FOR OFFICIAL USE

N5

National Qualifications 2017

Mark

X747/75/02

Mathematics Paper 2

FRIDAY, 5 MAY

2:20 PM – 3:50 PM

X7477502

Fill in these boxes and read what is printed below.

Full name of centre

WORKED SOLUTIONS

Town

Forename(s)

Surname

Number of seat

Date of birth

Day Month Year

Scottish candidate number

Total marks — 50

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give this book to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA

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FORMULAE LIST

The roots of $ax^2+bx+c=0$ are $x=\dfrac{-b\pm\sqrt{(b^2-4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Cosine rule: $a^2=b^2+c^2-2bc\cos A$ or $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

Area of a triangle: $A=\frac{1}{2}ab\sin C$

Volume of a sphere: $V=\frac{4}{3}\pi r^3$

Volume of a cone: $V=\frac{1}{3}\pi r^2 h$

Volume of a pyramid: $V=\frac{1}{3}Ah$

Standard deviation: $s=\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n-1}}$

or $s=\sqrt{\dfrac{\Sigma x^2-\dfrac{(\Sigma x)^2}{n}}{n-1}}$, where n is the sample size.

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Total marks — 50

Attempt ALL questions

1. Find $|\mathbf{v}|$, the magnitude of vector $\mathbf{v} = \begin{pmatrix} 18 \\ -14 \\ 3 \end{pmatrix}$. 2

$$|v| = \sqrt{18^2 + (-14)^2 + 3^2}$$

$$|v| = \sqrt{529}$$

$$|v| = 23 \text{ units}$$

2. A necklace is valued at £1200.
Its value is expected to increase by 4·5% per year over the next 3 years.
Calculate the expected value of the necklace after this time.
Give your answer to the nearest pound. 3

$$100\% + 4.5\% = 104.5\% = 1.045$$

$$\text{Expected value} = £1200 \times 1.045^3$$

$$= 1369.39935$$

Expected value = £1369 (nearest pound)

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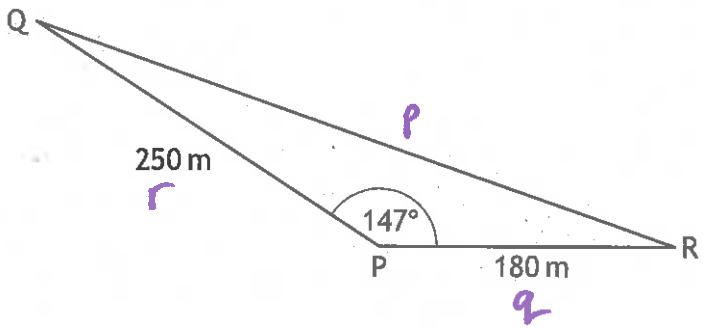
3. A piece of land is in the shape of a triangle as shown.



- PQ = 250 metres
- PR = 180 metres
- angle QPR = 147°

The owner wishes to build a fence along the side QR.

Calculate the length of the fence. 3

$p^2 = q^2 + r^2 - 2qr\cos P$

$p^2 = 180^2 + 250^2 - 2(180)(250)\cos 147°$

$p^2 = 170380.3511$

$p = \sqrt{ANS}$

$p = 412.77154483$

Length QR = 412.77m (2dp)

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4. Solve the equation $2x^2+5x-4=0$.

Give your answers correct to one decimal place. 3

Given $ax^2+bx+c=0$

$a=2$
$b=5$
$c=-4$

Discriminant $= b^2-4ac$
$= 5^2-4(2)(-4)$
$= 25+32$
$= 57$

$$x = \frac{-5 \pm \sqrt{57}}{4}$$

$x = \frac{-5-\sqrt{57}}{4}$ OR $x = \frac{-5+\sqrt{57}}{4}$

$x = -3.13745...$ $x = 0.63745...$

$x = -3.1, 0.6$ (1dp)

5. A theatre group sold 4830 tickets for their show.

This was 15% more than they sold last year.

How many tickets did they sell last year? 3

$100\% + 15\% = 115\%$

$115\% = 4830$

$1\% = \frac{4830}{115} = 42$

$100\% = 42 \times 100 = 4200$

Last year, they sold 4200 tickets

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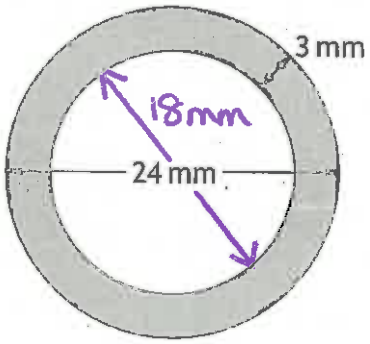
6. A spherical sweet is made by coating a caramel sphere evenly with chocolate.

A cross-section of the sweet is shown below.



The diameter of the sweet is 24 millimetres and the thickness of the chocolate coating is 3 millimetres.

Calculate the volume of the chocolate coating. 5

Give your answer correct to 3 significant figures.

OUTER SPHERE

$d = 24\text{mm}$

$r = 12\text{mm}$

$V_{out} = \frac{4}{3} \times \pi \times 12^3$

$= 2304\pi$

$= 7238.229474$

INNER SPHERE

$d = 24 - 3 - 3 = 18\text{mm}$

$r = 9\text{mm}$

$V_{IN} = \frac{4}{3} \times \pi \times 9^3$

$= 972\pi$

$= 3053.628059$

Volume of chocolate $= V_{out} - V_{in}$

$= 2304\pi - 972\pi$

$= 1332\pi$

$= 4184.601415$

Volume of chocolate $= 4180\ \text{mm}^3$ (3sf)

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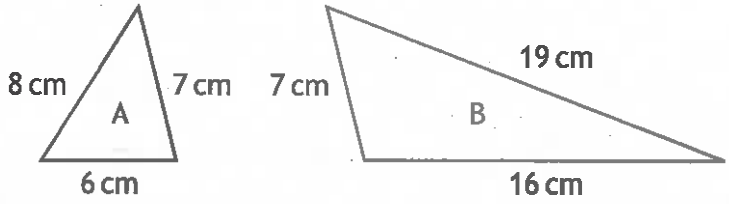
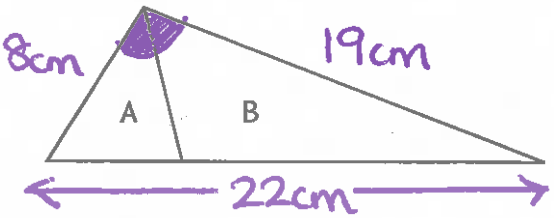
7. Triangles A and B are shown below.



The triangles are placed together to form the larger triangle shown below.



Is this larger triangle right-angled? 3

Justify your answer.

By Converse of Pythagoras' Theorem,
$c^2 = a^2 + b^2$ for any right angled triangle.

Let $a = 8$, $b = 19$ and $c = 22$.

$c^2 = 22^2$ $\qquad$ $a^2 + b^2 = 8^2 + 19^2$

$c^2 = 484$ $\qquad$ $a^2 + b^2 = 425$

Since $22^2 \neq 8^2 + 19^2$, then
the larger triangle is not right angled.

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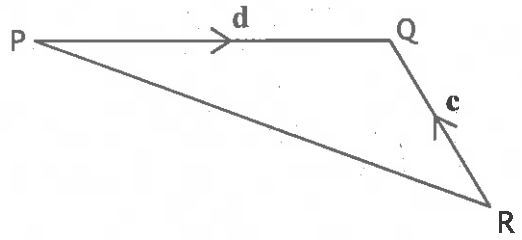
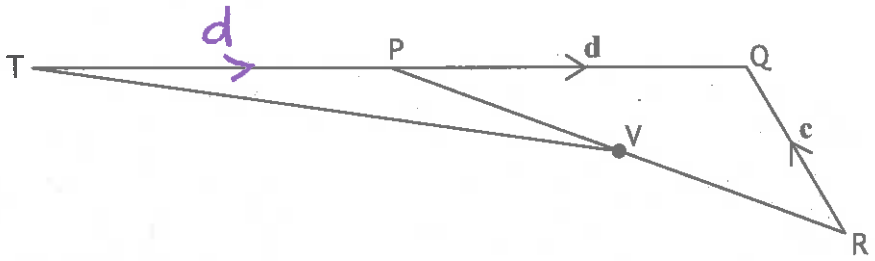
8. In the diagram below, $\overrightarrow{RQ}$ and $\overrightarrow{PQ}$ represent the vectors **c** and **d** respectively.



(a) Express $\overrightarrow{PR}$ in terms of **c** and **d**. 1

$\overrightarrow{PR} = d - c$

The line QP is extended to T.



- TP = PQ
- V is the midpoint of PR

(b) Express $\overrightarrow{TV}$ in terms of **c** and **d**.
Give your answer in simplest form. 2

$\overrightarrow{TV} = \overrightarrow{TP} + \frac{1}{2}\overrightarrow{PR}$

$= d + \frac{1}{2}(d - c)$

$= d + \frac{1}{2}d - \frac{1}{2}c$

$\overrightarrow{TV} = \frac{3}{2}d - \frac{1}{2}c$

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9. (a) Factorise $4x^2 - 25$. 1

$(2x - 5)(2x + 5)$

(b) Hence simplify $\frac{4x^2 - 25}{2x^2 - x - 10}$. 3

Factorise $2x^2 - x - 10$

$(2x - 5)(x + 2)$

$\frac{(2x - 5)(2x + 5)}{(2x - 5)(x + 2)} = \frac{2x + 5}{x + 2}$

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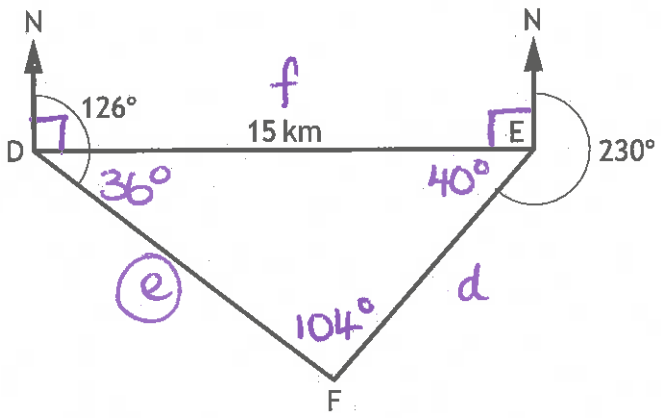
10. In the diagram below D, E and F represent the positions of Dunbridge, Earlsford and Fairtown respectively.



Dunbridge is 15 kilometres west of Earlsford.

From Dunbridge, the bearing of Fairtown is 126°.

From Earlsford the bearing of Fairtown is 230°.

Calculate the distance between Dunbridge and Fairtown. 4

Do not use a scale drawing.

$\angle EDF = 126 - 90 = 36°$ $\quad \angle DEF = 270 - 230 = 40°$.

$\angle DFE = 180 - 36 - 40 = 104°$.

$$\frac{d}{\sin D} = \frac{e}{\sin E} = \frac{f}{\sin F}$$

$$\frac{e}{\sin 40} = \frac{15}{\sin 104}$$

$$e = \frac{15 \sin 40}{\sin 104}$$

$$e = 9.93698507$$

Distance DF = 9.94 km (2dp)

MARKS | DO NOT WRITE IN THIS MARGIN

11. A straight line has equation $3x-5y-10=0$.

Find the gradient of this line. 2

$$3x - 10 = 5y$$

$$y = \frac{3}{5}x - 2$$

In the form $y = mx + c$

$$m = \frac{3}{5} \qquad c = -2$$

Gradient $= \frac{3}{5}$

12. Express $\frac{1}{\sqrt[3]{x}}$ in the form x^n. 2

$$\frac{1}{\sqrt[3]{x}} = \frac{1}{x^{1/3}} = x^{-1/3}$$

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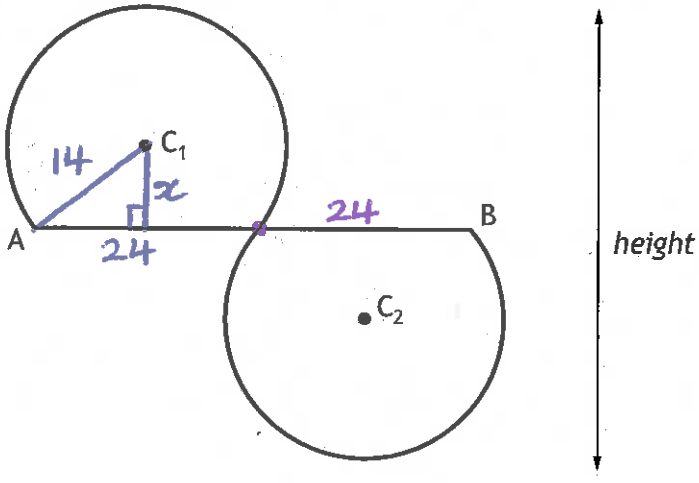
13. Two identical shapes are used to form a logo.
Each shape is part of a circle.



- The circles have centres C_1 and C_2.
- The radius of each circle is 14 centimetres.
- The logo has half-turn symmetry about the mid-point of AB.
- AB is 48 centimetres long.

Calculate the height of the logo. 4

$x^2 = 14^2 - 12^2$

$x^2 = 52$

$x = \sqrt{52}$

$x = 7.2111...$

$x = 7.21$ cm (2dp)

Height $= 2 \times (x + r)$

$= 2 \times (7.21 + 14)$

$= 2 \times 21.21$

Height $= 42.42$ cm (2dp)

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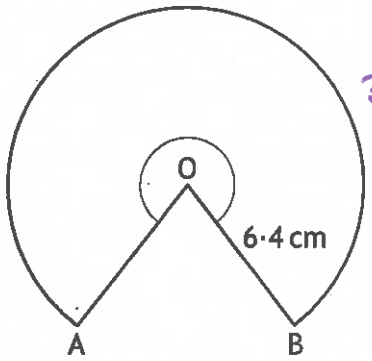
14. The diagram below shows part of a circle, centre O.



31.5cm

The radius of the circle is 6·4 centimetres.
Major arc AB has length 31·5 centimetres.
Calculate the size of the reflex angle AOB. 3

$r = 6.4$ cm
$d = 12.8$ cm

$$\frac{\theta}{360^\circ} = \frac{\text{Arc length}}{\pi d}$$

$$\frac{\theta}{360} = \frac{31.5}{12.8\pi}$$

$$\theta = \frac{31.5}{12.8\pi} \times 360$$

$$\theta = 282.00266\ldots$$

Reflex angle AOB = 282° (nearest degree)

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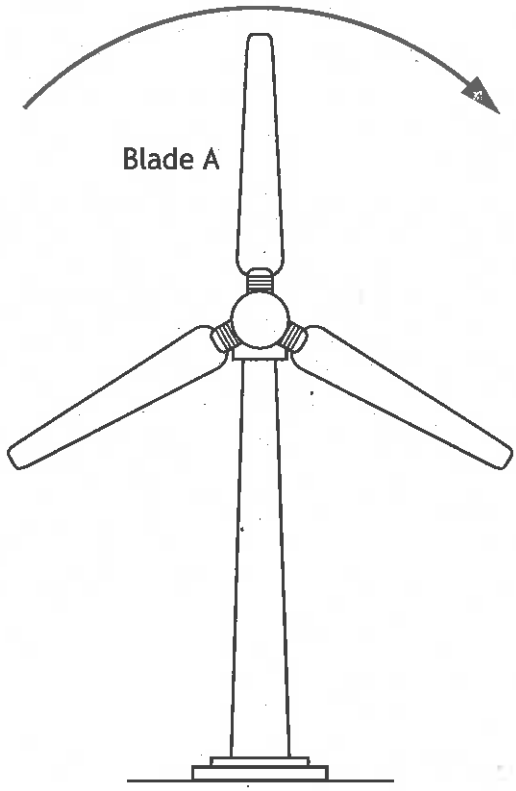
15. A wind turbine has three blades as shown below.



The height, h metres, of the tip of blade A above the ground in each rotation is given by

$$h = 40 + 23\cos x°, \qquad 0 \leq x < 360$$

where x is the angle blade A has turned clockwise from its vertical position.

(a) Calculate the height of the tip of blade A after it has turned through an angle of 60°. **1**

let $x = 60°$

$h = 40 + 23\cos 60°$

$h = 51.5\text{m}$

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15. (continued)

(b) Find the minimum height of the tip of blade A above the ground. 1

Minimum height when A is at bottom,

let $x = 180°$.

$$h = 40 + 23\cos 180°$$

$$h = 17\text{ m}$$

(c) Calculate the values of x for which the tip of blade A is 61 metres above the ground. 4

let $h = 61$ m

$$40 + 23\cos x° = 61$$

$$23\cos x° = 21$$

$$\cos x° = \frac{21}{23} \quad \text{positive}$$

S	A ✓
T	C ✓

$ra = \cos^{-1}(21/23) = 24.1°$ (1dp)

$$x = 24.1°,\ 360° - 24.1°$$

$$x = 24.1°,\ 335.9°$$

[END OF QUESTION PAPER]

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ADDITIONAL SPACE FOR ANSWERS