



FOR OFFICIAL USE

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National
Qualifications
2015

Mark

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X747/75/02

Mathematics
Paper 2

TUESDAY, 19 MAY
10:20AM – 11:50AM



* X 7 4 7 7 5 0 2 *

Fill in these boxes and read what is printed below.

Full name of centre

WORKED SOLUTIONS

Town

#6

Forename(s)

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Surname

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Number of seat

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Date of birth

Day

Month

Year

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Scottish candidate number

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Total marks — 50

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting.

Use blue or black ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



* X 7 4 7 7 5 0 2 0 1 *

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $A = \frac{1}{2} ab \sin C$

Volume of a sphere: $V = \frac{4}{3} \pi r^3$

Volume of a cone: $V = \frac{1}{3} \pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3} Ah$

Standard deviation: $s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}} = \sqrt{\frac{\Sigma x^2 - (\Sigma x)^2/n}{n-1}}$, where n is the sample size.



* X 7 4 7 7 5 0 2 0 2 *

1. A house is valued at £240 000.
 Its value is predicted to rise by 2.8% per annum.
 Calculate its predicted value after 2 years.

3

$$100\% + 2.8\% = 102.8\% = \underline{1.028}$$

$$\begin{aligned} \text{Predicted value} &= £240\,000 \times 1.028^2 \\ &= \underline{\underline{£253,628.16}} \end{aligned}$$

2. A function is defined as $f(x) = 3x + 2$.

Given that $f(a) = 23$, calculate a .

2

$$\text{If } f(a) = 3a + 2$$

$$\text{let } 3a + 2 = 23$$

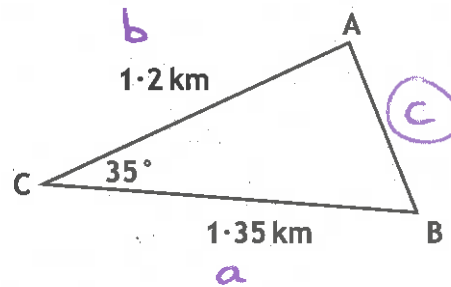
$$3a = 21$$

$$\underline{\underline{a = 7}}$$

[Turn over



3. Triangle ABC is shown below.



Calculate the length of AB.

3

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 1.35^2 + 1.2^2 - 2(1.35)(1.2) \cos 35^\circ$$

$$c^2 = 0.608447376$$

$$c = \sqrt{\text{ANS}}$$

$$c = 0.780030369$$

Length AB = 0.78 km (2dp)



4. Find $|u|$, the magnitude of vector $u = \begin{pmatrix} 6 \\ -13 \\ 18 \end{pmatrix}$.

$$|u| = \sqrt{6^2 + (-13)^2 + 18^2}$$

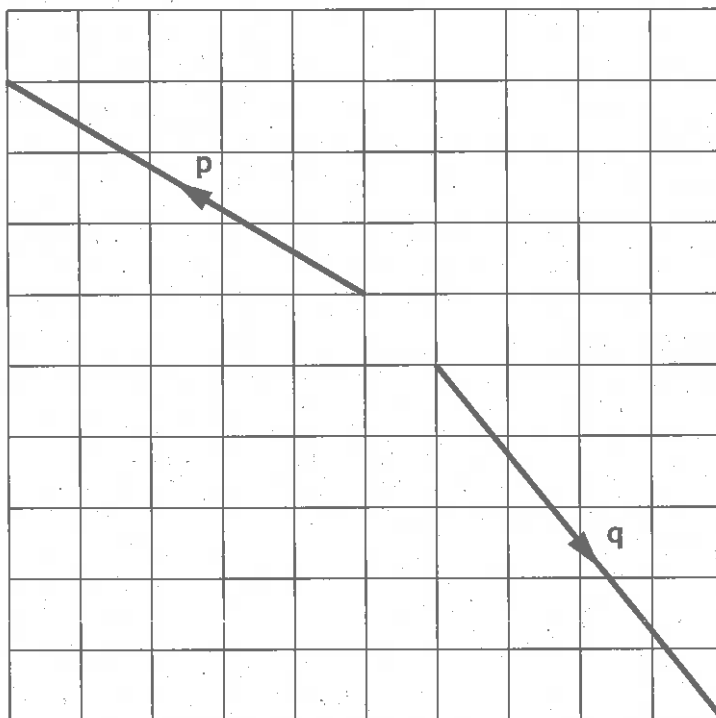
$$= \sqrt{529}$$

$|u| = 23$ units.

[Turn over



5. The vectors \underline{p} and \underline{q} are shown in the diagram below.
 Find the resultant vector $\underline{p} + \underline{q}$.
 Express your answer in component form.



2

$$\underline{p} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} \quad \text{and} \quad \underline{q} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$$\underline{p} + \underline{q} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$$\underline{p} + \underline{q} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$



6. (a) The Earth is approximately spherical with a radius of 6400 kilometres. Calculate the volume of the Earth giving your answer in scientific notation, correct to 2 significant figures.



3

$$V = \frac{4}{3} \times \pi \times 6400^3$$

$$V = 1.098066219 \times 10^{12}$$

$$\underline{\underline{V = 1.1 \times 10^{12} \text{ km}^3}} \quad (2 \text{ sf})$$

- (b) The approximate volume of the Moon is 2.2×10^{10} cubic kilometres. Calculate how many times the Earth's volume is greater than the Moon's.

2

$$\frac{\text{Earth}}{\text{Moon}} = \frac{1.1 \times 10^{12}}{2.2 \times 10^{10}} = 0.5 \times 10^2$$

50 times greater

[Turn over



7. Express $\frac{5t}{s} \div \frac{t}{2s^2}$ in its simplest form.

3

$$\frac{5t}{s} \times \frac{2s^2}{t}$$

$$\frac{10s^2t}{st} = \underline{\underline{10s}}$$

8. James paid £297.50 for a laptop in a sale.
The discount in the sale was 15%.
Calculate the original price of the laptop.

3

$$100\% - 15\% = 85\%$$

$$85\% = £297.50$$

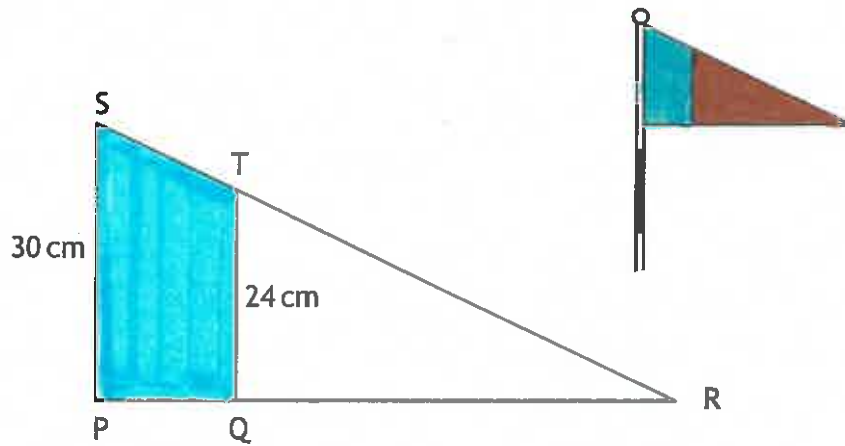
$$1\% = \frac{297.50}{85} = 3.5$$

$$100\% = 3.5 \times 100 = 350$$

Original Price was £350



9. The flag at each hole on a golf course is coloured red and blue.
 The diagram below represents a flag.
 Triangle QRT represents the red section.
PQTS represents the blue section.



Triangles PRS and QRT are mathematically similar.
 The area of triangle QRT is 400 square centimetres.
 Calculate the area of PQTS, the blue section of the flag.

4

$$SFE = \frac{\text{big}}{\text{wee}} = \frac{30}{24} = \frac{5}{4}$$

$$\text{Large } \Delta \text{ Area RPS} = \left(\frac{5}{4}\right)^2 \times 400 = 625 \text{ cm}^2$$

$$\text{Blue Area PQTS} = 625 - 400$$

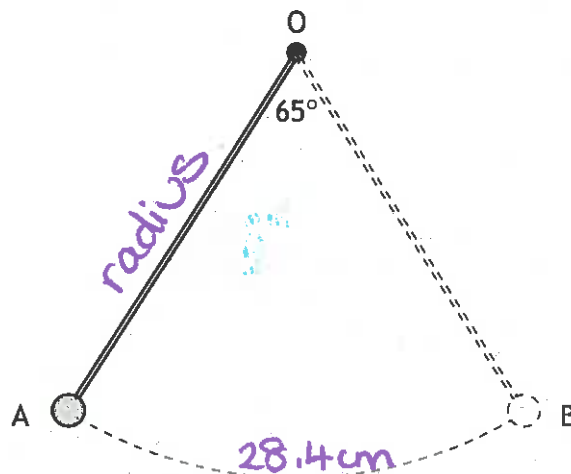
$$\underline{\underline{\text{Area PQTS} = 225 \text{ cm}^2}}$$

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* X 7 4 7 7 5 0 2 0 9 *

10. The pendulum of a clock swings along an arc of a circle, centre O.



The pendulum swings through an angle of 65°, travelling from A to B.

The length of the arc AB is 28.4 centimetres.

Calculate the length of the pendulum.

4

$$\frac{\theta}{360} = \frac{\text{Arc length}}{\pi d}$$

$$\frac{65}{360} = \frac{28.4}{\pi d}$$

$$65\pi d = 28.4 \times 360$$

$$d = \frac{10224}{65\pi}$$

$$d = 50.06769656$$

$$r = 25.03384828$$

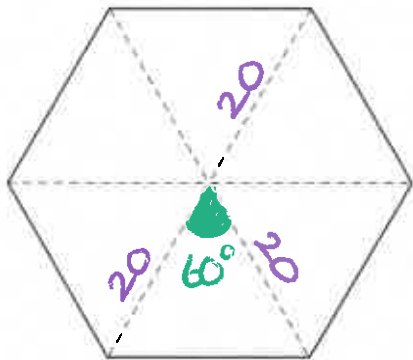
Length of pendulum = 25.03 cm (2dp)



* X 7 4 7 7 5 0 2 1 0 *

11. The top of a table is in the shape of a regular hexagon.

The three diagonals of the hexagon which are shown as dotted lines in the diagram below each have length 40 centimetres.



Calculate the area of the top of the table.

4

$$\text{Angle} = 360 \div 6 = \underline{60^\circ}$$

$$\begin{aligned} \text{Area } \Delta &= \frac{1}{2} \times 20 \times 20 \times \sin 60^\circ \\ &= \underline{173.2050808 \text{ cm}^2} \end{aligned}$$

$$\begin{aligned} \text{Area of Table} &= 6 \times \text{Area } \Delta \\ &= 6 \times 173.2050808 \\ &= 1039.230485 \end{aligned}$$

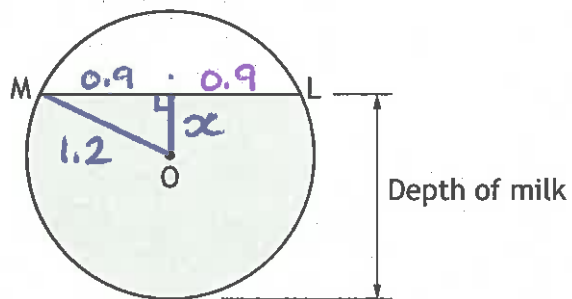
$$\underline{\underline{\text{Area of Table} = 1039.23 \text{ cm}^2 \text{ (2dp)}}}$$

[Turn over



* X 7 4 7 7 5 0 2 1 1 *

12. The diagram below shows the circular cross-section of a milk tank.



The radius of the circle, centre O, is 1.2 metres.

The width of the surface of the milk in the tank, represented by ML in the diagram, is 1.8 metres.

Calculate the depth of the milk in the tank.

4

$$x^2 = 1.2^2 - 0.9^2$$

$$x^2 = 0.63$$

$$x = \sqrt{0.63}$$

$$x = 0.793725\dots$$

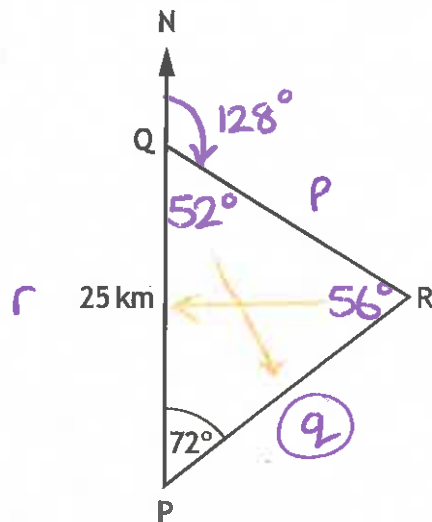
$$\underline{x = 0.8\text{m}} \text{ (1dp)}$$

$$\begin{aligned} \text{Depth of milk} &= x + r \\ &= 0.8 + 1.2 \\ &= \underline{\underline{2.0\text{m}}} \end{aligned}$$



* X 7 4 7 7 5 0 2 1 2 *

13. In the diagram below P, Q and R represent the positions of Portlee, Queenstown and Rushton respectively.



$$\angle PQR = 180 - 128 = 52^\circ$$

$$\angle PRQ = 180 - 72 - 52 = 56^\circ$$

Portlee is 25 kilometres due South of Queenstown.
 From Portlee, the bearing of Rushton is 072° .
 From Queenstown, the bearing of Rushton is 128° .

Calculate the distance between Portlee and Rushton.

Do not use a scale drawing.

4

$$\frac{P}{\sin P} = \frac{2}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{2}{\sin 52} = \frac{25}{\sin 56}$$

$$2 = \frac{25 \sin 52}{\sin 56}$$

$$2 = 23.7628 \dots$$

Distance PR = 23.76 km (2dp)

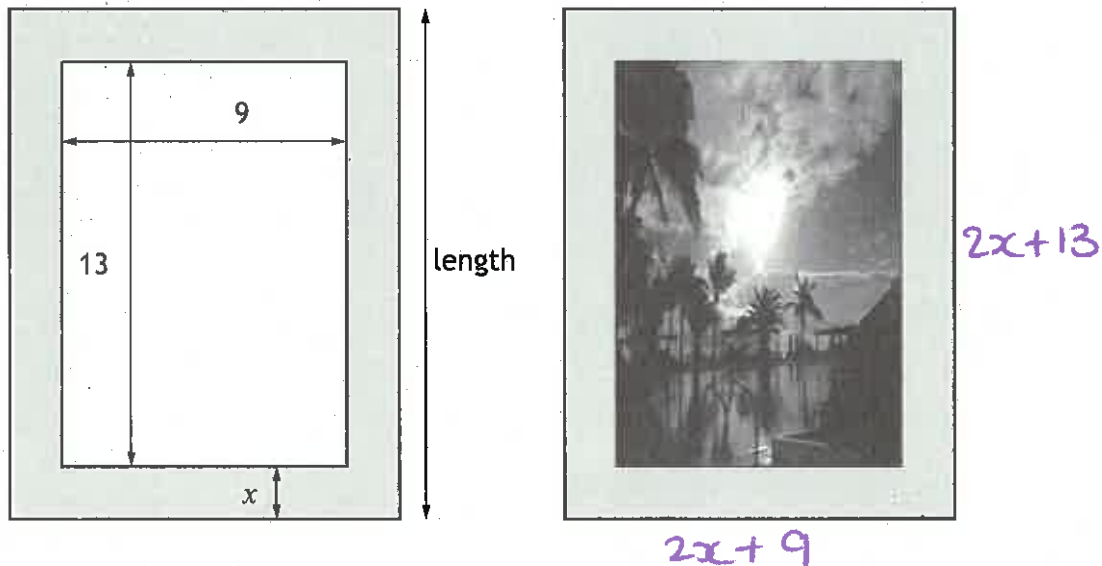
[Turn over



14. A rectangular picture measuring 9 centimetres by 13 centimetres is placed on a rectangular piece of card.

The area of the card is 270 square centimetres.

There is a border x centimetres wide on all sides of the picture.



- (a) (i) Write down an expression for the length of the card in terms of x . 1

$$\text{Length} = x + 13 + x$$

$$\underline{\underline{\text{Length} = 2x + 13}}$$

- (ii) Hence show that $4x^2 + 44x - 153 = 0$. 2

$$\text{Area} = (2x + 13)(2x + 9)$$

$$= 4x^2 + 18x + 26x + 117$$

$$\text{Area} = 4x^2 + 44x + 117$$

$$\text{let } 4x^2 + 44x + 117 = 270$$

$$\text{so } \underline{\underline{4x^2 + 44x - 153 = 0}} \text{ as required.}$$



14. (continued)

(b) Calculate x , the width of the border.

Give your answer correct to one decimal place.

4

Solve $4x^2 + 44x - 153 = 0$ for $x > 0$.

for $ax^2 + bx + c = 0$

$a = 4$

$b = 44$

$c = -153$

Discriminant = $b^2 - 4ac$

= $(44)^2 - 4(4)(-153)$

= 4384

$x = \frac{-44 \pm \sqrt{4384}}{8}$

$x = \frac{-44 - \sqrt{4384}}{8}$

(OR)

$x = \frac{-44 + \sqrt{4384}}{8}$

~~$x = -13.77647\dots$~~

$x = 2.77647\dots$

~~$x > 0$~~

~~no solution~~

$x = 2.8 \text{ cm (1dp)}$

[END OF QUESTION PAPER]



* X 7 4 7 7 5 0 2 1 5 *

ADDITIONAL SPACE FOR ANSWERS

MARKS

DO NOT
WRITE IN
THIS
MARGIN



* X 7 4 7 7 5 0 2 1 6 *