FOR OFFICIAL USE

N5

National Qualifications

Mark 60

X847/75/02

Mathematics
Paper 2

Duration — 1 hour 50 minutes

X8477502

2021 Paper 2 Worked Solutions

HMWallace

MARKS | DO NOT WRITE IN THIS MARGIN

Total marks — 60

Attempt ALL questions

1. A housing development is being built.

   The price of a house built in 2020 is £250 000.

   This price is expected to increase by 4% each year.

   Calculate the expected price of a house built in 2022. **3**

2 years later!

$100\% + 4\% = 104\% = 1.04$

$\text{Expected price} = 250\,000 \times 1.04^2$

$= £270,400.$

2. Light travels at 3×10^8 metres per second.

   A star is $4{\cdot}2 \times 10^{17}$ metres away from Earth.

   Calculate the number of seconds it takes for light from this star to reach Earth.

   Give your answer in scientific notation. **2**

$D = 4.2 \times 10^{17}\ m$

$S = 3 \times 10^8\ m/s$

$T = ?$

$T = \frac{D}{S} = \frac{4.2 \times 10^{17}}{3 \times 10^8}$

$\text{Time} = 1.4 \times 10^9 \text{ seconds.}$

MARKS | DO NOT WRITE IN THIS MARGIN

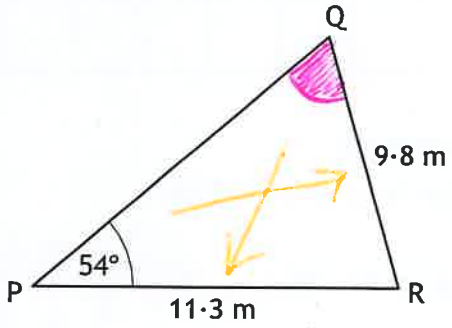
3. Factorise fully $3a^2 - 75$. 2

HCF $3(a^2 - 25)$

D2S $3(a-5)(a+5)$

4. In triangle PQR

- PR = 11·3 metres
- QR = 9·8 metres
- angle QPR = 54°.



Calculate the size of acute angle PQR. 3

$$\frac{\sin Q}{11.3} = \frac{\sin 54}{9.8}$$

$$\sin Q = \frac{11.3 \sin 54}{9.8}$$

$$\sin Q = 0.932846...$$

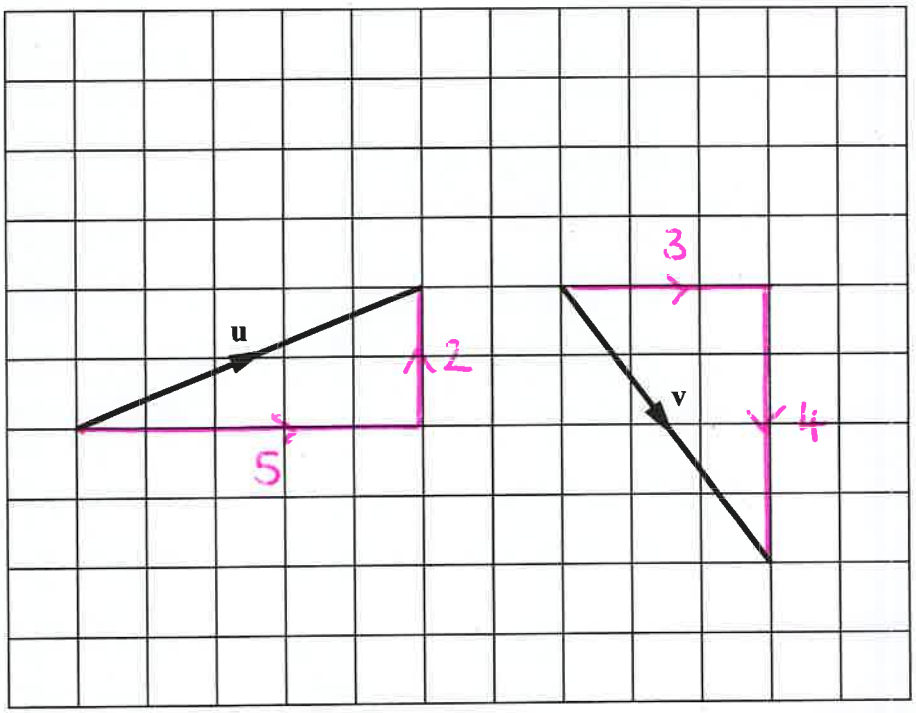
$$Q = \sin^{-1}(\text{ANS})$$

$$Q = 68.8829...$$

$\angle PQR = 68.9°$

MARKS | DO NOT WRITE IN THIS MARGIN

5. The vectors **u** and **v** are shown in the diagram below.



Find the resultant vector $\mathbf{u} - \mathbf{v}$.

Express your answer in component form. 2

$$u - v = \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$u - v = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

MARKS | DO NOT WRITE IN THIS MARGIN

6. A company operates a bus route from the city centre to the airport.
The number of passengers on six of its buses on a Monday was

32 27 34 29 31 33.

(a) Calculate the mean and standard deviation of the number of passengers. 4

$\Sigma x = 32 + 27 + 34 + 29 + 31 + 33 = 186$

$\Sigma x^2 = 32^2 + 27^2 + 34^2 + 29^2 + 31^2 + 33^2 = 5800$

$\text{Mean} = \frac{\Sigma x}{n} = \frac{186}{6} = 31$ passengers

$sd = \sqrt{\frac{5800 - (186)^2/6}{5}}$

$sd = 2.60768...$

$sd = 2.6$

(b) The mean number of passengers the following Saturday was 28 and the standard deviation was 3·2.

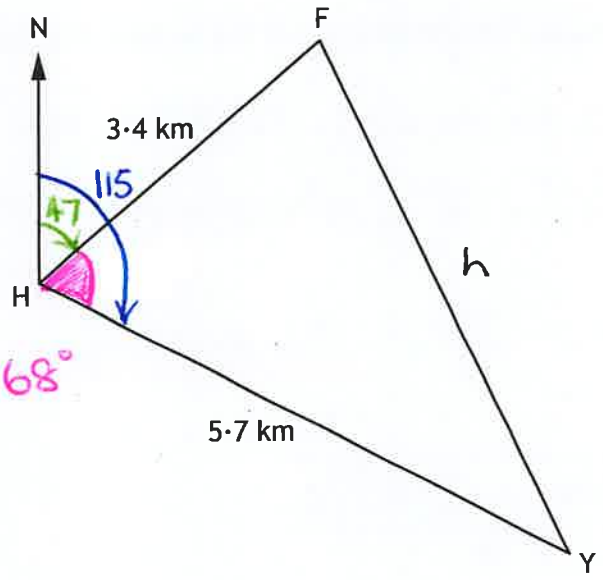
Make two valid comments comparing the number of passengers on each bus on Monday and Saturday. 2

On average, more passengers travelled to the airport on a Monday than a Saturday.

The number of passengers on a Monday were more consistent than a Saturday.

MARKS | DO NOT WRITE IN THIS MARGIN

7. A fishing boat and a yacht left a harbour at the point H.
The fishing boat travelled 3·4 kilometres on a bearing of 047° to the point F.
The yacht travelled 5·7 kilometres on a bearing of 115° to the point Y.



115 − 47 = 68°

Calculate the distance between the fishing boat at F and the yacht at Y. 4

$h^2 = 3.4^2 + 5.7^2 - 2(3.4)(5.7)\cos 68°$

$h^2 = 29.5302...$

$h = \sqrt{29.5302}$

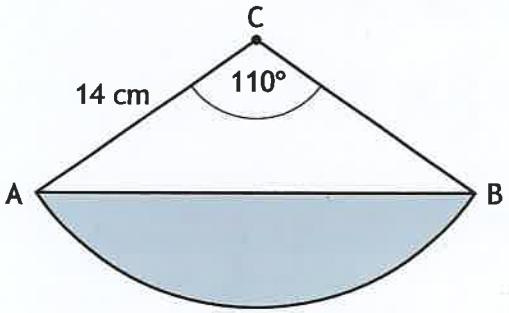
$h = 5.43417...$

Distance FY = 5.4 km (1dp)

MARKS | DO NOT WRITE IN THIS MARGIN

8. The diagram shows a sector of a circle, with centre C and radius 14 centimetres.

Angle ACB is 110°.



AB splits the sector into the shaded segment and triangle ABC.

Find the area of the shaded segment. **5**

$\frac{\theta}{360} = \frac{\text{Sector Area}}{\pi r^2}$

$\text{Sector Area} = \frac{110}{360} \times \pi \times 14^2$

$\text{Sector} = 188.1464934\text{cm}^2$

$\text{Area } \Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 14 \times 14 \times \sin 110$

$\Delta = 92.08987684\text{cm}^2$

Shaded Area = Sector − Triangle

$= 188.1464934 - 92.08987684$

$= 96.0566\ldots$

Area $= 96.06\text{ cm}^2$ (2dp)

MARKS | DO NOT WRITE IN THIS MARGIN

9. A straight line has equation $3x+4y-8=0$.

(a) Find the gradient of the line. 2

$4y = -3x + 8$

$y = -\frac{3}{4}x + 2$

$y = mx + c$

Gradient, $m = -3/4$

(b) State the coordinates of the point where the line crosses the y-axis. 1

y-intercept, $c = 2$

$\therefore$ Point $(0, 2)$

10. Change the subject of the formula $d = \sqrt{\frac{3h}{2}}$ to h. 3

$d^2 = \frac{3h}{2}$

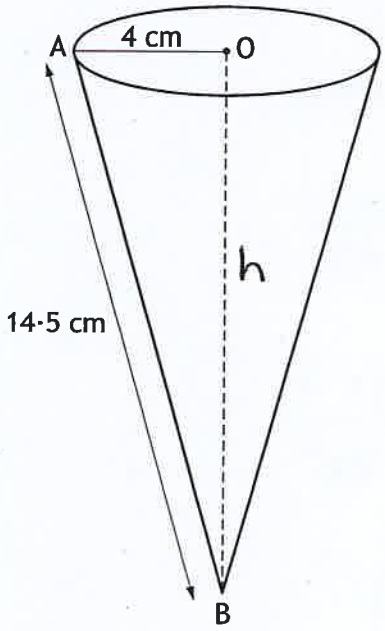
$2d^2 = 3h$

$\frac{2d^2}{3} = h$

$\hookrightarrow \quad h = \frac{2d^2}{3}$

MARKS | DO NOT WRITE IN THIS MARGIN

11. The base of an ice cream cone has centre O and radius 4 centimetres.
The length of AB is 14·5 centimetres.



Calculate the volume of the cone.
Give your answer correct to 2 significant figures. 5

Pythagoras for height

$h^2 = 14.5^2 - 4^2$

$h^2 = 194.25$

$h = \sqrt{194.25}$

$h = 13.93735986\text{ cm}$

Volume of cone

$V_{cone} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 4^2 \times 13.93735...$

$= 233.5227059$

Volume = 230 cm^3 (2sf)

MARKS | DO NOT WRITE IN THIS MARGIN

12. Express

$$\frac{6x}{y} \div \frac{2x^2}{y+5}, \quad x \neq 0, y \neq 0, y \neq -5$$

as a single fraction in its simplest form. 3

$$\frac{6x}{y} \div \frac{2x^2}{y+5}$$

$$= \frac{6x}{y} \times \frac{y+5}{2x^2}$$

$$= \frac{6x(y+5)}{2x^2y}$$

$$= \frac{3(y+5)}{xy}$$

MARKS | DO NOT WRITE IN THIS MARGIN

13. The two photographs shown are mathematically similar.

12 cm

width

The small photograph has an area of 80 square centimetres, and is 12 centimetres wide.

The large photograph has an area of 500 square centimetres.

Calculate the width of the large photograph. 3

Area SF$_E$ = $\frac{\text{big}}{\text{wee}} = \frac{500}{80} = \frac{50}{8} = \frac{25}{4}$

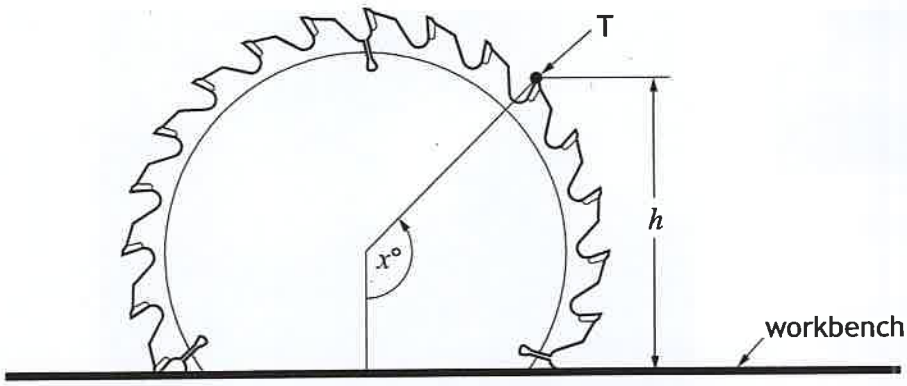
Linear SF$_E$ = $\sqrt{\frac{25}{4}} = \frac{5}{2}$

Width = $\frac{5}{2} \times 12$

Width = 30cm

MARKS | DO NOT WRITE IN THIS MARGIN

14. The diagram shows the part of the blade of a circular saw above a workbench.



As the blade rotates, the height, h millimetres, of point T above the workbench is given by

$$h = 57 - 85\cos x°$$

where x is the angle the blade has turned anti-clockwise from a starting position.

(a) Calculate the value of x when point T is first at a height of 115 millimetres above the workbench. **3**

let $h = 115$mm

$115 = 57 - 85\cos x$

$58 = -85\cos x$

$-\frac{58}{85} = \cos x$

$\cos x = -\frac{2}{3}$

$ra = \cos^{-1}(\frac{2}{3}) = 48.2°$

$x = 180 - ra,\ 180 + ra$

$x = 180 - 48.2,\ 180 + 48.2$

$x = 131.8°,\ 228.2°$

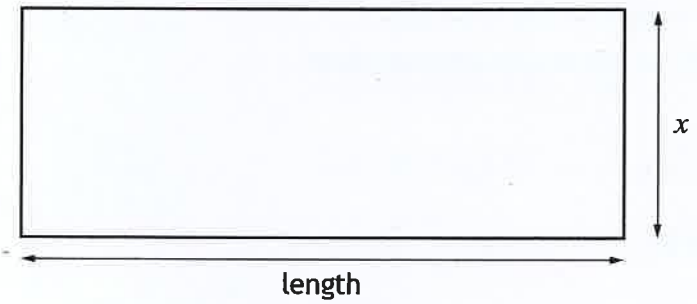
Firstly at $x = 131.8°$

(b) Calculate the value of x when point T is next at this height. **1**

Next at height 115m at $x = 228.2°$

MARKS | DO NOT WRITE IN THIS MARGIN

15. The diagram shows a rectangle with breadth x centimetres.



The length of the rectangle is 5 centimetres more than its breadth.

(a) Write down an expression for its length in terms of x. **1**

length $= x + 5$

The rectangle has an area of 20 square centimetres.

(b) Show that $x^2 + 5x - 20 = 0$. **2**

Area $= l \times b$

$\therefore \quad (x+5)(x) = 20$

$x^2 + 5x = 20$

$x^2 + 5x - 20 = 0$ as required

MARKS | DO NOT WRITE IN THIS MARGIN

15. (continued)

(c) Calculate x, the breadth of the rectangle.

Give your answer correct to one decimal place. 4

$x^2 + 5x - 20 = 0$

$ax^2 + bx + c = 0$

$a = 1$
$b = 5$
$c = -20$

$b^2 - 4ac = (5)^2 - 4(1)(-20)$
$= 25 + 80$
$= 105.$

$$x = \frac{-5 \pm \sqrt{105}}{2}$$

$$x = \frac{-5 - \sqrt{105}}{2} \quad \text{or} \quad x = \frac{-5 + \sqrt{105}}{2}$$

$x = -7.623475...$ $x = 2.623475...$

$x = -7.6, 2.6$ (1dp)

but $x > 0$ for breadth

↳ $x = 2.6$ cm only.

MARKS | DO NOT WRITE IN THIS MARGIN

16. Expand and simplify

$$\cos x^\circ \left(\tan x^\circ + 1\right).$$

2

Show your working.

$\cos x \left(\tan x + 1\right)$

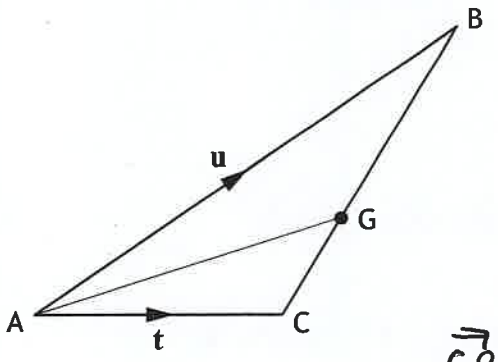
$\cos x \tan x + \cos x.$

$\cos x \left(\frac{\sin x}{\cos x}\right) + \cos x$

$\sin x + \cos x.$

MARKS | DO NOT WRITE IN THIS MARGIN

17. The triangle ABC is shown below



$\overrightarrow{AB} = \mathbf{u}$ and $\overrightarrow{AC} = \mathbf{t}$.

G is the point such that $CG = \frac{1}{3}CB$.

Express $\overrightarrow{AG}$ in terms of $\mathbf{u}$ and $\mathbf{t}$.

Give your answer in simplest form. **3**

$\vec{CB} = \vec{CA} + \vec{AB}$

$\vec{CB} = -t + u$

$\vec{CB} = u - t$

$\vec{AG} = \vec{AC} + \vec{CG}$

$= \vec{AC} + \frac{1}{3}\vec{CB}$

$= t + \frac{1}{3}(u - t)$

$= t + \frac{1}{3}u - \frac{1}{3}t$

$\vec{AG} = \frac{2}{3}t + \frac{1}{3}u$ or $\frac{1}{3}(u + 2t)$

[END OF QUESTION PAPER]