

FOR OFFICIAL USE



National  
Qualifications  
2016

Mark

**X747/75/02**

**Mathematics  
Paper 2**

THURSDAY, 12 MAY

2:20 PM – 3:50 PM



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## National 5 Mathematics 2016 Paper 2

### WORKED SOLUTIONS

H Wallace

Kilmarnock Academy

**Total marks — 50**

Attempt ALL questions.

**You may use a calculator.**

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



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Total marks — 50  
Attempt ALL questions

1. A drinks manufacturer is reducing the sugar content of one of their fizzy drinks by 8% per year over the next 3 years.

The sugar content of a standard can is currently 35 grams.

Calculate the sugar content of a standard can after 3 years.

3

$$100\% - 8\% = 92\% = 0.92$$

3 years

$$35 \times 0.92^3 = 27.25408$$

$$= \underline{\underline{27 \text{ grams}}}$$

2. A pollen sample weighs 12 grams and contains  $1.5 \times 10^9$  pollen grains.



Calculate the weight of one pollen grain in grams.

Give your answer in scientific notation.

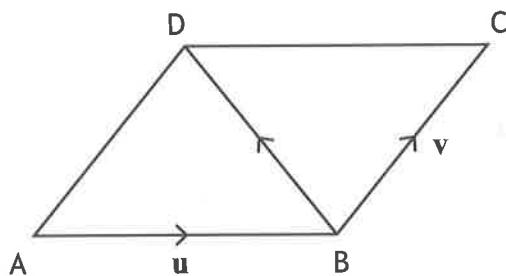
2

$$12 \div 1.5 = 8$$

$$\underline{\underline{8 \times 10^{-9} \text{ grams.}}}$$



3. The diagram below shows parallelogram ABCD.



$\vec{AB}$  represents vector  $\mathbf{u}$  and  $\vec{BC}$  represents vector  $\mathbf{v}$ .

Express  $\vec{BD}$  in terms of  $\mathbf{u}$  and  $\mathbf{v}$ .

1

$$\underline{\underline{\vec{BD} = \mathbf{v} - \mathbf{u}}}$$

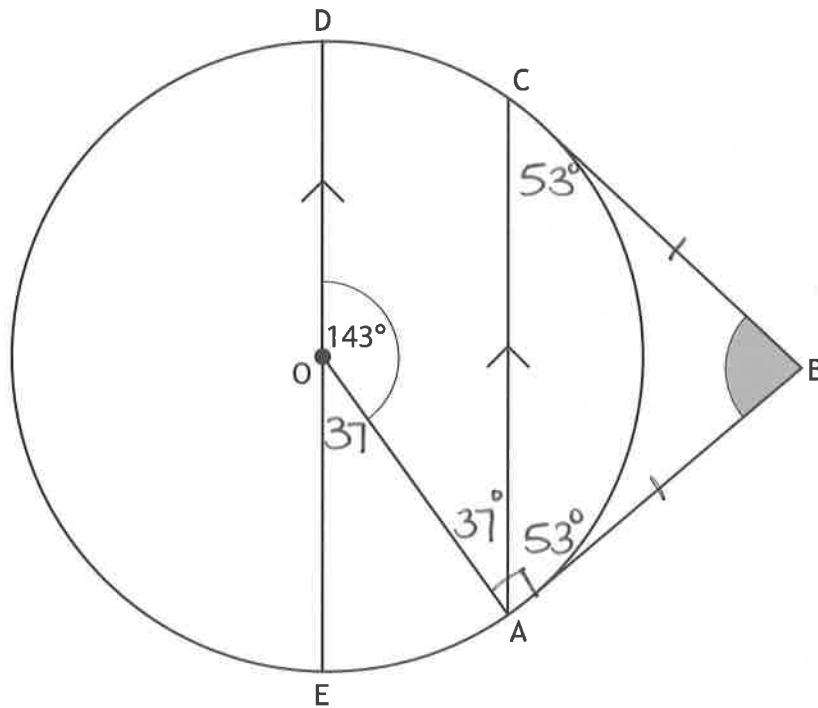
4. Factorise fully  $3x^2 - 48$ .

2

$$\begin{aligned} &3(x^2 - 16) \\ &\underline{\underline{3(x - 4)(x + 4)}} \end{aligned}$$



5. The diagram below shows a circle, centre O.



- AB and CB are tangents to the circle.
- AC and ED are parallel.
- Angle AOD is  $143^\circ$ .

Calculate the size of angle ABC.

3

$$180 - 143 = 37^\circ$$

$$\text{Angle } \text{BAO} = 90^\circ$$

$$90 - 37 = 53^\circ$$

$$53 + 53 = 106^\circ$$

$$180 - 106 = \underline{74^\circ}$$

$$\underline{\underline{\text{Angle } \text{ABC} = 74^\circ}}$$



6. Jack called his internet provider on six occasions to report connection problems.

On each occasion he noted the length of time he had to wait before speaking to an adviser.

The times (in minutes) were as follows:

13    16    10    22    5    12

- (a) Calculate the mean and standard deviation of these times.

4

$$\Sigma x = 78$$

$$\Sigma x^2 = 1178$$

$$\text{Mean} = \frac{78}{6} = \underline{\underline{13}} \text{ minutes}$$

$$\text{sd} = \sqrt{\frac{1178 - (78)^2/6}{5}}$$

$$= 5.727128$$

$$\underline{\underline{\text{sd} = 5.7 \text{ minutes (1dp)}}$$



## 6. (continued)

- (b) Sophie also called the same internet provider, on several occasions, to report connection problems.

Her mean waiting time was 15 minutes and the standard deviation was 4.3 minutes.

Make two valid comments comparing Sophie's waiting times with Jack's waiting times.

2

On average, Sophie had to wait 2 minutes longer than Jack.

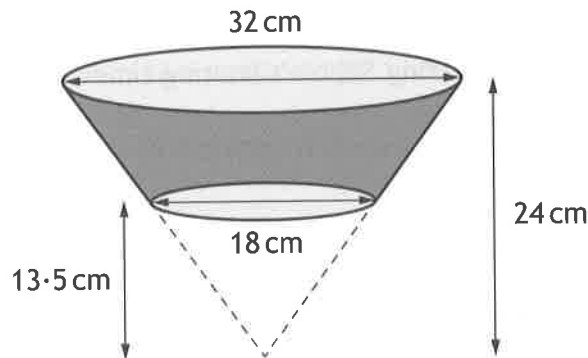
The length of time Sophie had to wait was less variable than the time Jack had to wait.

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7. A carton is in the shape of a large cone with a small cone removed. The large cone has diameter of 32 cm and height 24 cm. The small cone has diameter of 18 cm and height 13.5 cm.



Calculate the volume of the carton.

Give your answer correct to 2 significant figures.

5

Large cone

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 16^2 \times 24 \\ &= 6433.981755 \end{aligned}$$

Small cone

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 9^2 \times 13.5 \\ &= 1145.110522 \end{aligned}$$

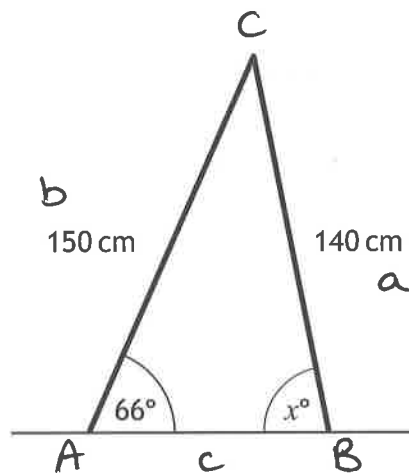
$$\begin{aligned} \text{Volume of carton} &= V_{\text{large}} - V_{\text{small}} \\ &= 6433.981755 - 1145.110522 \\ &= 5288.871232 \\ &= \underline{\underline{5300 \text{ cm}^3}} \quad 2 \text{ sig figs} \end{aligned}$$



8. A set of stepladders has legs 150 centimetres and 140 centimetres long.



When the stepladder is fully open, the angle between the longer leg and the ground is  $66^\circ$ .



Calculate  $x^\circ$ , the size of the angle between the shorter leg and the ground.

3

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin 66}{140} = \frac{\sin x}{150}$$

$$\sin x^\circ = \frac{150 \sin 66}{140}$$

$$x^\circ = \sin^{-1} \left( \frac{150 \sin 66}{140} \right)$$

$$x^\circ = 78.18077$$

$$x^\circ = \underline{\underline{78.2^\circ}} \quad (1 \text{ dp})$$



\* X 7 4 7 7 5 0 2 0 9 \*

9. Express  $x^2 + 8x - 7$  in the form  $(x+a)^2 + b$ .

2

completing the square

$$\left[ (x+4)^2 - 4^2 \right] - 7$$

$$(x+4)^2 - 16 - 7$$

$$\underline{\underline{(x+4)^2 - 23}}$$

10. Simplify  $(n^2)^3 \times n^{-10}$ .

Give your answer with a positive power.

3

$$(n^2)^3 \times n^{-10}$$

$$= n^6 \times n^{-10}$$

$$= n^{-4}$$

$$= \underline{\underline{\frac{1}{n^4}}}$$



11. Two pictures are mathematically similar in shape.



100 cm



60 cm

The cost of each picture is proportional to its area.

The large picture costs £13.75.

Find the cost of the small picture.

3

$$SFR = \frac{\text{wee}}{\text{big}} = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

$$SFR (\text{AREA}) = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$\begin{aligned} \text{Small cost} &= \frac{9}{25} \times \text{£}13.75 \\ &= \underline{\underline{\text{£}4.95}} \end{aligned}$$



12. Change the subject of the formula  $L = \sqrt{4kt - p}$  to  $k$ .

3

$$L^2 = 4kt - p$$

$$4kt - p = L^2$$

$$4kt = L^2 + p$$

$$k = \frac{L^2 + p}{4t}$$


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13. Express

$$\frac{3}{x-2} + \frac{5}{x+1}, \quad x \neq 2, x \neq -1$$

as a single fraction in its simplest form.

3

$$\frac{3(x+1)}{(x-2)(x+1)} + \frac{5(x-2)}{(x-2)(x+1)}$$

$$\frac{3x + 3 + 5x - 10}{(x-2)(x+1)}$$

$$\frac{8x - 7}{(x-2)(x+1)}$$


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14. Solve the equation  $2 \tan x^\circ + 5 = -4$ , for  $0 \leq x \leq 360$ .

$$2 \tan x^\circ + 5 = -4$$

$$2 \tan x^\circ = -9$$

$$\tan x^\circ = \frac{-9}{2} \quad \text{negative!}$$

$$\begin{aligned} \text{baa} = \tan^{-1}\left(\frac{9}{2}\right) &= 77.471 \quad \checkmark \begin{array}{c|c} \text{S} & \text{A} \\ \hline \text{T} & \text{C} \end{array} \checkmark \\ &= \underline{77.5^\circ} \end{aligned}$$

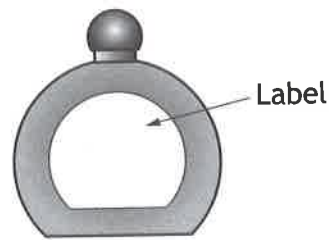
$$x^\circ = 180 - 77.5, \quad 360 - 77.5$$

$$\underline{\underline{x^\circ = 102.5^\circ, \quad 282.5^\circ}}$$

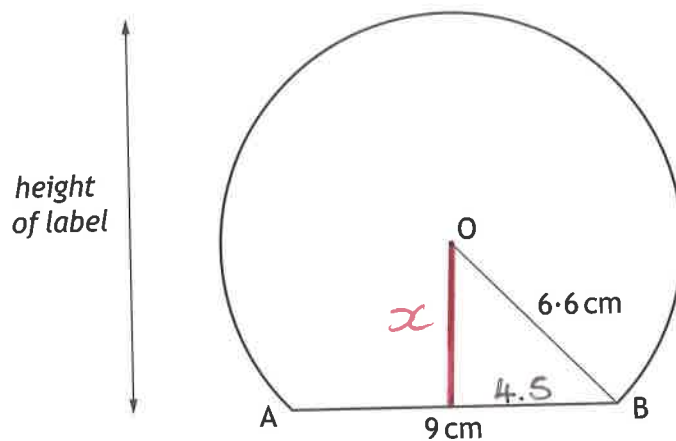
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15. This perfume bottle has a label in the shape of part of a circle.



A diagram of the label is shown below.



- The centre of the circle is O.
- The chord AB is 9 centimetres.
- The radius OB is 6.6 centimetres.

Find the height of the label.

Pythagoras

$$x^2 = 6.6^2 - 4.5^2$$

$$x^2 = 23.31$$

$$x = 4.8280$$

$$x = 4.8 \text{ cm (1dp)}$$

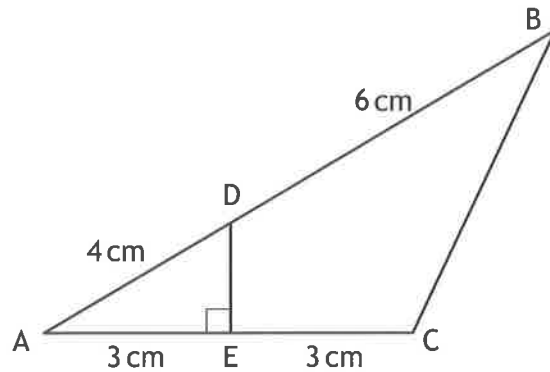
$$\begin{aligned} \text{height} &= x + r \\ &= 4.8 + 6.6 \\ &= \underline{\underline{11.4 \text{ cm}}} \end{aligned}$$

4



16. In the diagram below:

- DE is perpendicular to AC.
- AD = 4 centimetres.
- DB = 6 centimetres.
- AE = EC = 3 centimetres.

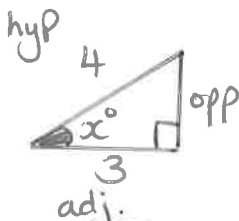


Calculate the length of BC.

Give your answer correct to one decimal place.

4

SOH CAH TOA then Cosine Rule

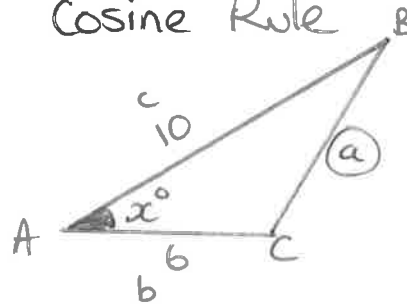


SOH CAH TOA

$$\cos x^\circ = \frac{3}{4}$$

$$x^\circ = \cos^{-1}\left(\frac{3}{4}\right)$$

$$x^\circ = 41.4096^\circ$$



$$a^2 = b^2 + c^2 - 2bc \cos x$$

$$a^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \frac{3}{4}$$

$$a^2 = 46$$

$$a = \sqrt{46}$$

$$a = 6.782329983$$

$$\text{length BC} = 6.8 \text{ cm (1dp)}$$

[END OF QUESTION PAPER]



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