

FOR OFFICIAL USE



National
Qualifications
2018

Mark

X847/75/01

**Mathematics
Paper 1 (Non-Calculator)**

FRIDAY, 4 MAY
9:00 AM – 10:15 AM



Fill in these boxes and read what is printed below.

Full name of centre

Town

WORKED SOLUTIONS

Forename(s)

Surname

Number of seat

Date of birth

Day

Month

Year

Scottish candidate number

Total marks — 50

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use blue or black ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



Total marks — 50
Attempt ALL questions

1. Evaluate $2\frac{1}{3} + \frac{4}{5}$.

$$\frac{7}{3} + \frac{4}{5}$$

$$\frac{35}{15} + \frac{12}{15}$$

$$\frac{47}{15} = \underline{\underline{3\frac{2}{15}}}$$

2

2. Expand and simplify $(3x+1)(x-1) + 2(x^2-5)$.

$$3x^2 - 3x + x - 1 + 2x^2 - 10$$

$$\underline{\underline{5x^2 - 2x - 11}}$$

3



3. Solve, algebraically, the system of equations

$$\begin{aligned} 4x + 5y &= -3 && \textcircled{\times 2} \\ 6x - 2y &= 5 && \textcircled{\times 5} \end{aligned}$$

3

$$\begin{aligned} 8x + 10y &= -6 \\ \textcircled{+} \quad 30x - 10y &= 25 \\ \hline 38x &= 19 \\ x &= \frac{19}{38} \\ x &= \underline{\underline{\frac{1}{2}}} \end{aligned}$$

$$\begin{aligned} 4x + 5y &= -3 \\ 4\left(\frac{1}{2}\right) + 5y &= -3 \\ 2 + 5y &= -3 \\ 5y &= -5 \\ y &= \underline{\underline{-1}} \end{aligned}$$

4. Two vectors are given by $\mathbf{u} = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$ and $\mathbf{u} + \mathbf{v} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix}$.

Find vector \mathbf{v} .

Express your answer in component form.

2

$$\mathbf{v} = \begin{pmatrix} 6 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 \\ -9 \\ 2 \end{pmatrix}}}$$



5. Solve

$$x^2 - 11x + 24 = 0.$$

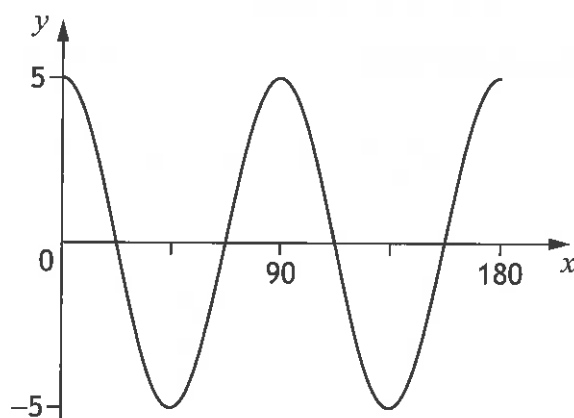
$$(x - 3)(x - 8) = 0$$

$$x - 3 = 0 \text{ or } x - 8 = 0$$

$$\underline{x = 3 \text{ or } x = 8}$$

2

6. Part of the graph of $y = a \cos bx^\circ$ is shown in the diagram.



State the values of a and b .

2

$$a = \text{Amplitude} = 5$$

$$\text{Period} = 90^\circ$$

$$b = 4$$

$$y = 5 \cos 4x^\circ$$

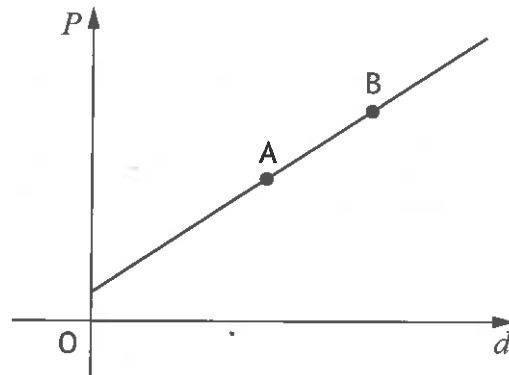
$$\underline{a = 5 \text{ and } b = 4}$$



* X 8 4 7 7 5 0 1 0 5 *

7. The cost of a journey with Tom's Taxis depends on the distance travelled.

The graph below shows the cost, P pounds, of a journey with Tom's Taxis against the distance travelled, d miles.



Point A represents a journey of 8 miles which costs £14.

Point B represents a journey of 12 miles which costs £20.

(a) Find the equation of the line in terms of P and d .

Give the equation in its simplest form.

3

$$A(8, 14) \quad B(12, 20)$$

$$m = \frac{20 - 14}{12 - 8} = \frac{6}{4} = \frac{3}{2}$$

$$y - b = m(x - a)$$

$$y - 14 = \frac{3}{2}(x - 8)$$

$$y - 14 = \frac{3}{2}x - 12$$

$$y = \frac{3}{2}x + 2$$

$$P = \frac{3}{2}d + 2$$



7. (continued)

(b) Calculate the cost of a journey of 5 miles.

1

$$d = 5$$

$$P = \frac{3}{2}(5) + 2$$

$$= 7.5 + 2$$

$$= 9.5$$

$$\underline{\text{Cost} = \pounds 9.50}$$

8. Determine the nature of the roots of the function $f(x) = 2x^2 + 4x + 5$.

2

Let $2x^2 + 4x + 5 = 0$ for roots.

$$a = 2$$

$$b = 4$$

$$c = 5$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= 4^2 - 4(2)(5)$$

$$= 16 - 40$$

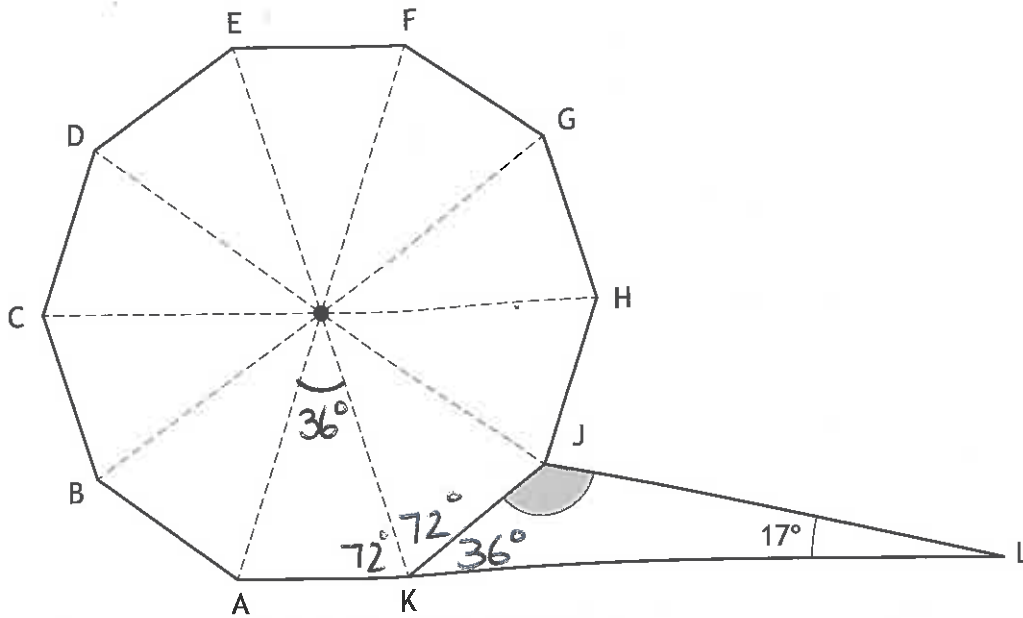
$$= -24$$

Since $b^2 - 4ac < 0$, there are
no real roots for function $f(x)$.



9. In the diagram shown below, ABCDEFGHJK is a regular decagon.

- Angle KLJ is 17° .
- AKL is a straight line.



Calculate the size of shaded angle KJL.

2

$$360^\circ \div 10 = 36^\circ$$

$$180 - 36 = 144^\circ$$

$$144 \div 2 = 72^\circ$$

$$180 - (72 + 72) = 36^\circ$$

$$180 - (36 + 17)$$

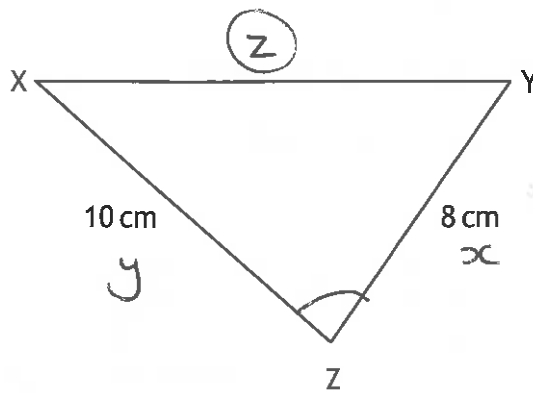
$$180 - 53 = 127^\circ$$

Shaded angle = 127°



10. In triangle XYZ:

- $XZ = 10$ centimetres
- $YZ = 8$ centimetres
- $\cos Z = \frac{1}{8}$.



Calculate the length of XY.

3

$$Z^2 = 8^2 + 10^2 - 2(8)(10)\left(\frac{1}{8}\right)$$

$$= 64 + 100 - 20$$

$$Z^2 = 144$$

$$Z = \sqrt{144}$$

$$Z = 12$$

Length XY = 12 cm



11. Express $\frac{9}{\sqrt{6}}$ with a rational denominator.

Give your answer in its simplest form.

2

$$\frac{9}{\sqrt{6}} = \frac{9\sqrt{6}}{6} = \underline{\underline{\frac{3\sqrt{6}}{2}}}$$

12. Given that $\cos 60^\circ = 0.5$, state the value of $\cos 240^\circ$.

1

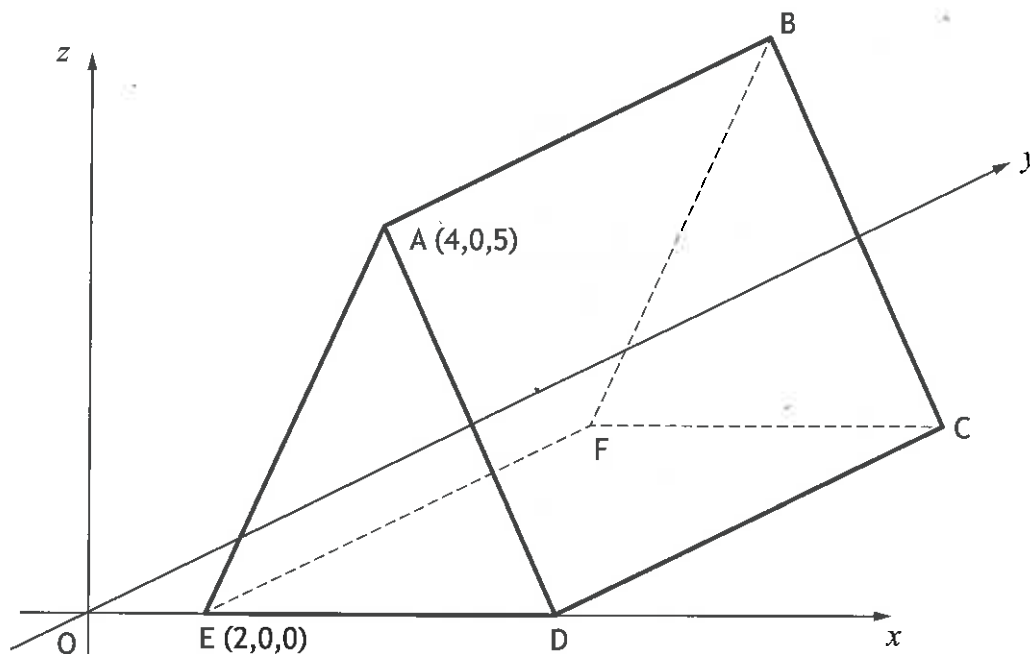


$$\cos 240^\circ = -\cos 60^\circ$$

$$\underline{\underline{\cos 240^\circ = -0.5}}$$



13. The diagram shows a triangular prism, ABCDEF, relative to the coordinate axes.



- $AD = AE$.
- $DC = 8$ units.
- Edges EF , DC and AB are parallel to the y -axis.

Write down the coordinates of B and C.

2

$B(4, 8, 5)$ and $C(6, 8, 0)$



* X 8 4 7 7 5 0 1 1 1 *

14. Change the subject of the formula $y = g\sqrt{x+h}$ to x .

3

$$\begin{aligned}g\sqrt{x} + h &= y \\g\sqrt{x} &= y - h \\\sqrt{x} &= \frac{y - h}{g} \\x &= \left(\frac{y - h}{g}\right)^2\end{aligned}$$

15. Remove the brackets and simplify $\left(\frac{2}{3}p^4\right)^2$.

2

$$\left(\frac{2}{3}p^4\right)^2 = \frac{4}{9}p^8$$



16. Sketch the graph of $y = (x-6)(x+4)$.

On your sketch, show clearly the points of intersection with the x -axis and the y -axis, and the coordinates of the turning point.

3

Roots: $(x-6)(x+4) = 0$

$$x-6=0 \text{ or } x+4=0$$

$$x=6 \qquad \qquad \qquad x=-4.$$

Roots are $(-4, 0)$ and $(6, 0)$

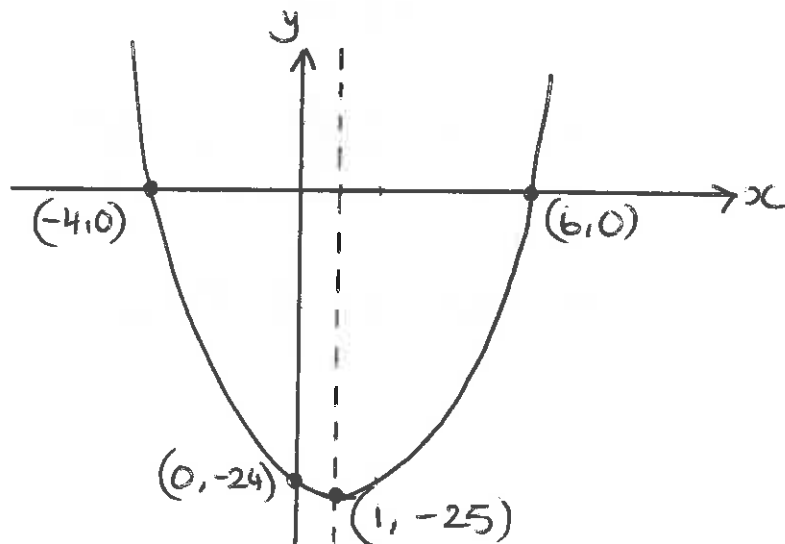
Axis of symmetry occurs at $x = 1$

Turning Point $y = (1-6)(1+4)$
 $y = (-5)(5) = -25$

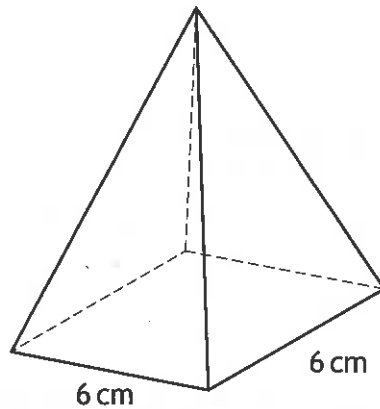
Minimum TP $(1, -25)$

y -intercept @ $x = 0$
 $y = (0-6)(0+4) = -24$
 y -intercept $(0, -24)$

Sketch



17. A square based pyramid is shown in the diagram below.



The square base has length 6 centimetres.

The volume is 138 cubic centimetres.

Calculate the height of the pyramid.

3

$$V_{\text{pyramid}} = \frac{1}{3} A_{\text{base}} H$$

$$A_{\text{base}} = 6 \times 6 = 36 \text{ cm}^2$$

$$\hookrightarrow V = \frac{1}{3} A \cdot H$$

$$138 = \frac{1}{3} (36) \times H$$

$$138 = 12 H$$

$$H = \frac{138}{12} = \frac{69}{6} = 11 \frac{3}{6}$$

$$\underline{\text{Height} = 11.5 \text{ cm}}$$



18. Express $\sin x^\circ \cos x^\circ \tan x^\circ$ in its simplest form.
Show your working.

2

$$\sin x^\circ \cos x^\circ \tan x^\circ$$

$$\text{but } \tan x = \frac{\sin x}{\cos x}$$

$$\sin x^\circ \cos x^\circ \frac{\sin x^\circ}{\cos x^\circ}$$

$$\rightarrow \underline{\underline{\sin^2 x}}$$



19. (a) (i) Express $x^2 - 6x - 81$ in the form $(x - p)^2 + q$.

2

$$[x^2 - 6x] - 81$$

$$(x - 3)^2 - 3^2 - 81$$

$$(x - 3)^2 - 9 - 81$$

$$\underline{\underline{(x - 3)^2 - 90}}$$

(ii) Hence state the equation of the axis of symmetry of the graph of $y = x^2 - 6x - 81$.

1

Axis of symmetry

$$\underline{\underline{x = 3}}$$



19. (continued)

(b) The roots of the equation $x^2 - 6x - 81 = 0$ can be expressed in the form $x = d \pm d\sqrt{e}$.

Find, algebraically, the values of d and e .

4

$$a = 1$$

$$b = -6$$

$$c = -81$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} \text{but } b^2 - 4ac &= (-6)^2 - 4(1)(-81) \\ &= 36 + 324 \\ &= 360 \end{aligned}$$

$$x = \frac{6 \pm \sqrt{360}}{2}$$

$$\begin{aligned} &\sqrt{360} \\ &= \sqrt{36} \sqrt{10} \\ &= 6\sqrt{10} \end{aligned}$$

$$x = \frac{6 \pm 6\sqrt{10}}{2}$$

$$\underline{x = 3 \pm 3\sqrt{10}}$$

$$\underline{d = 3 \text{ and } e = 10}$$

[END OF QUESTION PAPER]



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