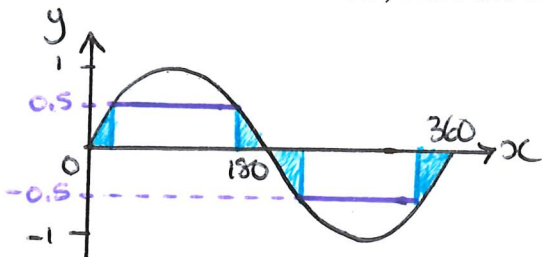
$$\text{width} = 90 \text{ cm} \checkmark$$



* X 8 4 7 7 5 0 1 1 1 *

1

11. Given that $\sin 30^\circ = 0.5$, state the value of $\sin 330^\circ$.



Since $\sin 30^\circ = 0.5$
 then $\sin 330^\circ = -0.5$ ✓

12. Simplify $\frac{5c^{-2}}{c^3 \times c^4}$.

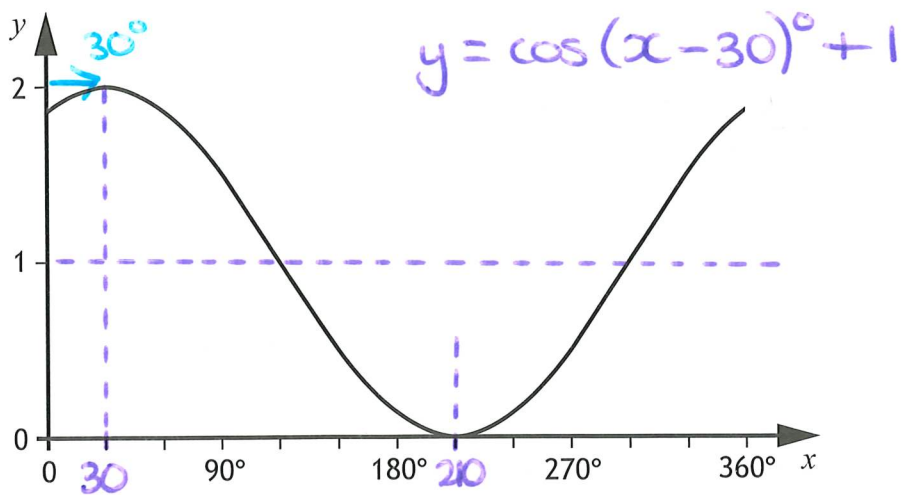
Give your answer with a **positive** power.

3

$$\frac{5c^{-2}}{c^3 \times c^4} = \frac{5c^{-2}}{c^7} \checkmark = 5c^{-9} \checkmark = \underline{\underline{\frac{5}{c^9}}} \checkmark$$



13. Part of the graph of $y = \cos(x + a)^\circ + b$ is shown.



(a) State the value of a .

1

$a = -30^\circ$ ✓

(b) State the value of b .

1

$b = 1$ ✓

[Turn over

14. Solve, algebraically, the inequation $\frac{x+1}{3} - 2 > \frac{3x}{5}$.

$$\frac{x+1}{3} - 2 > \frac{3x}{5}$$

x15 x15 x15

$$5(x+1) - 30 > 9x \quad \checkmark$$

$$5x + 5 - 30 > 9x$$

$$5x - 25 > 9x$$

$$-25 > 4x \quad \checkmark$$

$$-\frac{25}{4} > x$$

$$\hookrightarrow \underline{\underline{x < -\frac{25}{4}}} \quad \checkmark$$

[END OF QUESTION PAPER]



* X 8 4 7 7 5 0 1 1 4 *