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N5

National Qualifications 2019

Mark

X847/75/02

Mathematics Paper 2

FRIDAY, 3 MAY

10:45 AM – 12:35 PM

X8477502

Fill in these boxes and read what is printed below.

Full name of centre: WORKED SOLUTIONS

Town: HL

Forename(s)

Surname

Number of seat

Date of birth

Day | Month | Year

Scottish candidate number

Total marks — 60

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

X8477502 01

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle $A = \frac{1}{2}ab\sin C$

Volume of a sphere $V = \frac{4}{3}\pi r^3$

Volume of a cone $V = \frac{1}{3}\pi r^2 h$

Volume of a pyramid $V = \frac{1}{3}Ah$

Standard deviation $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n - 1}}$

or $s = \sqrt{\dfrac{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}{n - 1}}$, where n is the sample size.

MARKS | DO NOT WRITE IN THIS MARGIN

Total marks — 60

Attempt ALL questions

1. A charity distributed 80 000 emergency packages during 2018.

   This number is expected to increase by 15% each year.

   Calculate how many emergency packages the charity expects to distribute in 2021. **3**

2018 → 2021 = 3 years

100% + 15% = 115% = 1.15.

80000×1.15^3

121,670 packages in 2021.

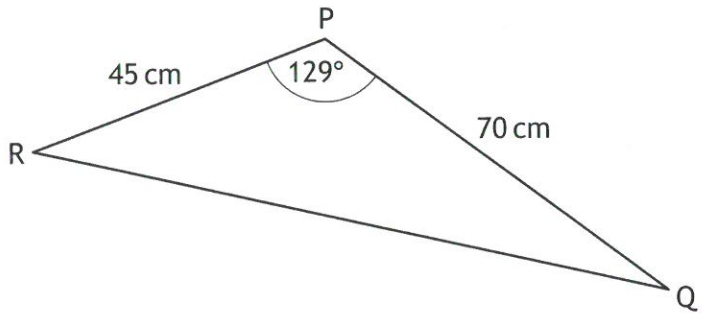
2. Find $|\mathbf{p}|$, the magnitude of vector $\mathbf{p} = \begin{pmatrix} 6 \\ 27 \\ -18 \end{pmatrix}$. **2**

$|p| = \sqrt{6^2 + 27^2 + (-18)^2}$

$= \sqrt{1089}$

$|p| = 33$ units

3. The diagram shows triangle PQR.



- PR = 45 centimetres
- PQ = 70 centimetres
- Angle QPR = 129°

Calculate the area of triangle PQR. **2**

Area = $\frac{1}{2} \times 45 \times 70 \times \sin 129$

= 1224.004889

Area = 1224 cm^2

4. A sesame seed weighs $3{\cdot}6 \times 10^{-6}$ kilograms.
The weight of a poppy seed is 8% of the weight of a sesame seed.
Calculate the weight of a poppy seed in kilograms.
Give your answer in scientific notation. **2**

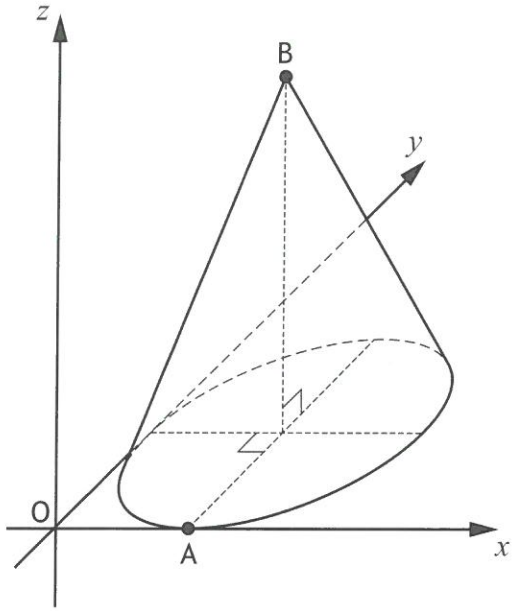
8% of 3.6×10^{-6}

$\frac{8}{100} \times 3.6 \times 10^{-6}$

2.88×10^{-7} kg.

MARKS | DO NOT WRITE IN THIS MARGIN

5. The diagram shows a cone with diameter 6 units and height 8 units.



- The x-axis and the y-axis are tangents to the base
- A is the point of contact between the base and the x-axis
- B is directly above the centre of the base

Write down the coordinates of A and B. **2**

A (3, 0, 0)

B (3, 3, 8)

MARKS | DO NOT WRITE IN THIS MARGIN

6. Solve the equation $3x^2 + 9x - 2 = 0$.

Give your answers correct to 1 decimal place. **3**

$$ax^2 + bx + c = 0$$

$a = 3$

$b = 9$

$c = -2$

$b^2 - 4ac$

$9^2 - 4(3)(-2)$

$81 + 24$

105

$$x = \frac{-9 \pm \sqrt{105}}{6}$$

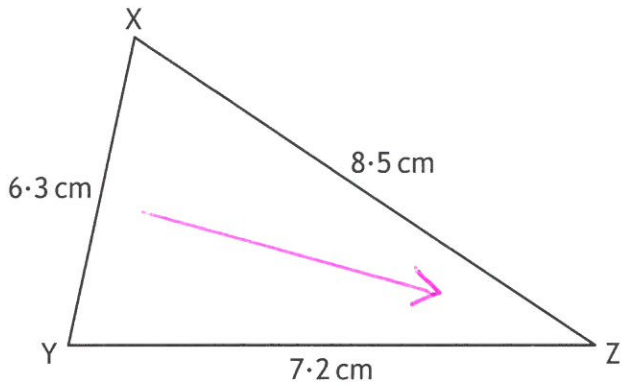
$$x = \frac{-9 + \sqrt{105}}{6} \qquad x = \frac{-9 - \sqrt{105}}{6}$$

$$x = 0.2078... \qquad x = -3.2078...$$

$$x = -3.2, \ 0.2 \quad (1dp)$$

MARKS | DO NOT WRITE IN THIS MARGIN

7. Triangle XYZ is shown below.



smallest side

smallest angle.

Calculate the size of the smallest angle in triangle XYZ. **3**

$$\cos Z = \frac{7.2^2 + 8.5^2 - 6.3^2}{2(7.2)(8.5)}$$

$$\cos Z = \frac{211}{306}$$

$$Z = \cos^{-1}\left(\frac{211}{306}\right)$$

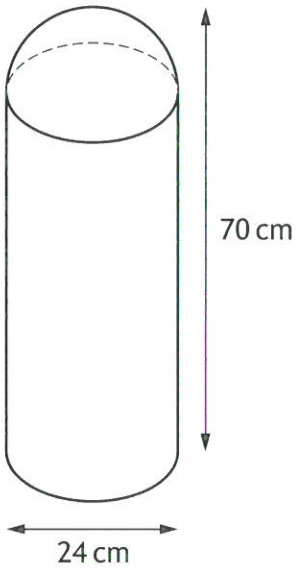
$$Z = 46.40609661$$

Angle XYZ = 46.4° (1dp)

8. A traffic bollard is in the shape of a cylinder with a hemisphere on top.

The bollard has

- diameter 24 centimetres
- height 70 centimetres.



radius = 12cm.

Calculate the volume of the bollard.

Give your answer correct to 3 significant figures. **5**

Cylinder

$h = 70 - 12 = 58\text{cm}$

$V_{cyl} = \pi r^2 h$

$= \pi \times 12^2 \times 58$

$= 8352\pi$

$= 26,238.5818...$

Hemi-sphere

$V_{HS} = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 12^3$

$= 1152\pi$

$= 3619.1147...$

Total Volume $= 26238.5818 + 3619.1147$

$= 29857.6965...$

$= 29,900\ \text{cm}^3$. (3sf)

MARKS | DO NOT WRITE IN THIS MARGIN

9. Georgie had her roof repaired.

She was charged an extra 2·5% for late payment.

She had to pay a total of £977·85.

Calculate how much she would have **saved** if she had paid on time. 3

$100\% + 2.5\% = 102.5\%$

$102.5\% = £977.85$

$1\% = \frac{977.85}{102.5} = 9.54$

$100\% = 9.54 \times 100 = £954$

Saving $= £977.85 - £954$

$= £23.85$

10. Express $x^2 + 10x - 15$ in the form $(x+p)^2 + q$. 2

$[x^2 + 10x] - 15$

$(x+5)^2 - 25 - 15$

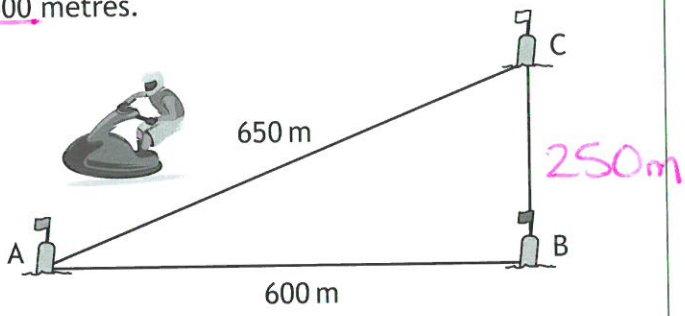
$(x+5)^2 - 40$

11. The diagram shows the course for a jet-ski race.

The course is indicated by markers A, B and C.

The total length of the course is 1500 metres.

- B is 600 metres from A
- C is 650 metres from A
- C is due north of B



1000 − 600 − 650

BC = 250 m

Determine whether B is due east of A.

Justify your answer. 4

By Converse of Pythagoras' Theorem,

$c^2 = a^2 + b^2$ for any right-angled Δ.

$LHS = c^2$

$= 650^2$

$LHS = 422,500$

$RHS = a^2 + b^2$

$= 600^2 + 250^2$

$RHS = 422,500$

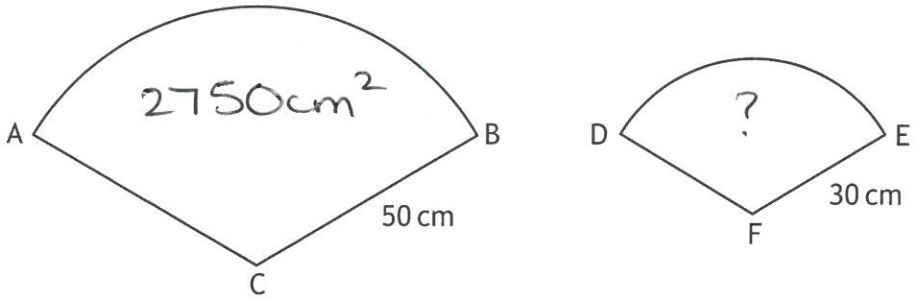
since $650^2 = 600^2 + 250^2$,

then triangle ABC is right-angled at B, and therefore A is due West of B, meaning B is due East of A. ☺

MARKS | DO NOT WRITE IN THIS MARGIN

12. In the diagram

- ABC is a sector of a circle, centre C
- DEF is a sector of a circle, centre F.



The sectors are mathematically similar.

The area of the larger sector, ABC, is 2750 square centimetres.

(a) Calculate the area of the smaller sector, DEF. **3**

$$\text{Linear SF}_R = \frac{30}{50} = \frac{3}{5}$$

$$\text{Small Area} = \left(\frac{3}{5}\right)^2 \times 2750$$

$$= 990\,\text{cm}^2$$

(b) Calculate the size of angle ACB. **3**

$$\frac{\theta}{360} = \frac{\text{Sector Area}}{\pi r^2}$$

$$\frac{\theta}{360} = \frac{2750}{\pi \times 50^2}$$

$$\theta = \frac{2750 \times 360}{2500\pi}$$

$$\theta = 126.0507149$$

$$\text{Angle ACB} = 126.1^\circ \quad (1\text{dp})$$

13. Find an expression for the gradient of the line joining point A(6,9) to point B($4p$,$4p^2$).

Give your answer in its simplest form. **3**

$$m = \frac{4p^2 - 9}{4p - 6} = \frac{(2p-3)(2p+3)}{2(2p-3)}$$

$$m = \frac{2p+3}{2}$$

$$m = \frac{1}{2}(2p+3)$$

$$m = p + 3/2$$

14. Solve the equation $5\cos x° + 2 = 1$, $0 \leq x < 360$. **3**

$$5\cos x + 2 = 1$$
$$5\cos x = -1$$
$$\cos x = -\frac{1}{5}$$

S	A
T	C

$$ra = \cos^{-1}\left(\tfrac{1}{5}\right) = 78.5° \text{ (1dp)}$$

$$x = 180 - 78.5,\ 180 + 78.5$$

$$x = 101.5°,\ 258.5°$$

MARKS | DO NOT WRITE IN THIS MARGIN

15. Express

$$\frac{4}{x-2}-\frac{3}{x+5}, \quad x \neq 2, x \neq -5$$

as a single fraction in its simplest form. **3**

$$\frac{4(x+5) - 3(x-2)}{(x-2)(x+5)}$$

$$\frac{4x + 20 - 3x + 6}{(x-2)(x+5)}$$

$$\frac{x + 26}{(x-2)(x+5)}$$

16. Simplify $\frac{a^4 \times 3a}{\sqrt{a}}$. **3**

$$\frac{a^4 \times 3a}{\sqrt{a}} = \frac{3a^5}{a^{1/2}} = 3a^{9/2}$$

$$3a^{9/2} = 3\sqrt{a^9}$$

17. Expand and simplify

$$(\sin x° + \cos x°)^2.$$

Show your working. 2

$(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x$

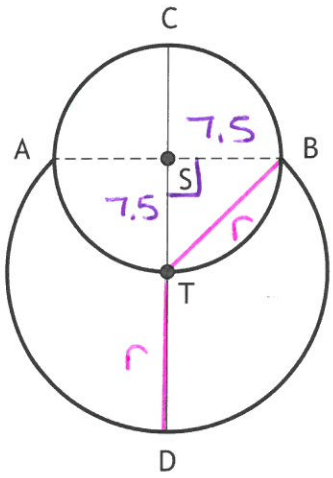
$\sin^2 x + \cos^2 x + 2 \sin x \cos x$

$\sin^2 x + \cos^2 x = 1$

$1 + 2 \sin x \cos x$

18. The picture shows a cartoon snowman.

The diagram below represents the snowman.



small radius
7.5cm

large radius
r

- The head is a small circle, centre S, with diameter 15 centimetres
- The body is part of a larger circle, centre T
- The point T lies on the circumference of the small circle
- The points A and B lie on the circumferences of both circles

Calculate CD, the height of the snowman. **4**

$r^2 = 7.5^2 + 7.5^2$

$r^2 = 112.5$

$r = \sqrt{112.5}$

$r = 10.6066$

$r = 10.6$ cm (1dp)

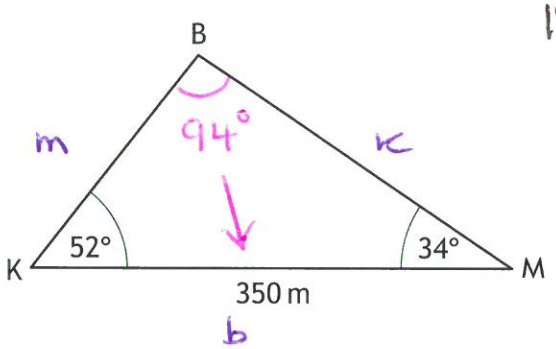
$CD = 15 + r$

$= 15 + 10.6$

$CD = 25.6$cm

19. Katy and Mona are looking up at a hot-air balloon.

In the diagram below, K, M and B represent the positions of Katy, Mona and the balloon respectively.



180 − 52 − 34
94°.

- The angle of elevation of the balloon from Katy is 52°
- The angle of elevation of the balloon from Mona is 34°
- Katy and Mona are 350 metres apart on level ground

Calculate the height of the hot-air balloon above the ground. **5**

Must calculate KB or BM first.

Sine Rule

$$\frac{b}{\sin B} = \frac{k}{\sin K} = \frac{m}{\sin M}$$

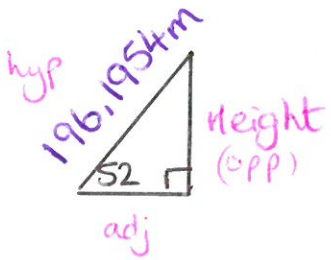
$$\frac{350}{\sin 94} = \frac{m}{\sin 34}$$

$$m = \frac{350 \sin 34}{\sin 94}$$

$$m = 196.1954\ldots$$

KB = 196.1954 m

SOH CAH TOA.



$$\sin 52 = \frac{H}{196.1954}$$

$$H = 196.1954 \sin 52$$

$$H = 154.60411\ldots$$

Height 154.6 m

[END OF QUESTION PAPER]