FOR OFFICIAL USE

National Qualifications 2016

Mark

X747/75/01

Mathematics Paper 1 (Non-Calculator)

THURSDAY, 12 MAY
1:00 PM – 2:00 PM

X7477501

National 5 Mathematics 2016 Paper 1

WORKED SOLUTIONS

H Wallace

Kilmarnock Academy

Total marks — 40

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA

X747750101

A/HTP

MARKS | DO NOT WRITE IN THIS MARGIN

FORMULAE LIST

The roots of $ax^2+bx+c=0$ are $x=\dfrac{-b\pm\sqrt{(b^2-4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Cosine rule: $a^2=b^2+c^2-2bc\cos A$ or $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

Area of a triangle: $A=\frac{1}{2}ab\sin C$

Volume of a sphere: $V=\frac{4}{3}\pi r^3$

Volume of a cone: $V=\frac{1}{3}\pi r^2 h$

Volume of a pyramid: $V=\frac{1}{3}Ah$

Standard deviation: $s=\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n-1}}$

or $s=\sqrt{\dfrac{\Sigma x^2-\dfrac{(\Sigma x)^2}{n}}{n-1}}$, where n is the sample size.

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Total marks — 40

Attempt ALL questions

1. Given $\mathbf{p} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$.

Find the resultant vector $\frac{1}{2}\mathbf{p} + \mathbf{q}$.

Express your answer in component form. **2**

$\frac{1}{2}p = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\frac{1}{2}p + q = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

2. Evaluate $\frac{3}{4}\left(\frac{1}{3} + \frac{2}{7}\right)$.

Give your answer in its simplest form. **2**

$\frac{1}{3} + \frac{2}{7}$

$\frac{7}{21} + \frac{6}{21}$

$\frac{13}{21}$

$\frac{3}{4}\left(\frac{13}{21}\right)$

$\frac{39}{84} = \frac{13}{28}$

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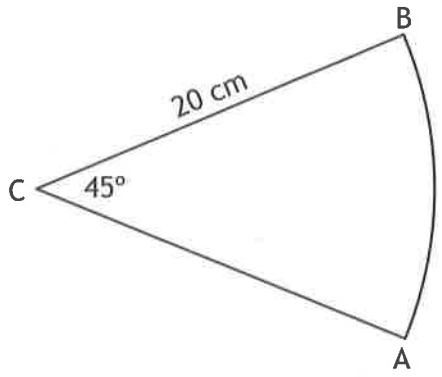
3. The diagram shows a sector of a circle, centre C.



The radius of the circle is 20 centimetres and angle ACB is 45°.

Calculate the area of the sector.

Take $\pi = 3{\cdot}14$. 3

Sector Area $= \frac{\theta}{360} \times \pi r^2$

$= \frac{45}{360} \times \pi \times 20^2$

$= \frac{1}{8} \times \pi \times 400$

$= 50\pi.$

$= 50 \times 3.14$

$= 5 \times 31.4$

Sector Area $= 157.0 \text{ cm}^2$

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4. Charlie is making costumes for a school show.
One day he made 2 cloaks and 3 dresses.
The total amount of material he used was 9·6 square metres.

(a) Write down an equation to illustrate this information. 1

$2c + 3d = 9.6$

(b) The following day Charlie made 3 cloaks and 4 dresses.
The total amount of material he used was 13·3 square metres.
Write down an equation to illustrate this information. 1

$3c + 4d = 13.3$

(c) Calculate the amount of material required to make one cloak and the amount of material required to make one dress. 4

$2c + 3d = 9.6$ (×3)
$3c + 4d = 13.3$ (×2)

$6c + 9d = 28.8$
$\ominus\ 6c + 8d = 26.6$
$d = 2.2$

$2c + 3(2.2) = 9.6$
$2c + 6.6 = 9.6$
$2c = 3$
$c = 1.5$

Cloak requires $1.5m^2$
Dress requires $2.2m^2$

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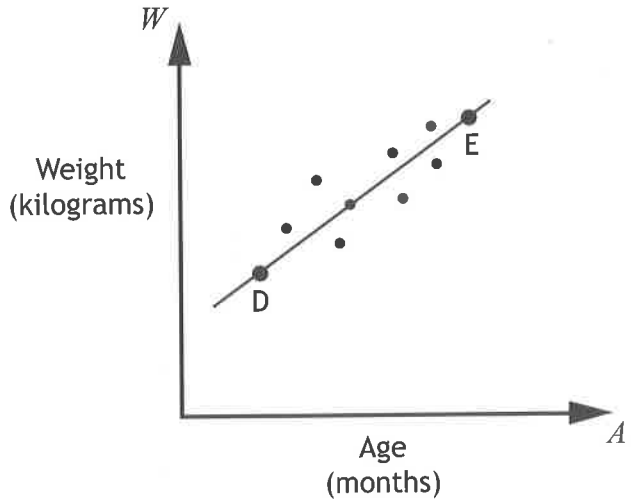
5. A cattle farmer records the weight of some of his calves.

The scattergraph shows the relationship between the age, A months, and the weight, W kilograms, of the calves.



A line of best fit is drawn.

Point D represents a 3 month old calf which weighs 100 kilograms.

Point E represents a 15 month old calf which weighs 340 kilograms.

(a) Find the equation of the line of best fit in terms of A and W.
Give the equation in its simplest form. **3**

D(3, 100) E(15, 340)

$$m = \frac{340-100}{15-3} = \frac{240}{12} = 20$$

$$y - b = m(x - a)$$
$$y - 100 = 20(x - 3)$$
$$y - 100 = 20x - 60$$
$$y = 20x - 60 + 100$$
$$y = 20x + 40$$
$$W = 20A + 40$$

MARKS | DO NOT WRITE IN THIS MARGIN

5. (continued)

(b) Use your equation from part (a) to estimate the weight of a one **year** old calf.

Show your working. 1

$W = 20A + 40$

$W = 20(12) + 40$

$= 240 + 40$

$= 280$ kg.

one year
12 months

6. Determine the nature of the roots of the function $f(x) = 7x^2 + 5x - 1$. 2

$a = 7 \quad b = 5 \quad c = -1$

$b^2 - 4ac$

$(5)^2 - 4(7)(-1)$

$25 + 28$

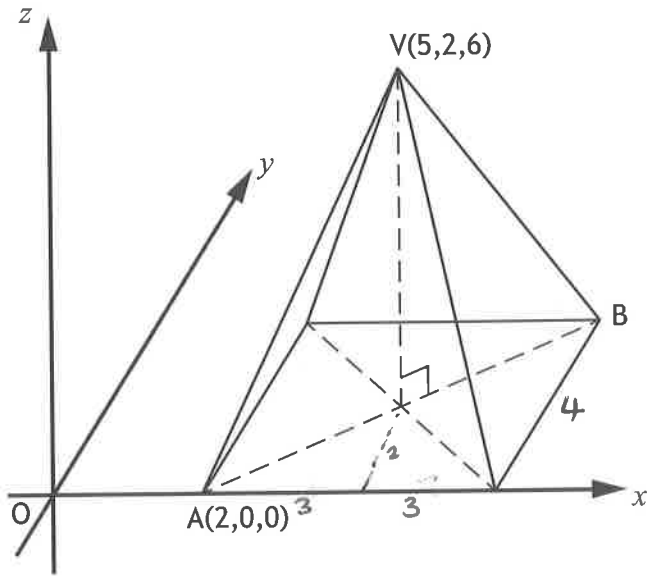
53

since $b^2 - 4ac > 0$

Two real and distinct roots.

MARKS | DO NOT WRITE IN THIS MARGIN

7. The diagram shows a rectangular based pyramid, relative to the coordinate axes.



- A is the point (2,0,0).
- V is the point (5,2,6).

(a) Write down the coordinates of B. 1

$(8, 4, 0)$

(b) Calculate the length of edge AV of the pyramid. 3

$$\overrightarrow{AV} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$$

$$|\overrightarrow{AV}| = \sqrt{(3)^2 + (2)^2 + (6)^2}$$

$$= \sqrt{9 + 4 + 36}$$

$$= \sqrt{49}$$

Length AV. = 7 units

8. Solve the equation

$$\frac{2x}{3} - \frac{5}{6} = 2x.$$

Give your answer in its simplest form. **3**

$$\frac{2x}{3} - \frac{5}{6} = 2x.$$

$$\frac{12x}{3} - 5 = 12x.$$

$$4x - 5 = 12x$$

$$-5 = 8x.$$

$$x = \frac{-5}{8}$$

9. The function $f(x)$ is defined by $f(x) = \frac{2}{\sqrt{x}}, \quad x > 0.$

Express $f(5)$ as a fraction with a rational denominator. **2**

$$f(5) = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

MARKS | DO NOT WRITE IN THIS MARGIN

10. Sketch the graph of $y=(x-3)^2+1$.

On your sketch, show clearly the coordinates of the turning point and the point of intersection with the y–axis. **3**

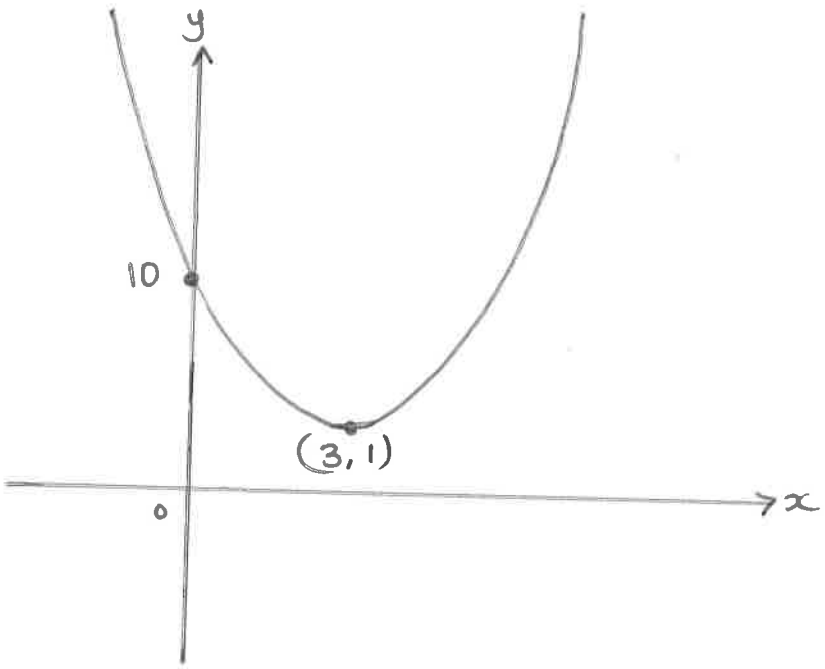
Min TP (3, 1)

y-intercept occurs @ $x=0$.

$y = (0-3)^2 + 1$

$= 9+1$

$y = 10$ (0, 10).



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11. Simplify

$$\tan^2 x° \cos^2 x° .$$

Show your working. 2

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$. from $\tan x = \frac{\sin x}{\cos x}$

$\frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x .$

$\sin^2 x .$

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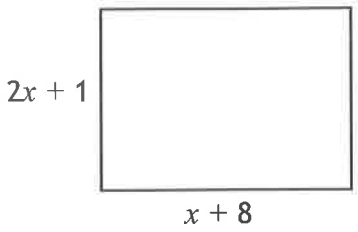
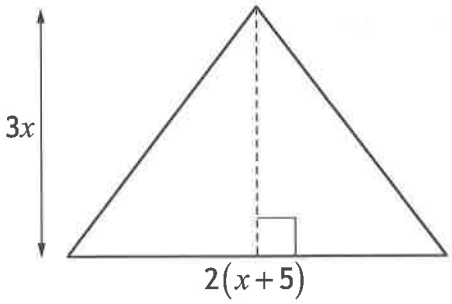
12. The diagrams below show a rectangle and a triangle.

All measurements are in centimetres.





(a) Find an expression for the area of the **rectangle.** 1

Area = l x b

= (x+8)(2x+1)

= $2x^2 + x + 16x + 8$

Area = $2x^2 + 17x + 8$.

(b) Given that the area of the rectangle is equal to the area of the triangle, show that $x^2 - 2x - 8 = 0$. 3

Area Δ = $\frac{1}{2}$ bh

= $\frac{1}{2}$ x 2(x+5) x 3x.

= 3x(x+5)

= $3x^2 + 15x$.

$2x^2 + 17x + 8 = 3x^2 + 15x$

$0 = x^2 - 2x - 8$

MARKS | DO NOT WRITE IN THIS MARGIN

12. (continued)

(c) Hence find, **algebraically**, the length and breadth of the rectangle. **3**

$x^2 - 2x - 8 = 0.$

$(x-4)(x+2) = 0.$

$m \Rightarrow -8$

$A \Rightarrow -2$

$x - 4 = 0 \qquad x + 2 = 0$

$x = 4 \qquad x = -2.$

1×8

$2 \times 4.$

but $x + 8 > 0$ and $2x + 1 > 0.$

So $x = 4$

Length $= x + 8 = 4 + 8 = 12\text{cm}.$

Breadth $= 2x + 1 = 2(4) + 1 = 9\text{cm}.$

[END OF QUESTION PAPER]