

STUDENT MATERIALS

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Specimen Assessment Questions

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ALGEBRAIC OPERATIONS

By the end of this set of exercises, you should be able to

- (a) reduce an algebraic fraction to its simplest form
- (b) apply the four rules to algebraic fractions
- (c) change the subject of a formula
- (d) simplify surds
- (e) rationalise a surd denominator
- (f) simplify expressions using the laws of indices.

ALGEBRAIC OPERATIONS

A. Reducing algebraic fractions to their simplest form

Exercise 1

1. Simplify these fractions:

(a) $\frac{6}{10}$	(b) $\frac{3}{9}$	(c) $\frac{21}{28}$	(d) $\frac{33}{44}$
(e) $\frac{100}{400}$	(f) $\frac{4a}{5a}$	(g) $\frac{2p}{3p}$	(h) $\frac{5v}{5v}$
(i) $\frac{rs}{rt}$	(j) $\frac{ab}{ac}$	(k) $\frac{a}{ad}$	(l) $\frac{m}{m^2}$
(m) $\frac{a^2}{4a}$	(n) $\frac{8z}{z^2}$	(o) $\frac{x^2}{x}$	(p) $\frac{d}{d^2}$
(q) $\frac{5x^2}{6x}$	(r) $\frac{3v}{6w}$	(s) $\frac{8xy}{4x}$	(t) $\frac{a^2}{3ab}$
(u) $\frac{2pq}{6p}$	(v) $\frac{5b}{5b^2}$	(w) $\frac{xyz}{xz}$	(x) $\frac{3ef}{7ef}$
(y) $\frac{5pq}{2pq^2}$	(z) $\frac{8x^2y}{4xy^2}$	(aa) $\frac{(x+1)^2}{(x+1)^3}$	(ab) $\frac{(x-5)^2}{(x-5)^5}$

2. Factorise either the numerator or the denominator, then simplify:

(a) $\frac{2x+8}{4}$	(b) $\frac{3x+6}{9}$	(c) $\frac{2x-8}{2}$	(d) $\frac{5x-10}{25}$
(e) $\frac{2}{2x+6}$	(f) $\frac{4}{4x-12}$	(g) $\frac{5}{5x+15}$	(h) $\frac{8}{4x-10}$
(i) $\frac{x^2-xy}{x}$	(j) $\frac{pq+p}{p}$	(k) $\frac{v^2+v}{v}$	(l) $\frac{a-a^2}{a}$

3. Factorise the numerator and/or the denominator, then simplify:

(a) $\frac{2a+6}{a+3}$	(b) $\frac{3c+9}{c+3}$	(c) $\frac{d-2}{3d-6}$	(d) $\frac{g^2+g}{g+1}$
(e) $\frac{2x+2y}{5x+5y}$	(f) $\frac{3p+3q}{7p+7q}$	(g) $\frac{4-4w}{1-w}$	(h) $\frac{x^2-xy}{8x-8y}$
(i) $\frac{x^2-1}{x-1}$	(j) $\frac{y^2-9}{y+3}$	(k) $\frac{a^2-25}{a-5}$	(l) $\frac{w+10}{w^2-100}$
(m) $\frac{x^2-1}{x^2+2x+1}$	(n) $\frac{3v^2-5v-2}{v^2-4}$	(o) $\frac{2y^2+y-1}{2y^2+5y-3}$	(p) $\frac{6x^2-13x+6}{3x^2+10x-8}$

B. Multiplying, dividing, adding and subtracting algebraic fractions

Exercise 2

1. Simplify these fractions by multiplying:

(a) $\frac{1}{3} \times \frac{3}{5}$	(b) $\frac{2}{3} \times \frac{1}{6}$	(c) $\frac{5}{6} \times \frac{3}{5}$	(d) $\frac{3}{10} \times \frac{20}{3}$
(e) $\frac{1}{6} \times 12$	(f) $\frac{9}{11} \times \frac{33}{9}$	(g) $\frac{7}{10} \times \frac{5}{1}$	(h) $8 \times \frac{2}{3}$
(i) $\frac{a}{b} \times \frac{c}{d}$	(j) $\frac{x}{y} \times \frac{v}{w}$	(k) $\frac{m}{n} \times \frac{m}{n}$	(l) $\frac{a}{b} \times \frac{b}{a}$
(m) $\frac{x}{2} \times \frac{x}{5}$	(n) $x \times \frac{x}{7}$	(o) $\frac{a}{5} \times \frac{a}{5}$	(p) $\frac{n}{3} \times \frac{3}{n}$
(q) $\frac{a}{6} \times \frac{6}{d}$	(r) $x^2 \times \frac{1}{x}$	(s) $\frac{a^2}{c} \times \frac{c}{a}$	(t) $\frac{a^3}{9} \times \frac{9}{a}$

2. Change these divisions to multiplications and simplify:

(a) $\frac{3}{7} \div \frac{6}{7}$	(b) $\frac{2}{3} \div \frac{8}{3}$	(c) $\frac{3}{8} \div \frac{9}{4}$	(d) $\frac{9}{10} \div \frac{18}{5}$
(e) $\frac{x}{2} \div \frac{x}{3}$	(f) $\frac{a}{8} \div \frac{a}{2}$	(g) $\frac{d}{3} \div \frac{d}{6}$	(h) $\frac{m}{10} \div \frac{m}{50}$
(i) $\frac{a^2}{3} \div \frac{a}{3}$	(j) $\frac{b^2}{6} \div \frac{b}{2}$	(k) $\frac{r^4}{6} \div \frac{r^2}{2}$	(l) $\frac{a}{b} \div \frac{a}{b}$
(m) $\frac{a^2}{d} \div \frac{a}{d^2}$	(n) $\frac{1}{w^2} \div \frac{5}{w}$	(o) $\frac{1}{a^3} \div \frac{1}{a^2}$	(p) $\frac{x^2}{y} \div \frac{2x}{d}$
(q) $\frac{a^2}{b} \div \frac{a}{b}$	(r) $\frac{2a^2}{5d^2} \div \frac{a^2}{d^2}$		

3. Do the following additions and subtractions:

(a) $\frac{1}{3} + \frac{1}{4}$	(b) $\frac{1}{4} + \frac{2}{3}$	(c) $\frac{3}{4} - \frac{1}{5}$	(d) $\frac{1}{5} + \frac{1}{3}$
(e) $\frac{1}{3} - \frac{1}{5}$	(f) $\frac{4}{7} - \frac{1}{2}$	(g) $\frac{1}{2} + \frac{1}{5}$	(h) $\frac{1}{2} - \frac{1}{5}$
(i) $\frac{5}{8} - \frac{1}{4}$	(j) $\frac{7}{10} + \frac{1}{5}$	(k) $\frac{x}{3} + \frac{a}{2}$	(l) $\frac{c}{5} + \frac{d}{2}$
(m) $\frac{e}{3} - \frac{h}{4}$	(n) $\frac{m}{4} - \frac{n}{8}$	(o) $\frac{2x}{3} + \frac{k}{2}$	(p) $\frac{u}{2} - \frac{2w}{5}$
(q) $\frac{4r}{5} + \frac{s}{2}$	(r) $\frac{a}{3} - \frac{2d}{5}$	(s) $\frac{2x}{3} + \frac{3y}{2}$	(t) $\frac{3x}{4} + \frac{2u}{5}$

4. By finding a common denominator with letters, work out these additions/subtractions:

(a) $\frac{2}{x} + \frac{3}{y}$ (b) $\frac{5}{a} - \frac{2}{b}$ (c) $\frac{4}{c} + \frac{1}{d}$ (d) $\frac{1}{p} - \frac{2}{q}$

(e) $\frac{2}{v} + \frac{2}{w}$ (f) $\frac{1}{g} - \frac{1}{h}$ (g) $\frac{7}{k} + \frac{1}{n}$ (h) $\frac{1}{x} - \frac{8}{y}$

5. Add or subtract these fractions:

(a) $\frac{x+1}{3} + \frac{x+1}{2}$ (b) $\frac{x+2}{4} + \frac{x-1}{5}$ (c) $\frac{x+3}{2} + \frac{x+1}{4}$ (d) $\frac{2x-3}{5} + \frac{x+1}{3}$

(e) $\frac{x+1}{2} - \frac{x+1}{3}$ (f) $\frac{x+2}{2} - \frac{x+1}{5}$ (g) $\frac{2x+1}{2} - \frac{x+1}{4}$ (h) $\frac{x+1}{2} - \frac{x-1}{5}$

C. Changing the subject of a formula

Exercise 3A

This exercise has a mixed selection of formulae.

Change the subject of each formula to the letter shown in the brackets.

ALL WORKING and ALL STEPS SHOULD BE SHOWN.

- | | | |
|------------------------|--------------------------|------------------------------|
| 1. $x + 2 = c$ (x) | 2. $x - 4 = c$ (x) | 3. $x + p = q$ (x) |
| 4. $x - p = q$ (x) | 5. $x/2 = a$ (x) | 6. $x/7 = a$ (x) |
| 7. $x/y = a$ (x) | 8. $x/p = m$ (x) | 9. $x/r = s$ (x) |
| 10. $4x = 20$ (x) | 11. $4x = a$ (x) | 12. $gx = h$ (x) |
| 13. $nx = t$ (x) | 14. $2x + 1 = 5$ (x) | 15. $2x + 1 = b$ (x) |
| 16. $2x + c = b$ (x) | 17. $ax + c = b$ (x) | 18. $px + q = r$ (x) |
| 19. $vx - w = y$ (x) | 20. $D = S \times T$ (S) | 21. $C = \pi d$ (d) |
| 22. $x^2 = 16$ (x) | 23. $x^2 = y$ (x) | 24. $A = \pi r^2$ (r) |
| 25. $T = D/S$ (S) | 26. $A = y^2$ (y) | 27. $P = 3\pi r^2$ (π) |
| 28. $P = 5\pi r^2$ (r) | 29. $h - p = q$ (h) | 30. $h - p = q$ (p) |
| 31. $2h - 5p = q$ (h) | 32. $2h - 5p = q$ (p) | 33. $b - c = ax$ (x) |

Exercise 3B

1. Change the subject of each formula to h .

(a) $g = hf$ (b) $e = g + h$ (c) $k = h/f(d)$ $e = g - h$

2. Change the subject of each formula to r .

(a) $Q = r^2$ (b) $N = \pi r^2$ (c) $M = 2 r^2$ (d) $P = r^2 w$

3. Change the subject of each formula to m .

(a) $A = klm$ (b) $B = Km$ (c) $C = \pi mr^2$ (d) $D = \frac{1}{3pm}$

4. Change the subject to x .

(a) $p = q + x$ (b) $r = s - x$ (c) $r = s - 5x$ (d) $r = 7x - 3$
(e) $m = 2(x + 1)$ (f) $m = \frac{1}{2}(x - 5)$ (g) $n = \frac{1}{2}(x + 2)$ (h) $p = \frac{1}{2}(x + q)$

5. Change the subject of the formula to the letter in brackets.

(a) $P/Q = R$ (P) (b) $t = 1/s$ (s) (c) $M = P/Q^2$ (Q) (d) $v = \sqrt{\frac{w}{z}}$ (w)
(e) $d = \frac{e}{5f}$ (f) (f) $\frac{K}{mn} = T$ (n) (g) $R = \frac{7}{9s^2}$ (s) (h) $a^2 + b^2 = c^2$ (a)

6. Harder examples. Change the subject of the formula to the letter in brackets.

(a) $A + d = V/T$ (T) (b) $px + qx = r$ (x) (c) $ax = bx + c$ (x)
(d) $m = \frac{r-s}{s}$ (s) (e) $x = \frac{v-w}{v+w}$ (w) (f) $p = 2\sqrt{r} - 1$ (r)

D. Simplifying surds

Exercise 4

1. Express each of the following in its simplest form:

(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{27}$ (d) $\sqrt{20}$ (e) $\sqrt{50}$ (f) $\sqrt{28}$
(g) $\sqrt{18}$ (h) $\sqrt{24}$ (i) $\sqrt{200}$ (j) $\sqrt{75}$ (k) $\sqrt{45}$ (l) $\sqrt{72}$
(m) $\sqrt{300}$ (n) $\sqrt{147}$ (o) $\sqrt{54}$ (p) $7\sqrt{8}$ (q) $5\sqrt{32}$ (r) $6\sqrt{40}$

2. Add or subtract the following:

(a) $3\sqrt{2} + 5\sqrt{2}$ (b) $6\sqrt{5} - 5\sqrt{5}$ (c) $8\sqrt{10} + 5\sqrt{10}$ (d) $9\sqrt{20} - 9\sqrt{20}$
(e) $\sqrt{6} - 3\sqrt{6}$ (f) $\sqrt{3} + \sqrt{3} - 3\sqrt{3}$ (g) $5\sqrt{7} - 8\sqrt{7} + 3\sqrt{7}$ (h) $10\sqrt{2} + 10\sqrt{3}$

3. Simplify:

(a) $\sqrt{8} - \sqrt{2}$ (b) $\sqrt{18} - \sqrt{2}$ (c) $\sqrt{125} + 5\sqrt{5}$ (d) $\sqrt{48} + \sqrt{12}$
(e) $\sqrt{45} + \sqrt{20}$ (f) $\sqrt{63} - \sqrt{28}$ (g) $\sqrt{50} + \sqrt{18}$ (h) $\sqrt{72} - \sqrt{32}$

Exercise 5

1. Simplify:

(a) $\sqrt{3} \times \sqrt{3}$ (b) $\sqrt{5} \times \sqrt{5}$ (c) $\sqrt{6} \times \sqrt{6}$ (d) $\sqrt{1} \times \sqrt{1}$
(e) $\sqrt{x} \times \sqrt{x}$ (f) $\sqrt{3} \times \sqrt{2}$ (g) $\sqrt{4} \times \sqrt{5}$ (h) $\sqrt{16} \times \sqrt{a}$
(i) $\sqrt{2} \times \sqrt{c}$ (j) $\sqrt{x} \times \sqrt{y}$ (k) $\sqrt{2} \times \sqrt{8}$ (l) $\sqrt{2} \times \sqrt{32}$
(m) $\sqrt{6} \times \sqrt{3}$ (n) $\sqrt{20} \times \sqrt{10}$ (o) $3\sqrt{2} \times \sqrt{2}$ (p) $3\sqrt{2} \times 2\sqrt{3}$

contd.

2. Multiply out the brackets:

- (a) $\sqrt{2}(1 + \sqrt{2})$ (b) $\sqrt{3}(\sqrt{2} + 1)$ (c) $\sqrt{5}(\sqrt{5} + 1)$
(d) $\sqrt{7}(1 + \sqrt{7})$ (e) $\sqrt{2}(5 - \sqrt{2})$ (f) $\sqrt{2}(5 - 4\sqrt{2})$
(g) $(\sqrt{3} + 2)(\sqrt{3} - 1)$ (h) $(\sqrt{2} - 2)(\sqrt{2} + 1)$ (i) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$
(j) $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$ (k) $(\sqrt{3} + \sqrt{2})^2$ (l) $(\sqrt{5} - \sqrt{3})^2$

3. If $a = 1 + \sqrt{2}$ and $b = 1 - \sqrt{2}$, simplify:

- (a) $3a + 3b$ (b) $2ab$ (c) $a^2 + b^2$

4. If $r = \sqrt{5} + \sqrt{3}$ and $s = \sqrt{5} - \sqrt{3}$, simplify:

- (a) $2r - 2s$ (b) $5rs$ (c) $r^2 - s^2$

5. A rectangle has sides of length $(2\sqrt{2} + 2)$ cm and $(2\sqrt{2} - 2)$ cm.

Calculate:

- (a) its area (b) the length of a diagonal

$$2\sqrt{2} - 2$$

$$2\sqrt{2} + 2$$

E. Rationalising a surd denominator

Exercise 6

1. Rationalise the denominators in the following and simplify where possible:

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$
(e) $\frac{1}{\sqrt{7}}$ (f) $\frac{10}{\sqrt{5}}$ (g) $\frac{2}{\sqrt{3}}$ (h) $\frac{3}{\sqrt{5}}$
(i) $\frac{20}{\sqrt{2}}$ (j) $\frac{6}{\sqrt{3}}$ (k) $\frac{12}{\sqrt{6}}$ (l) $\frac{3}{2\sqrt{5}}$
(m) $\frac{4}{5\sqrt{2}}$ (n) $\frac{1}{\sqrt{20}}$ (o) $\frac{1}{\sqrt{50}}$ (p) $\frac{4}{\sqrt{8}}$

2. Express each of the following in its simplest form with a rational denominator:

- (a) $\frac{\sqrt{4}}{\sqrt{3}}$ (b) $\frac{\sqrt{5}}{\sqrt{2}}$ (c) $\sqrt{\frac{4}{10}}$ (d) $\sqrt{\frac{1}{11}}$ (e) $\sqrt{\frac{3}{5}}$ (f) $\sqrt{\frac{a}{b}}$

3. Simplify the following by rationalising the denominator:

- (a) $\frac{1}{\sqrt{2} - 1}$ (b) $\frac{1}{\sqrt{7} - 1}$ (c) $\frac{1}{2 + \sqrt{2}}$ (d) $\frac{4}{\sqrt{5} + 1}$
(e) $\frac{3}{2 - \sqrt{3}}$ (f) $\frac{1}{\sqrt{3} - \sqrt{2}}$ (g) $\frac{2}{\sqrt{5} + \sqrt{3}}$ (h) $\frac{9}{\sqrt{5} - \sqrt{2}}$

F. Simplifying expressions using the laws of indices

Exercise 7

$$\text{Rule 1 } a^m \times a^n = a^{m+n}$$

1. Use Rule 1 to write down the simplest form of the products in the following:

- | | | | |
|----------------------|----------------------------|-----------------------|------------------------------|
| (a) $2^3 \times 2^4$ | (b) $3^5 \times 3^3$ | (c) $8^2 \times 8^5$ | (d) $10^{10} \times 10^{20}$ |
| (e) $a^3 \times a^4$ | (f) $b^5 \times b^3$ | (g) $c^2 \times c^6$ | (h) $d^5 \times d^5$ |
| (i) $v^3 \times v^8$ | (j) $x^2 \times x^{10}$ | (k) $w^3 \times w$ | (l) $z \times z^5$ |
| (m) $f^4 \times f^7$ | (n) $g^{10} \times g^{10}$ | (o) $k \times k^{11}$ | (p) $m^{100} \times m$ |

$$\text{Rule 2 } a^m \div a^n = a^{m-n}$$

2. Use Rule 2 to write down the simplest form of the quotients in the following:

- | | | | |
|--------------------------|--------------------------|---------------------|----------------------------|
| (a) $2^4 \div 2^3$ | (b) $3^5 \div 3^3$ | (c) $8^5 \div 8^2$ | (d) $10^{20} \div 10^{10}$ |
| (e) $a^7 \div a^6$ | (f) $b^5 \div b^3$ | (g) $c^9 \div c^4$ | (h) $d^5 \div d^5$ |
| (i) $v^{12} \div v^{11}$ | (j) $x^{10} \div x^2$ | (k) $w^3 \div w$ | (l) $z^2 \div z$ |
| (m) $f^7 \div f^4$ | (n) $g^{10} \div g^{10}$ | (o) $k^{11} \div k$ | (p) $m^{100} \div m$ |
| (q) $\frac{x^6}{x^2}$ | (r) $\frac{m^3}{m^2}$ | (s) $\frac{a^5}{a}$ | (t) $\frac{r^9}{r^8}$ |

$$\text{Rule 3 } (a^m)^n = a^{mn}$$

3. Simplify these. For example, $(a^3)^4 = a^{12}$.

- | | | | |
|---------------|---------------|---------------|---------------|
| (a) $(x^2)^3$ | (b) $(y^3)^5$ | (c) $(z^5)^2$ | (d) $(g^2)^8$ |
| (e) $(a^3)^7$ | (f) $(b^4)^4$ | (g) $(c^5)^6$ | (h) $(d^3)^7$ |

4. Express the following without brackets, writing answers in index form:

- | | | | |
|---------------|---------------|---------------|---------------|
| (a) $(2^5)^2$ | (b) $(7^3)^5$ | (c) $(6^5)^4$ | (d) $(8^3)^5$ |
| (e) $(2^7)^7$ | (f) $(3^4)^3$ | (g) $(9^5)^2$ | (h) $(2^5)^5$ |

5. Note $(ab)^m = a^m b^m$. Use this result to simplify:

- | | | | |
|--------------|--------------|-------------------|---------------|
| (a) $(ab)^3$ | (b) $(cd)^6$ | (c) $(x^2y)^{10}$ | (d) $(2pq)^2$ |
|--------------|--------------|-------------------|---------------|

6. Use the 3 rules learned so far to simplify the following:

- | | | | |
|-----------------------|-----------------------|----------------------------------|----------------------------------|
| (a) $y^3 \times y^2$ | (b) $t^4 \times t$ | (c) $x^5 \div x^2$ | (d) $v^7 \div v$ |
| (e) $3x^2 \times x^3$ | (f) $x^2 \times 4x^5$ | (g) $2x^2 \times 6x^2$ | (h) $8x^5 \div 2x^2$ |
| (i) $6x^3 \div 2x$ | (j) $x^2(x^3 + x^4)$ | (k) $x^3(x^3 - 4)$ | (l) $(uv)^7$ |
| (m) $(5y)^2$ | (n) $(mn^2)^8$ | (o) $\frac{x^3 \times x^4}{x^2}$ | (p) $\frac{u^7 \times u^2}{u^3}$ |

7. Find m . For example: if $2^m = 8$, $m = 3$ since $2^3 = 8$.

- (a) $2^m = 16$ (b) $2^m = 32$ (c) $4^m = 64$ (d) $5^m = 625$

Exercise 8

Rule 4 $a^0 = 1$

1. Write down the values of:

- (a) 2^0 (b) 12^0 (c) x^0 (d) $(32546)^0$

Rule 5 $a^{-m} = 1/a^m$

2. Write the following with positive indices. For example: $2^{-5} = 1/2^5$.

- (a) 3^{-2} (b) 5^{-7} (c) a^{-4} (d) b^{-9}
 (e) x^{-1} (f) $3y^{-2}$ (g) xy^{-3} (h) $\frac{1}{x^{-3}}$
 (i) $\frac{1}{t^{-5}}$ (j) $\frac{6}{c^{-3}}$ (k) $\frac{1}{2y^{-2}}$ (l) $\frac{1}{7x^{-3}}$

3. Express these in a form without indices:

- (a) 5^0 (b) 7^{-1} (c) 3^{-3} (d) 8^{-2} (e) $\frac{1}{2^{-3}}$ (f) $\left(\frac{2}{3}\right)^{-2}$

4. Simplify the following. For example: $x^{-2} \times x^5 = x^{-2+5} = x^3$.

- (a) $a^{-2} \times a^4$ (b) $b^6 \times b^{-4}$ (c) $c^{-1} \times c^{-1}$ (d) $d^{-6} \times d^6$
 (e) $e^6 \div e^{-3}$ (f) $g^{-4} \div g^4$ (g) $w^{-3} \div w^{-5}$ (h) $(x^{-2})^3$
 (i) $(y^5)^{-1}$ (j) $(z^{-4})^{-4}$ (k) $(klm)^0$.

5. Express with positive indices:

- (a) $4^2 \times 4^{-5}$ (b) $6^{-2} \times 6^{-3}$ (c) $2^7 \times 2^{-2}$ (d) $4^{-3} \times 5^2$
 (e) $3^4 \times 2^{-3}$ (f) $(9^0)^{-8}$ (g) $(w^{-3})^{-2}$ (h) $(x^{-2})^{\frac{1}{2}}$
 (i) $\frac{1}{2^{-2}}$ (j) $\frac{5}{2^{-3}}$ (k) $6h^{-2}$ (l) $9s^{-1}$
 (m) $\frac{1}{2}k^{-1}$ (n) $\frac{3}{4}m^{-3}$

6. Multiply out the brackets:

- (a) $x^5(x^2 + x^{-2})$ (b) $x^3(x - x^{-2})$ (c) $x^{-3}(x^4 + x)$ (d) $x^{-1}(x + x^2)$
 (e) $x(x^2 - x^{-7})$ (f) $x^{-5}(2 - x^5)$ (g) $2x^{-2}(3 - x^3)$ (h) $3x^2(2x - x^{-2})$

7. (a) Find the value of: 3^3 , 3^0 , 3^{-2} , 3^1 , 3^{-3} .

(b) Write in the form 4^{-p} : $1/4$, $1/16$, $1/64$.

Exercise 9

$$\text{Rule 6 } a^{m/n} = \sqrt[n]{a^m}$$

1. Write these in root form. For example: $x^{\frac{4}{5}} = \sqrt[5]{x^4}$

(a) $x^{\frac{3}{4}}$ (b) $m^{\frac{3}{5}}$ (c) $r^{\frac{2}{3}}$ (d) $w^{\frac{1}{2}}$ (e) $n^{\frac{1}{3}}$ (f) $r^{-\frac{4}{3}}$

2. Write these in index form. For example: $\sqrt[5]{x^4} = x^{4/5}$

(a) $\sqrt[3]{x^5}$ (b) $\sqrt[3]{b^4}$ (c) $\sqrt[3]{z^2}$ (d) $\sqrt[4]{w}$ (e) $\frac{1}{\sqrt[3]{x}}$ (f) \sqrt{u}

3. Evaluate the following. For example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (2)^2 = 4$

(a) $9^{\frac{1}{2}}$ (b) $64^{\frac{1}{2}}$ (c) $8^{\frac{1}{3}}$ (d) $64^{\frac{1}{3}}$ (e) $100^{\frac{1}{2}}$ (f) $27^{\frac{1}{3}}$
(g) $9^{\frac{3}{2}}$ (h) $16^{\frac{3}{4}}$ (i) $49^{\frac{3}{2}}$ (j) $16^{-\frac{1}{2}}$ (k) $27^{-\frac{1}{3}}$ (l) $25^{-\frac{3}{2}}$
(m) $81^{\frac{3}{4}}$ (n) $81^{-\frac{3}{4}}$ (o) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ (p) $\left(\frac{1}{8}\right)^{\frac{5}{3}}$

4. Simplify:

(a) $(x^4)^{\frac{1}{2}}$ (b) $(m^9)^{\frac{1}{3}}$ (c) $(c^6)^{\frac{2}{3}}$ (d) $(n^{-15})^{\frac{1}{5}}$
(e) $(n^{-2})^{\frac{1}{4}}$ (f) $\left(g^{-\frac{2}{3}}\right)^{-3}$ (g) $\left(b^{\frac{5}{2}}\right)^{-2}$ (h) $\left(z^{-\frac{3}{2}}\right)^{-2}$
(i) $\left(x^{\frac{4}{5}}\right)^{\frac{1}{2}}$ (j) $\left(x^2\right)^{\frac{4}{3}}$ (k) $(5^2 + 12^2)^{\frac{1}{2}}$ (l) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$

Exercise 10

Miscellaneous examples

1. Simplify:

(a) $a^{\frac{3}{2}} \times a^{\frac{1}{2}}$ (b) $b^{\frac{5}{4}} \times b^{-\frac{1}{4}}$ (c) $c^{\frac{1}{3}} \times c^{-\frac{1}{3}}$ (d) $d^{\frac{5}{2}} \div d^{\frac{1}{2}}$
(e) $e^{\frac{3}{5}} \div e^{-\frac{1}{5}}$ (f) $\left(z^{\frac{1}{2}}\right)^2$ (g) $\left(w^{-\frac{1}{2}}\right)^2$ (h) $\left(w^{-\frac{1}{2}}\right)^0$

2. Simplify:

(a) $2a^{1/2} \times 2a^{-1/2}$ (b) $3b^{2/3} \times 4b^{1/3}$ (c) $6c^{1/2} \times c^{1/2}$ (d) $2d^{3/2} \times d^{-1/2}$
(e) $4e^{3/2} \div 2e^{1/2}$ (f) $8v^{3/4} \div 2v^{-1/4}$ (g) $15z^{-1/3} \div 15z^{-1/3}$

3. Multiply out the brackets:

(a) $x^{1/2}(x^{1/2} - x^{-1/2})$ (b) $x^{2/3}(x^{4/3} + x^{1/3})$ (c) $x^{-3/5}(x^{6/5} + x^{-1/5})$

4. Evaluate the following for $x = 16$ and $y = 27$:

(a) $3x^{1/2}$ (b) $4x^{3/4}$ (c) $5y^{2/3}$ (d) $8y^{-2/3}$ (e) $x^{-1/4} \times y^{1/3}$

5. Simplify the following, expressing your answers with positive indices:

(a) $p^6 \times p^{1/2}$ (b) $p^5 \div p^{-1/2}$ (c) $(p^{2/3})^{-3/2}$ (d) $(p^{-3/4})^{12}$
 (e) $5p^{3/4} \div 5p^{-3/4}$ (f) $7p^{1/2} \times 7p^{-3/2}$

6. Simplify:

(a) $\frac{x^4 \times x^{-3}}{x}$ (b) $\frac{x^{-1} \times x^5}{x^{-2}}$ (c) $\frac{x^{-3} \times x^3}{x^{-1}}$ (d) $\frac{x^{\frac{1}{4}} \times x^{-\frac{3}{4}}}{x^4}$
 (e) $\frac{x^{-\frac{1}{2}} \times x^{\frac{3}{2}}}{x}$ (f) $\frac{x^{\frac{3}{5}} \times x^{-\frac{3}{5}}}{x^{-1}}$

7. Multiply out the brackets:

(a) $(x^3 - 1)(x^{-3} - 1)$ (b) $(x^{-1} + 2)(x^{-1} - 2)$ (c) $(x^{1/2} + 4)(x^{-1/2} - 4)$

Exercise 11

Express the following with x in the numerator in index form:

For example :- $\frac{2}{5\sqrt{x}} = \frac{2}{5x^{\frac{1}{2}}} = \frac{2x^{-\frac{1}{2}}}{5} = \frac{2}{5}x^{-\frac{1}{2}}$

1. $\frac{1}{x}$ 2. $\frac{1}{x^2}$ 3. $\frac{7}{x^2}$ 4. $\frac{5}{x^3}$ 5. $\frac{1}{3x}$ 6. $\frac{4}{5x}$
 7. $\frac{3}{2x^2}$ 8. $\frac{1}{\sqrt{x}}$ 9. $\frac{5}{2\sqrt{x}}$ 10. $\frac{3}{\frac{5x^3}{m}}$ 11. $\frac{1}{\sqrt{(3x^6)}}$ 12. $\frac{2}{\sqrt{(5x^3)}}$
 $\frac{a^3 \times a^2}{a}$

$\frac{1}{4x^{-2}}$

$$\frac{x^{-1/4} \times x^{3/4}}{x^{-1/2}}$$

Checkup for algebraic operations

1. Factorise the numerator and/or denominator if possible, then simplify:

(a) $\frac{7v}{14v}$ (b) $\frac{9a^2b}{3ab^2}$ (c) $\frac{6}{6x-18}$ (d) $\frac{v^2 - vw}{v}$
 (e) $\frac{3x+3y}{6x+9y}$ (f) $\frac{a-8}{a^2-64}$ (g) $\frac{v^2+3v+2}{v^2-4}$

2. Simplify:

(a) $\frac{b^3}{4} \times \frac{2}{b}$ (b) $\frac{a^2}{x} \times \frac{x}{a}$ (c) $\frac{z^2}{2} \div \frac{z}{6}$ (d) $\frac{a^2}{b} \div \frac{a}{b}$
 (e) $\frac{c}{4} + \frac{c}{8}$ (f) $\frac{3u}{4} - \frac{2v}{5}$ (g) $\frac{3}{k} + \frac{1}{m}$ (h) $\frac{x+1}{2} - \frac{x+2}{5}$

3. Change the subject of each formula to x .

(a) $5 + x = 6$ (b) $x - a = w$ (c) $ax + m = p$ (d) $x/z = p/w$
 (e) $N = 2\pi x^2$ (f) $T = 4x + 3$ (g) $w = 1/2(x + y)$ (h) $M = \frac{5}{8x^2}$

4. Express each of these in its simplest form:

(a) $\sqrt{32}$ (b) $\sqrt{1000}$ (c) $2\sqrt{45}$ (d) $\sqrt{45} - \sqrt{20}$
 (e) $\sqrt{50} - \sqrt{18}$ (f) $\sqrt{72} + \sqrt{50}$ (g) $\sqrt{\frac{9}{a^2}}$ (h) $2\sqrt{8} - \sqrt{2}$

5. If $x = 1 + \sqrt{3}$ and $y = 1 - \sqrt{3}$, simplify:

(a) $2x + 2y$ (b) $5xy$ (c) $x^2 + y^2$

6. Rationalise the denominators in the following and simplify where possible:

(a) $1/\sqrt{5}$ (b) $8/\sqrt{2}$ (c) $15/\sqrt{5}$ (d) $\sqrt{2}/\sqrt{6}$

7. Write in their simplest form:

(a) $5^6 \times 5^8$ (b) $x^8 \div x^6$ (c) (d) $(a^2)^3$
 (e) $(4a^3b)^2$ (f) (g) $(p^2)^0$ (h) $x^3(x^2 - x)$

8. Write with positive indices:

(a) 5^{-2} (b) ab^{-3} (c) $(y^{-3})^{-2}$ (d)

9. Write these in root form:

(a) $b^{1/2}$ (b) $c^{-3/2}$

10. Write these in index form:

(a) $\sqrt[3]{x^4}$ (b) $1/\sqrt{a^3}$

11. Evaluate:

(a) $36^{3/2}$ (b) $32^{-2/5}$ (c) $(x^8)^{1/2}$ (d) $(y^{7/2})^{-2}$
 (e) $a^{-3/2}(a^{1/2} - a^{-5/2})$ (f) $2s^{1/2} \div 4s^{-1/2}$ (g)

QUADRATIC FUNCTIONS

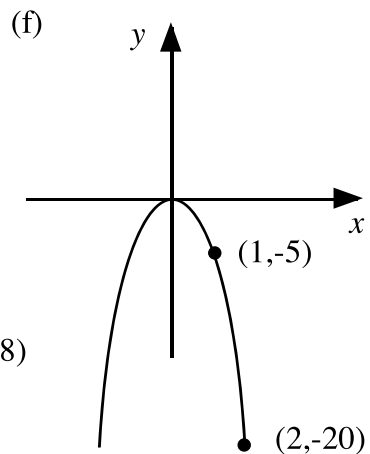
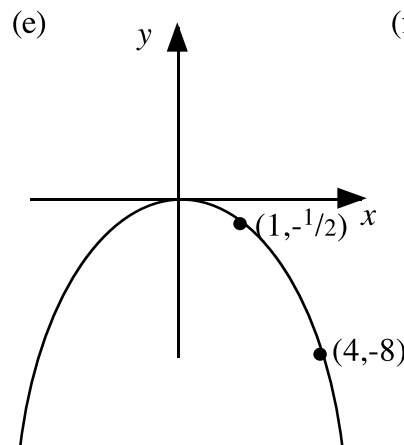
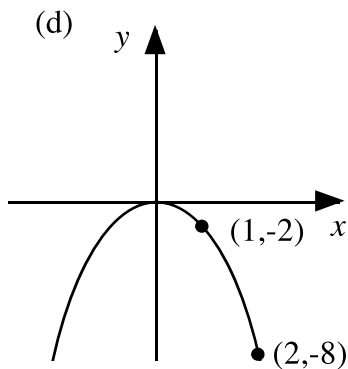
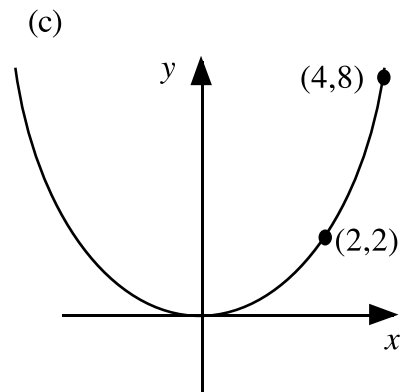
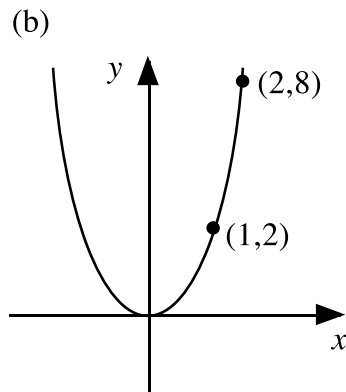
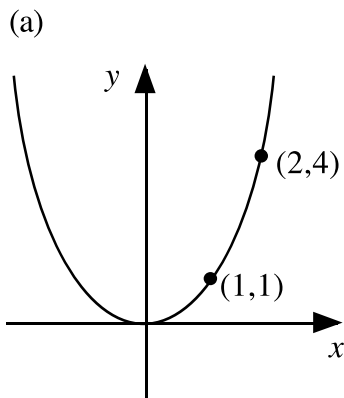
By the end of this unit, you should be able to:

- (a) recognise quadratics of the form $y = kx^2$ and $y = (x + a)^2 + b$; $a, b \in \mathbb{Z}$, from their graphs
- (b) identify the nature and coordinates of the turning points and the equation of the axis of symmetry of a quadratic of the form $y = k(x + a)^2 + b$; $a, b \in \mathbb{Z}$, $k = \pm 1$
- (c) know the meaning of 'root of a quadratic equation' and solve a quadratic equation graphically
- (d) solve quadratic equations by factorisation
- (e) solve quadratic equations by using the quadratic formula

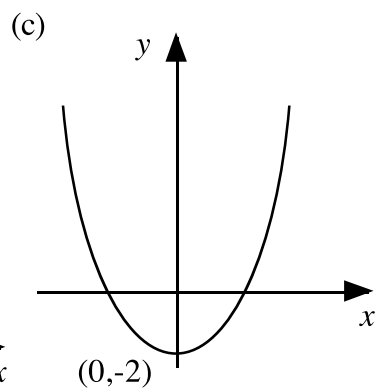
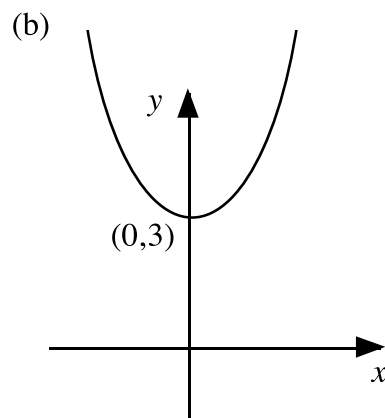
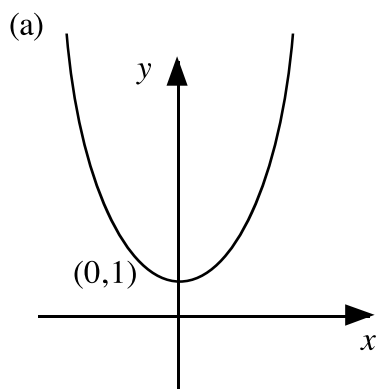
A. Recognising quadratics from their graphs

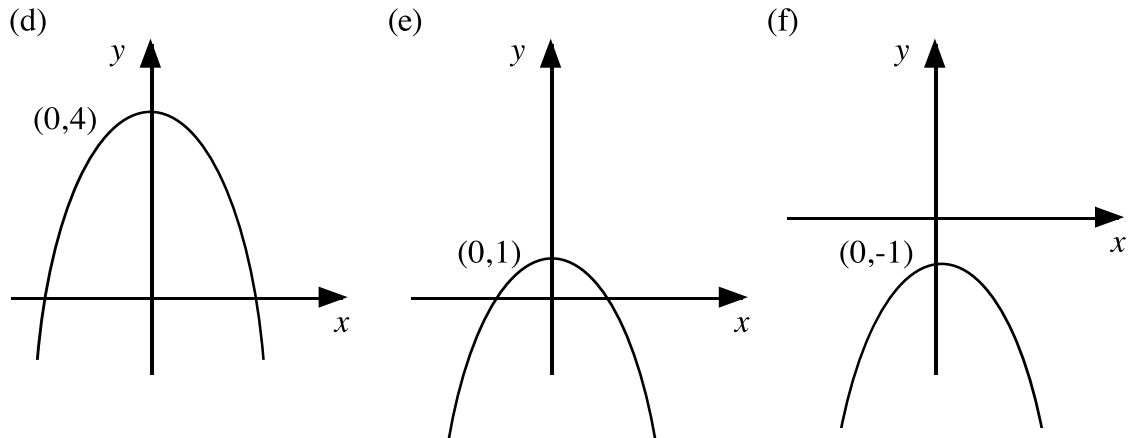
Exercise 1

1. Each of the following graphs represents a simple parabola of the form $y = kx^2$. Find k each time and hence write down the equation representing each parabola.

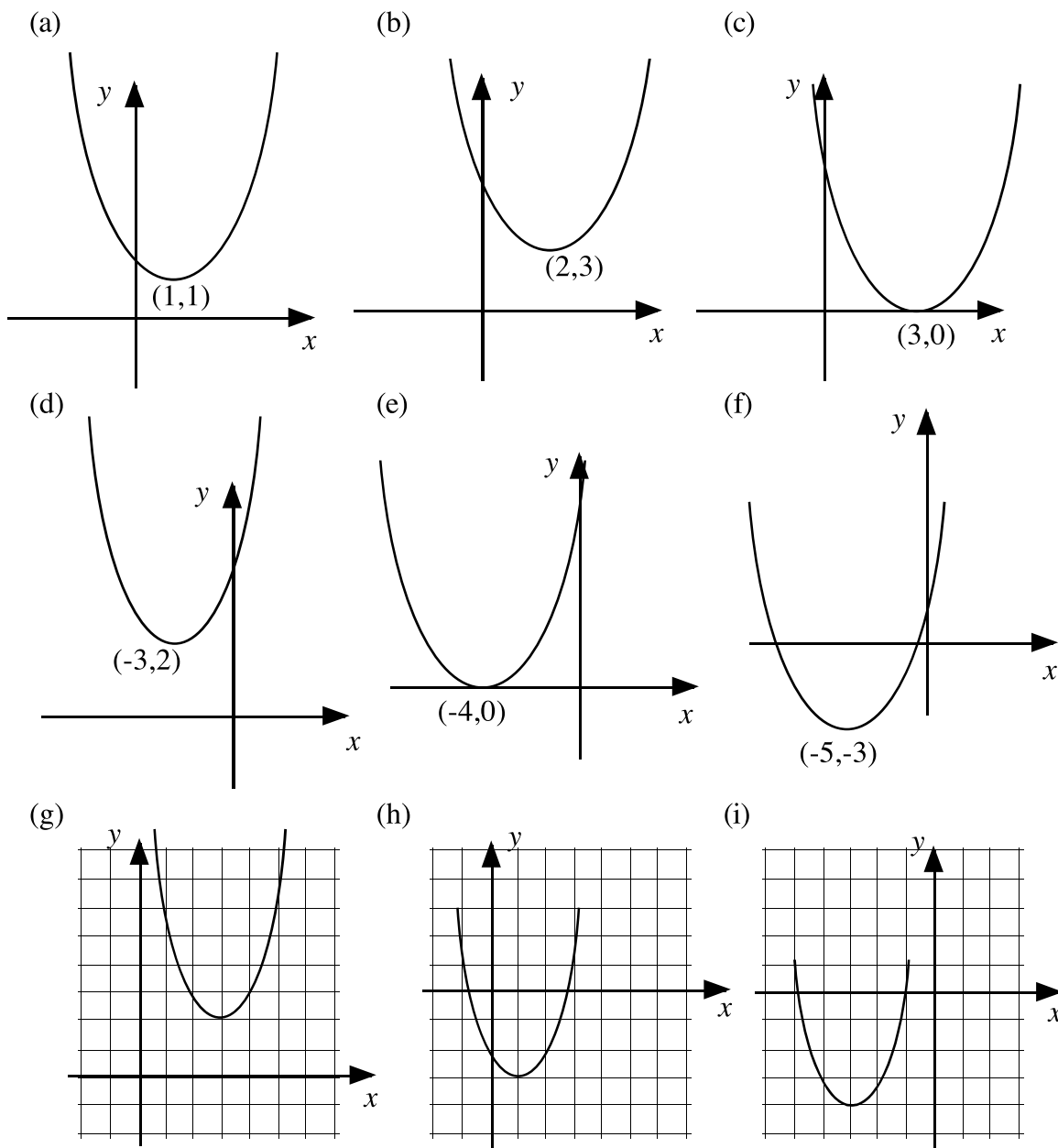


2. Each of the following parabolas can be represented by the equation $y = x^2 + b$ or $y = -x^2 + b$, where b is an integer. Write down their equations.





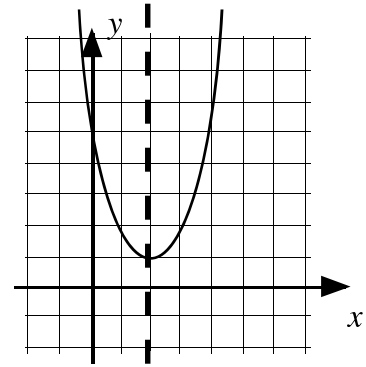
3. Each of the following parabolas can be represented by an equation of the form $y = (x + a)^2 + b$, (where a and b are integers). Write down the equation of each one.



B. The nature and coordinates of turning points

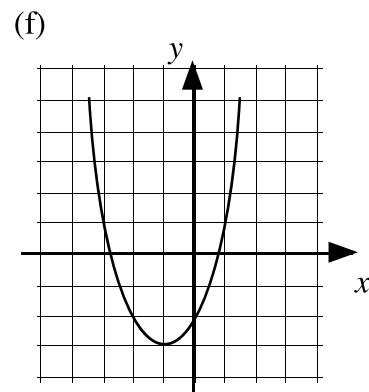
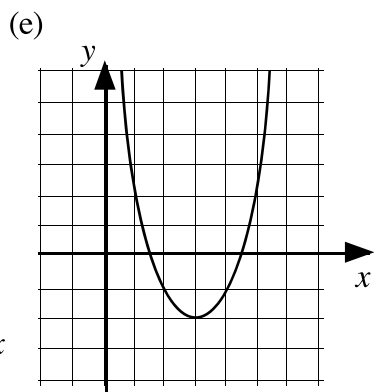
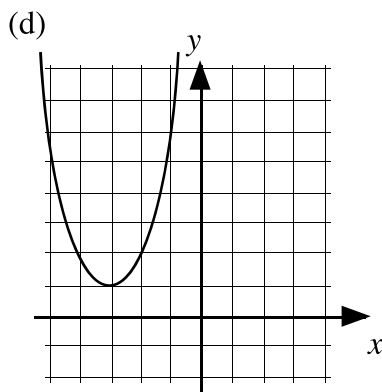
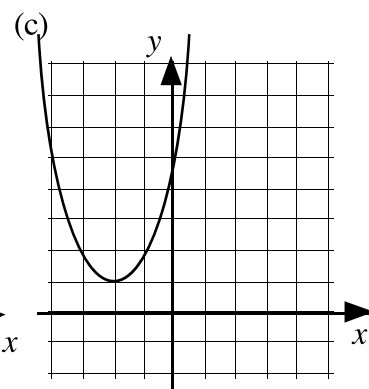
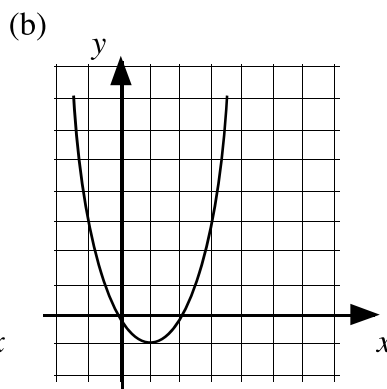
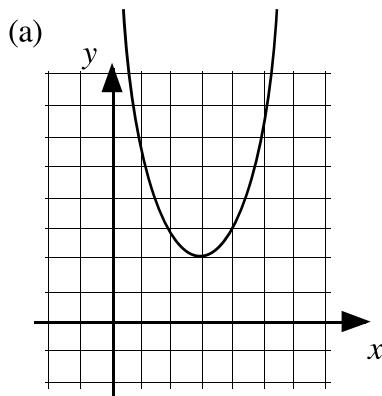
Exercise 2

- Write down the coordinates of the minimum turning point of this parabola.
 - Write down the equation of the axis of symmetry. (the dotted line).
 - The equation of the parabola is of the form $y = (x + a)^2 + b$.



Find the values of a and b and hence write down the equation of the parabola.

- For each of the following parabolas:
 - write down the coordinates of its minimum turning point.
 - write down the equation of its axis of symmetry.
 - write down the equation of the function represented by the parabola.



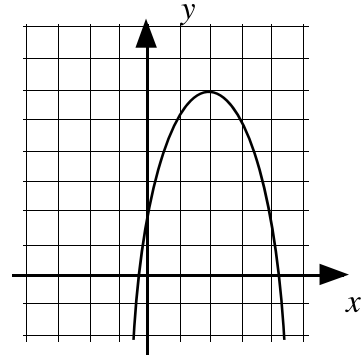
- Without making a sketch, write down the coordinates of the minimum turning points and the equation of the axes of symmetry of these parabolas.

(a) $y = (x - 4)^2 + 1$	(b) $y = (x - 2)^2 + 7$	(c) $y = (x - 8)^2 + 3$
(d) $y = (x + 1)^2 + 2$	(e) $y = (x - 1)^2 - 3$	(f) $y = (x + 3)^2 - 7$
(g) $y = (x - 5)^2$	(h) $y = (x + 2)^2$	(i) $y = x^2 + 3$

4. This parabola is of the form

$$y = -(x + a)^2 + b$$

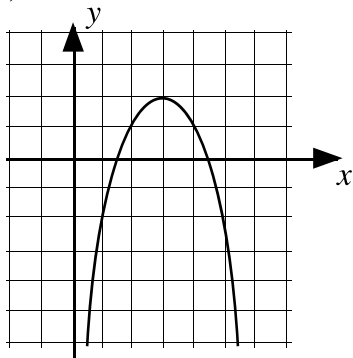
- Write down the coordinates of the maximum turning point.
- Write down the equation of the axes of symmetry.
- Find the values of a and b and hence write down the equation of the function represented by the parabola.



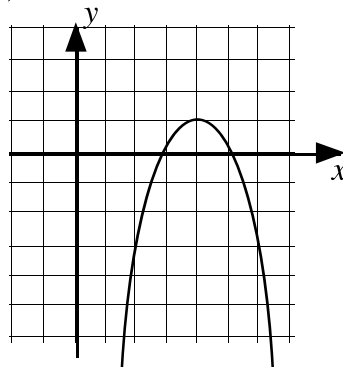
5. For each of the following parabolas:

- write down the coordinates of its maximum turning point.
- write down the equation of its axis of symmetry.
- write down the equation of the function represented by the parabola.

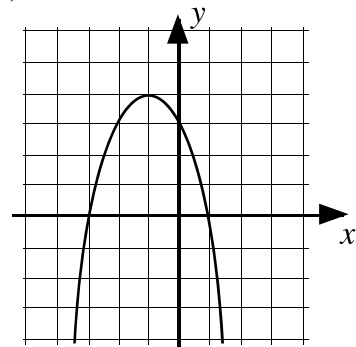
(a)



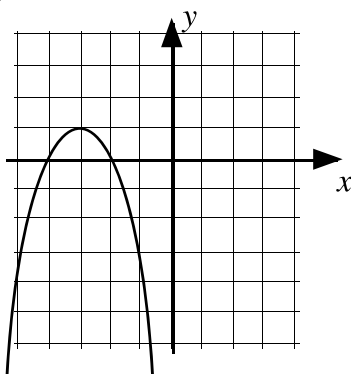
(b)



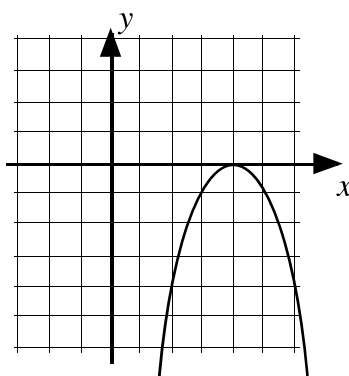
(c)



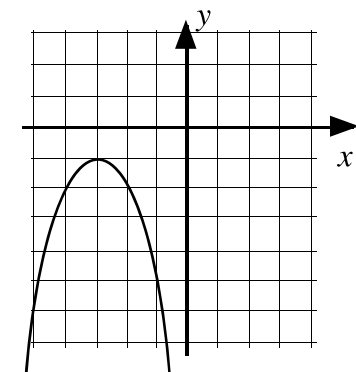
(d)



(e)



(f)



6. Without a sketch, write down the coordinates of the maximum turning points and the equation of the axes of symmetry of these parabolas.

(a) $y = -(x - 2)^2 + 6$

(b) $y = -(x - 5)^2 + 1$

(c) $y = -(x - 6)^2 - 2$

(d) $y = -(x + 1)^2 + 7$

(e) $y = -(x + 4)^2 - 5$

(f) $y = -(x + 3)^2$

(g) $y = 7 - (x - 1)^2$

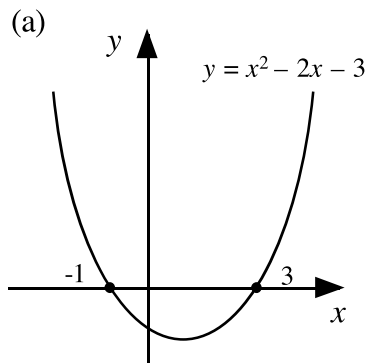
(h) $y = 1 - (x - 8)^2$

(i) $y = -2 - (x + 5)^2$

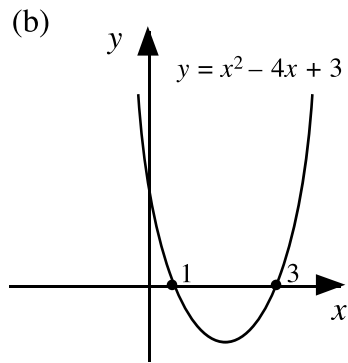
C. Solve quadratic equations graphically

Exercise 3

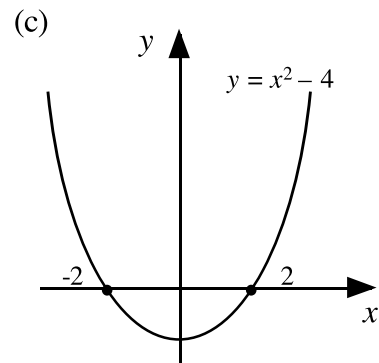
1. In each of the following, the graph has been sketched for you. Solve the quadratic equation associated with it.



Solve $x^2 - 2x - 3 = 0$



Solve $x^2 - 4x + 3 = 0$

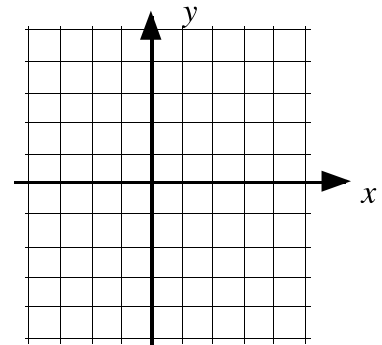


Solve $x^2 - 4 = 0$

2. For the quadratic function $y = x^2 - x - 2$

(a) Copy and complete this table.

x	-2	-1	0	1	2	3
$y = x^2 - x - 2$	4	...	-2



(b) Draw a set of axes, plot the 6 points above and draw a smooth (parabolic) curve through them.

(c) Use your graph to solve the quadratic equation, $x^2 - x - 2 = 0$ [i.e. find the 2 roots]

3. Solve each of the following quadratic equations by:

(i) completing the table

(ii) plotting the points and drawing the smooth parabola

(iii) reading off the roots from the graph.

(a) $x^2 - 4x = 0$	<table border="1"> <tr> <td>x</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>$y = x^2 - 4x$</td> <td>5</td> <td>...</td> <td>-3</td> <td>...</td> <td>...</td> <td>...</td> <td>...</td> </tr> </table>	x	-1	0	1	2	3	4	5	$y = x^2 - 4x$	5	...	-3		
x	-1	0	1	2	3	4	5												
$y = x^2 - 4x$	5	...	-3												
(b) $x^2 + x - 2 = 0$	<table border="1"> <tr> <td>x</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>$y = x^2 + x - 2$</td> <td>4</td> <td>...</td> <td>-2</td> <td>...</td> <td>...</td> <td>...</td> </tr> </table>	x	-3	-2	-1	0	1	2	$y = x^2 + x - 2$	4	...	-2				
x	-3	-2	-1	0	1	2													
$y = x^2 + x - 2$	4	...	-2													
(c) $x^2 - 6x + 8 = 0$	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>$y = x^2 - 6x + 8$</td> <td>8</td> <td>...</td> <td>...</td> <td>...</td> <td>...</td> <td>...</td> </tr> </table>	x	0	1	2	3	4	5	$y = x^2 - 6x + 8$	8				
x	0	1	2	3	4	5													
$y = x^2 - 6x + 8$	8													
(d) $6 - x - x^2 = 0$	<table border="1"> <tr> <td>x</td> <td>-4</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$y = 6 - x - x^2$</td> <td>-6</td> <td>...</td> <td>4</td> <td>...</td> <td>...</td> <td>...</td> <td>...</td> <td>...</td> </tr> </table>	x	-4	-3	-2	-1	0	1	2	3	$y = 6 - x - x^2$	-6	...	4
x	-4	-3	-2	-1	0	1	2	3											
$y = 6 - x - x^2$	-6	...	4											

(e) $x^2 - 4 = 0$	x	-3	-2	-1	0	1	2	3
	$y = x^2 - 4$	5	-4

(f) $5 + 4x - x^2 = 0$	x	-2	-1	0	1	2	3	4	5	6
	$y = 5 + 4x - x^2$

D. Solve quadratic equations by factorising

Exercise 4

In this exercise you are going to solve quadratic equations by factorisation.

1. By considering a common factor, factorise and solve the following:

- | | | |
|---------------------|----------------------|-----------------------|
| (a) $x^2 - 4x = 0$ | (b) $x^2 - 10x = 0$ | (c) $8x - x^2 = 0$ |
| (d) $x^2 + 6x = 0$ | (e) $x^2 + x = 0$ | (f) $x^2 - x = 0$ |
| (g) $2x^2 - 6x = 0$ | (h) $5x^2 + 15x = 0$ | (i) $12x^2 - 18x = 0$ |

2. By considering the difference of 2 squares, factorise and solve the following quadratic equations:

- | | | |
|--------------------|---------------------|----------------------|
| (a) $x^2 - 4 = 0$ | (b) $x^2 - 9 = 0$ | (c) $x^2 - 25 = 0$ |
| (d) $16 - x^2 = 0$ | (e) $x^2 - 100 = 0$ | (f) $x^2 = 49$ |
| (g) $x^2 = 81$ | (h) $x^2 - 9 = 0$ | (i) $25x^2 - 16 = 0$ |

3. Factorise the following trinomials and solve the quadratic equations:

- | | | |
|-------------------------|--------------------------|--------------------------|
| (a) $x^2 + 3x + 2 = 0$ | (b) $x^2 - 5x + 6 = 0$ | (c) $x^2 + 6x + 5 = 0$ |
| (d) $x^2 - 9x + 20 = 0$ | (e) $x^2 + 7x + 10 = 0$ | (f) $x^2 - 6x + 9 = 0$ |
| (g) $x^2 - 7x + 12 = 0$ | (h) $x^2 - 8x + 7 = 0$ | (i) $x^2 - 13x + 42 = 0$ |
| (j) $x^2 + 3x - 10 = 0$ | (k) $x^2 - 3x - 4 = 0$ | (l) $x^2 + 2x - 8 = 0$ |
| (m) $x^2 - x - 20 = 0$ | (n) $x^2 + x - 12 = 0$ | (o) $x^2 + 2x - 35 = 0$ |
| (p) $x^2 + 4x - 12 = 0$ | (q) $x^2 + 3x - 18 = 0$ | (r) $x^2 + 21x + 20 = 0$ |
| (s) $x^2 - 9x + 8 = 0$ | (t) $x^2 - 10x - 24 = 0$ | (u) $x^2 + 5x - 24 = 0$ |
| (v) $x^2 - 2x - 24 = 0$ | (w) $x^2 - 23x - 24 = 0$ | (x) $x^2 - 15x + 54 = 0$ |

4. The following are harder and take a little longer to do.
Solve:

(a) $2x^2 + 7x + 3 = 0$	(b) $2x^2 + 5x + 3 = 0$	(c) $3x^2 + 7x + 2 = 0$
(d) $2x^2 - 9x + 9 = 0$	(e) $3x^2 + 11x + 6 = 0$	(f) $5x^2 + 11x + 2 = 0$
(g) $3x^2 - 2x - 8 = 0$	(h) $3x^2 - 5x - 2 = 0$	(i) $3x^2 + 2x - 1 = 0$
(j) $2x^2 - 7x - 4 = 0$	(k) $5x^2 + 13x - 6 = 0$	(l) $2x^2 + 9x + 10 = 0$

5. Rearrange the following into quadratic equations of the form $ax^2 + bx + c = 0$ and solve them:

(a) $x(x + 2) = 3$	(b) $x(x - 1) = 20$	(c) $x(x - 3) = 10$
(d) $x(x - 5) = 6$	(e) $x(x + 3) = 70$	(f) $x(x + 1) = 56$
(g) $(x + 1)(x + 2) = 12$	(h) $(x - 1)(x + 2) = 28$	(i) $(x + 3)(x - 1) = 5$
(j) $(x - 1)(x + 1) = 8$	(k) $(x - 2)(x + 2) = 21$	(l) $(x - 2)(x - 3) = 2$
(m) $2x^2 + 3x - 1 = x^2 - x - 4$	(n) $3x^2 + 5x - 8 = 2x^2 + 2x + 2$	
(o) $2x(x + 2) = 16$	(p) $x(2x - 3) = 20$	(q) $x(3x - 1) = 10$
(r) $x(5x + 2) = 7$	(s) $(2x - 1)(x + 3) = 30$	(t) $(x + 1)(3x - 2) = 12$

E. Solve quadratic equations using the formula

Exercise 5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Solve the following quadratic equations using the formula, correct to 2 decimal places:

(a) $x^2 + 4x + 1 = 0$	(b) $x^2 + 3x + 1 = 0$	(c) $x^2 + 6x + 2 = 0$
(d) $x^2 + 10x + 7 = 0$	(e) $x^2 - 4x + 2 = 0$	(f) $x^2 - 5x + 5 = 0$
(g) $2x^2 + 7x + 2 = 0$	(h) $3x^2 + 10x + 5 = 0$	(i) $4x^2 - 7x + 2 = 0$

2. Solve the following quadratic equations using the formula, correct to 3 decimal places:

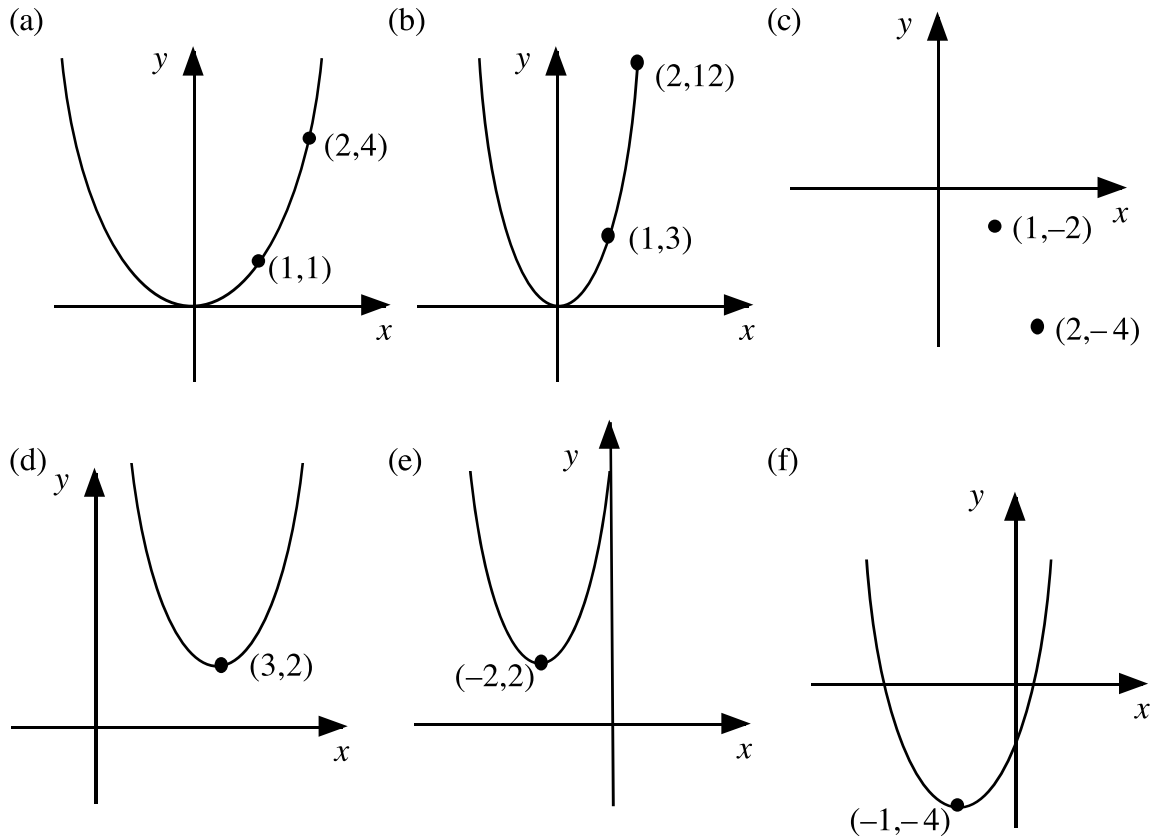
(a) $x^2 + 2x - 2 = 0$	(b) $x^2 + x - 5 = 0$	(c) $x^2 + 3x - 6 = 0$
(d) $x^2 - 2x - 5 = 0$	(e) $x^2 - 3x - 1 = 0$	(f) $x^2 + 8x - 2 = 0$
(g) $2x^2 + 3x - 4 = 0$	(h) $3x^2 - 2x - 2 = 0$	(i) $8x^2 - 3x - 1 = 0$

3. Rearrange the following and solve each equation giving the roots correct to 3 significant figures (3 figures of accuracy):

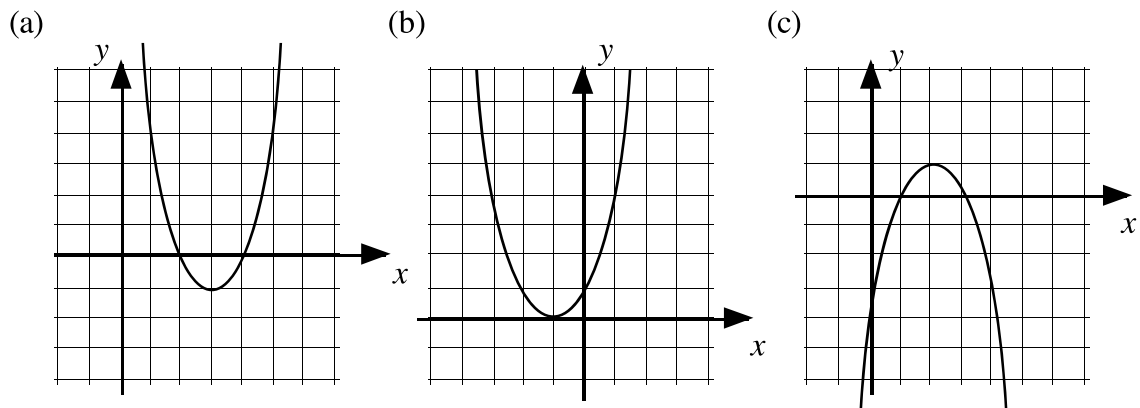
(a) $x(x + 5) = 2$	(b) $x(x - 3) = 1$	(c) $2x(x + 3) = 5$
(d) $3x(x - 1) = 10$	(e) $(x + 3)(x + 4) = 7$	(f) $(x - 1)^2 = 5$

Checkup for quadratic equations

1. Write down the equations of the quadratics corresponding to these parabolas:
 (each one is of the form $y = kx^2$ or $y = (x + a)^2 + b$)



2. (i) Write down the coordinates of the turning point for each of the following, stating whether it is a maximum or a minimum.
 (ii) Write down the equation of the axes of symmetry for each one.



3. Write down the coordinates of the turning point of each of the following.

State whether each is a maximum or a minimum.

Give also the equation of the axes of symmetry.

(a) $y = (x - 21)^2 + 5$ (b) $y = 3(x + 2)^2 - 1$ (c) $y = -2(x - 3)^2 + 2$

4. Solve each of the following quadratic equations by

(i) completing the table.

(ii) plotting the points and drawing the smooth parabola.

(iii) reading off the roots from the graph.

(a) $x^2 - 2x - 8 = 0$	x	-3	-2	-1	0	1	2	3	4	5
	$y = x^2 - 2x - 8$	7	-8

(b) $x^2 + 2x - 3 = 0$	x	-4	-3	-2	-1	0	1	2
	$y = x^2 + 2x - 3$	-3

5. Solve the following quadratic equations by factorising:

(a) $x^2 - 7x = 0$

(b) $x^2 - 9 = 0$

(c) $x^2 + 8x + 12 = 0$

(d) $6x^2 + 9x = 0$

(e) $25 - x^2 = 0$

(f) $x^2 - x - 30 = 0$

(g) $4x^2 - 9 = 0$

(h) $x^2 - 7x + 10 = 0$

(i) $2x^2 + 7x - 15 = 0$

(j) $x(x + 5) = 14$

(k) $(x - 1)(x + 2) = 18$

(l) $(x - 2)^2 = 16$

6. Solve the following quadratic equations using the formula.
(Give the answers correct to 2 decimal places):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) $x^2 + 6x + 3 = 0$

(b) $x^2 - 4x + 1 = 0$

(c) $3x^2 + 10x + 6 = 0$

(d) $x^2 + 5x - 2 = 0$

(e) $x^2 - 6x - 4 = 0$

(f) $2x^2 - x - 5 = 0$

FURTHER TRIGONOMETRY

By the end of this set of exercises, you should be able to

- (a) recognise the graphs of sine, cosine and tangent functions
- (b) sketch and identify other trigonometric functions
- (c) solve simple trigonometric equations (in degrees)
- (d) define the period of trigonometric functions, either from their graphs or from their equations
- (e) simplify trigonometric expressions using $\sin^2 x + \cos^2 x = 1$
and $\tan x = \frac{\sin x}{\cos x}$

A. Sine, cosine and tangent graphs

Exercise 1

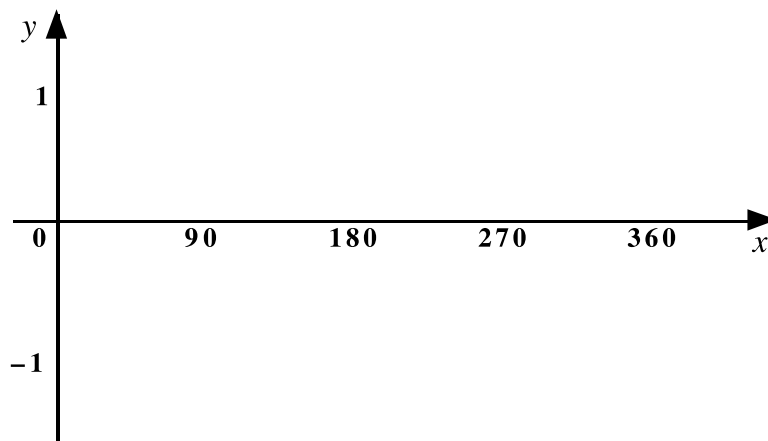
You may have drawn the sine, cosine and tangent graphs as part of the introduction to trigonometry in Maths 2 Intermediate 2. If you have retained the graphs, you may miss out questions 1 to 3 of Exercise 1 below.

1. The Sine Graph

- (a) Make a copy of this table and use your calculator to help fill it in, giving each answer correct to 2 decimal places.

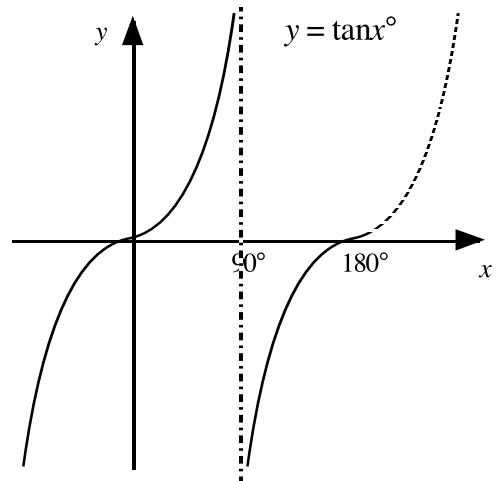
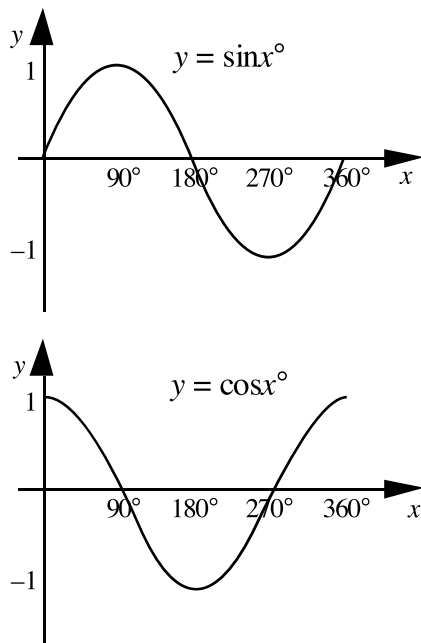
x	0°	20°	40°	60°	80°	90°	100°	120°	140°	160°	180°
$\sin x^\circ$	0.00	0.34	0.64	0.87	0.98	1.00
x	200°	220°	240°	260°	270°	280°	300°	320°	340°	360°	
$\sin x^\circ$	

- (b) Use a piece of 2 mm graph paper to draw a set of axes as illustrated below.



- (c) Plot as accurately as possible the 21 points from your table.
- (d) Join them up smoothly to create the graph of the function $y = \sin x^\circ$.
2. Repeat question 1 (a) to (d) for the function $y = \cos x^\circ$
3. Repeat for the graph of $y = \tan x^\circ$ (a different scale will be required for the vertical axis).
4. Study your three graphs carefully. You should now be able to sketch the sine, cosine (and tangent) graphs fairly quickly (about 30 seconds) indicating the main points where the graphs cut the x and y axes and the general shape of each graph.
- Try them now.

Sketches of the three trigonometric graphs, for comparison:

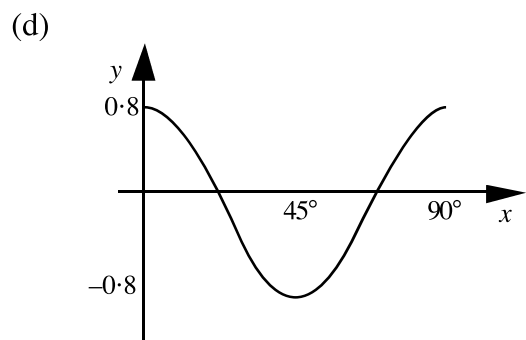
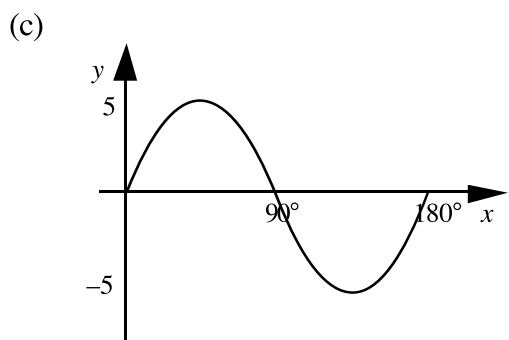
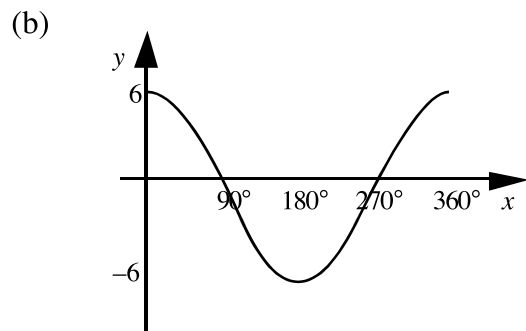
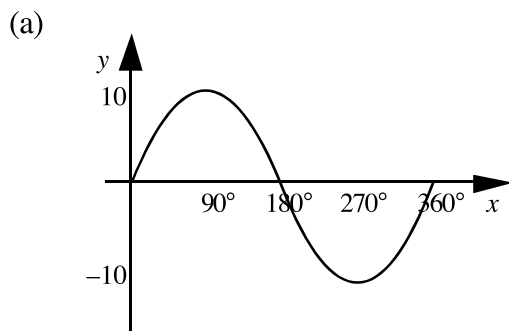


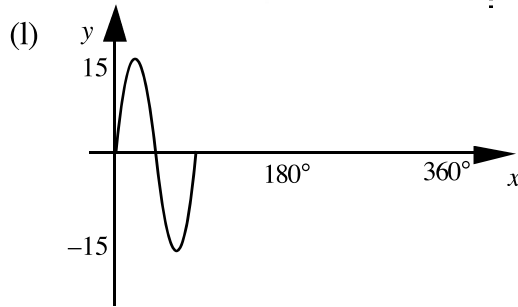
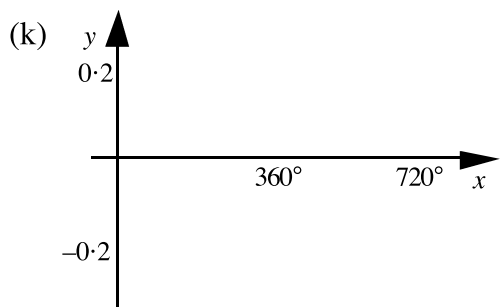
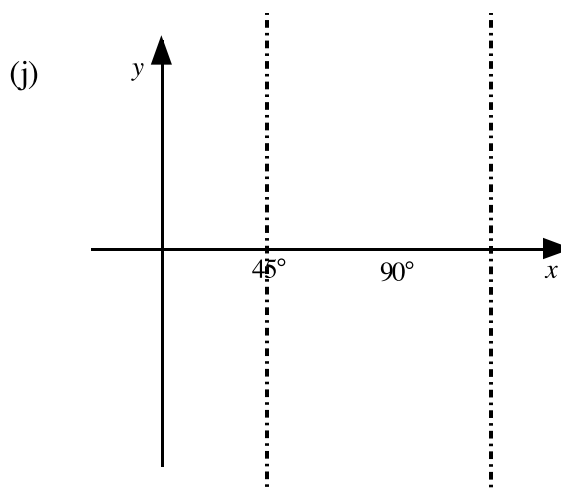
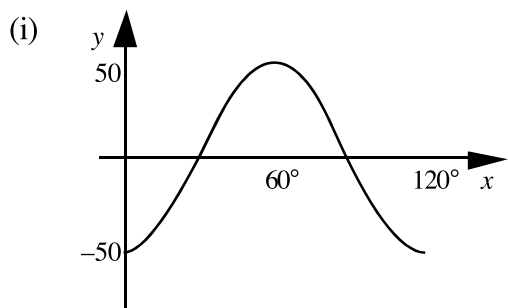
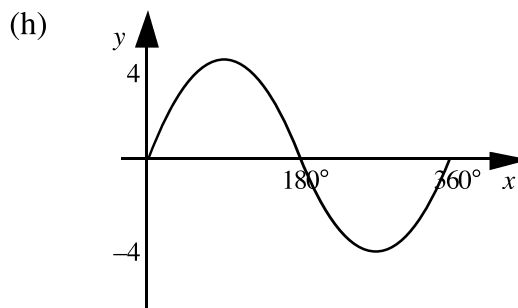
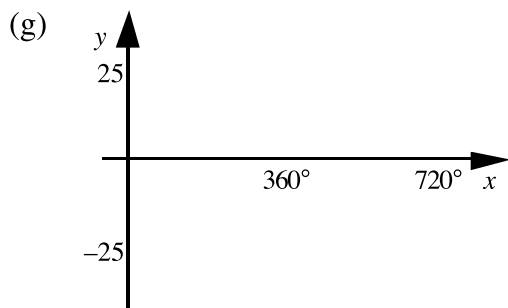
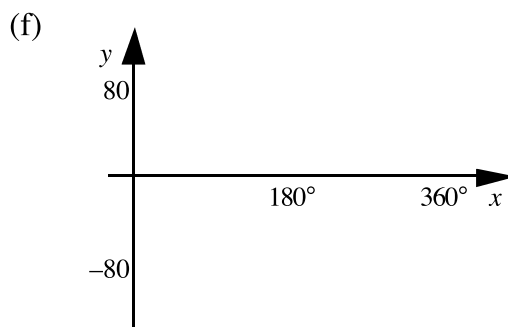
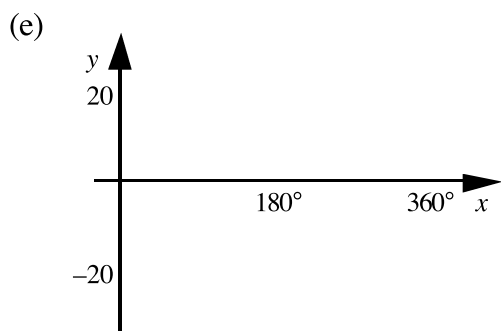
Check your graphs are similar to those shown above.

B. Sketching and identifying trigonometric functions

Exercise 2A

1. Write down the equations of the trigonometric functions represented by the following graphs:





2. Make neat sketches of the following trigonometric functions, clearly indicating
- (i) the shape of the graph (draw one 'cycle' of it only)
 - (ii) the important values on the horizontal axis
 - (iii) the maximum and minimum values of the function.

cont'd ...

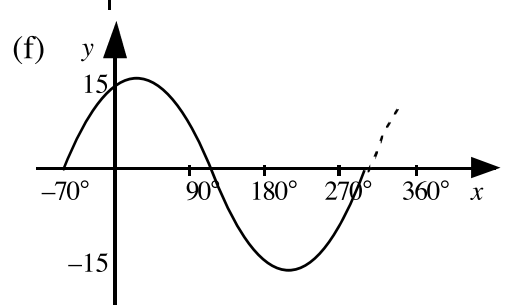
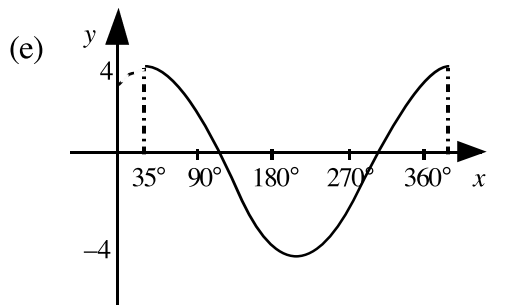
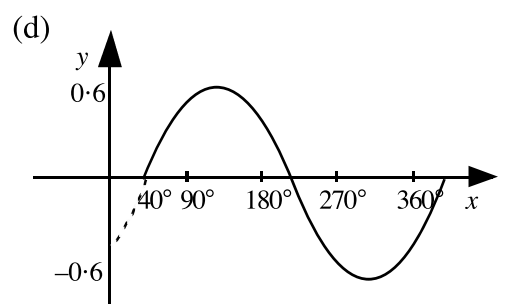
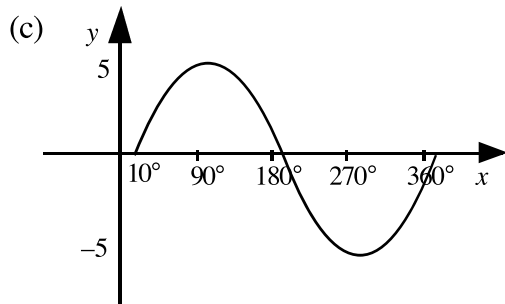
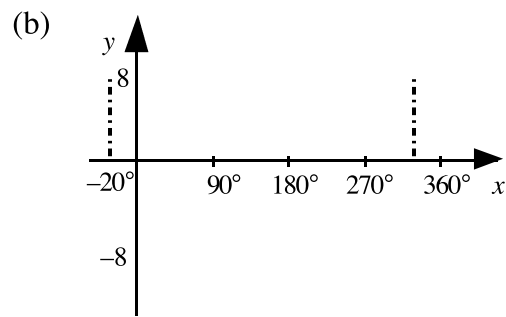
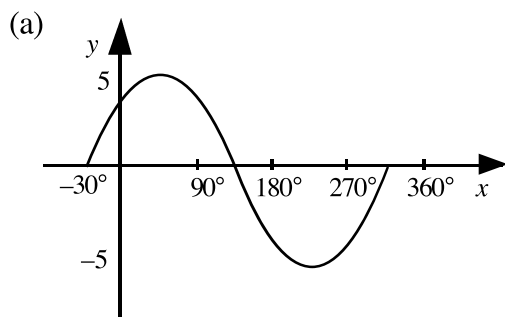
- (a) $y = 3\sin x^\circ$ (b) $y = 4\cos x^\circ$ (c) $y = \tan 3x^\circ$
 (d) $y = 10\sin 3x^\circ$ (e) $y = 12\cos 2x^\circ$ (f) $y = 0.7\sin 4x^\circ$
 (g) $y = 1.2\cos 4x^\circ$ (h) $y = 30\sin 6x^\circ$ (i) $y = 100\cos 5x^\circ$
 (j) $y = -\sin x^\circ$ (k) $y = -6\sin^{1/2} 2x^\circ$ (l) $y = -20\cos 3x^\circ$

3. Make neat sketches of the following over the given range of values:

- (a) $y = 60\sin 2x^\circ$ $0 \leq x \leq 360$ (b) $y = 2.5\cos 3x^\circ$ $0 \leq x \leq 360$
 (c) $y = 40\sin 4x^\circ$ $0 \leq x \leq 180$ (d) $y = -2\cos 6x^\circ$ $0 \leq x \leq 180$
 (e) $y = -15\sin 8x^\circ$ $0 \leq x \leq 180$ (f) $y = 1.8\cos 30x^\circ$ $0 \leq x \leq 12$

Exercise 2B

1. Write down the equations of the trigonometric functions in the form $y = k \sin(x - a)^\circ$ or $y = k \cos(x - a)^\circ$ represented by the following graphs:



2. Make neat sketches of the following trigonometric functions, showing clearly:

- (i) where each graph cuts the x -axis. (ii) the maximum and minimum values.

- (a) $y = 10\sin(x - 15)^\circ$ (b) $y = 1.4 \cos(x - 20)^\circ$ (c) $y = 15\sin(x + 10)^\circ$
 (d) $y = 2.4 \cos(x + 30)^\circ$ (e) $y = 300\sin(x - 80)^\circ$ (f) $y = \tan(x - 10)^\circ$

C. Solving trigonometric equations

S (in)	A (ll)
T (an)	C (os)

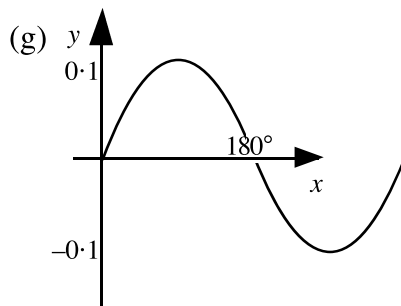
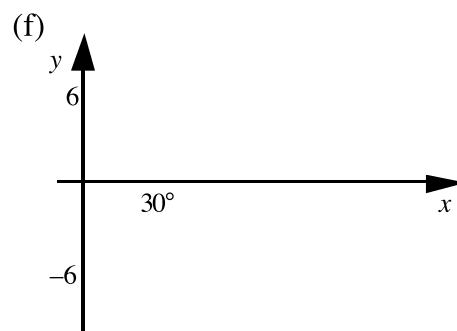
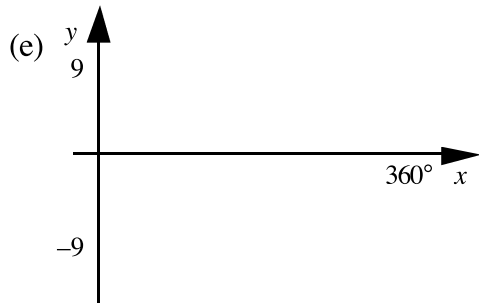
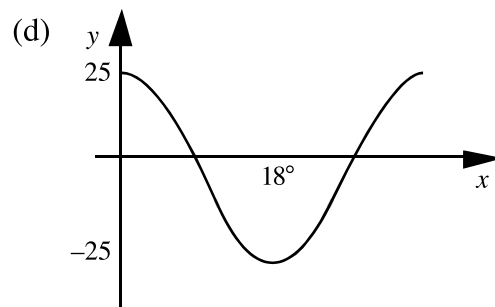
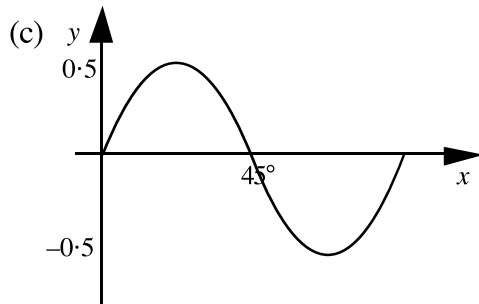
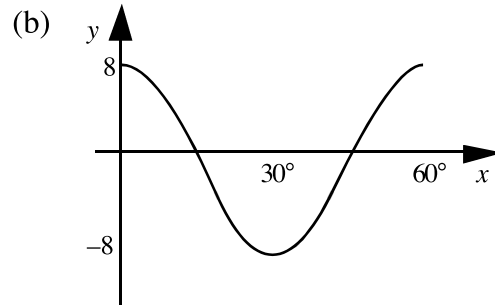
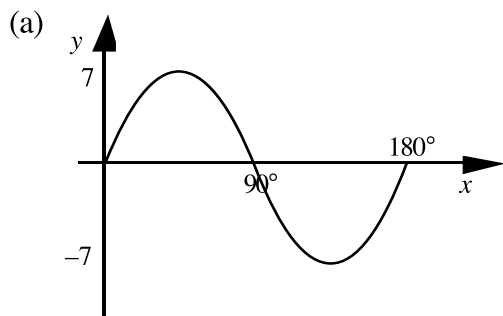
Exercise 3

- Find the two solutions for each of the following in the range $0 \leq x \leq 360$:
(Give each answer correct to the nearest whole degree).
 - $\sin x^\circ = 0.500$
 - $\cos x^\circ = 0.707$
 - $\tan x^\circ = 0.869$
 - $\cos x^\circ = 0.940$
 - $\tan x^\circ = 1.280$
 - $\sin x^\circ = 0.574$
 - $\sin x^\circ = 0.990$
 - $\tan x^\circ = 6.314$
 - $\cos x^\circ = 0.391$
 - $\cos x^\circ = 0.985$
 - $\sin x^\circ = 0.866$
 - $\tan x^\circ = 1.732$
- Rearrange each of the following and solve them in the range $0 \leq x \leq 360$.
(Give your answers correct to 1 decimal place).
 - $2\cos x^\circ - 1 = 0$
 - $5\sin x^\circ - 4 = 0$
 - $10\tan x^\circ - 7 = 0$
 - $1 - 3\sin x^\circ = 0$
 - $5 - 6\cos x^\circ = 0$
 - $3\tan x^\circ - 5 = 0$
- Find the two solutions for each of the following in the range $0 \leq x \leq 360$:
(Give each answer correct to the nearest whole degree).
 - $\sin x^\circ = -0.500$
 - $\cos x^\circ = -0.707$
 - $\tan x^\circ = -0.384$
 - $\cos x^\circ = -0.292$
 - $\tan x^\circ = -1.000$
 - $\sin x^\circ = -0.866$
 - $\tan x^\circ = -4$
 - $\sin x^\circ = -0.174$
 - $\cos x^\circ = -0.927$
- Rearrange each of the following and solve them in the range $0 \leq x \leq 360$.
(Give your answers correct to 1 decimal place).
 - $4\sin x^\circ + 1 = 0$
 - $5\cos x^\circ + 3 = 0$
 - $3\tan x^\circ + 1 = 0$
 - $7 + 8\cos x^\circ = 0$
 - $0.4\sin x^\circ + 0.3 = 0$
 - $5\tan x^\circ + 8 = 0$
- Solve the following mixture of trigonometric equations in the range $0 \leq x \leq 360$.
(Give your answers correct to 1 decimal place).
 - $\sin x^\circ = 0.323$
 - $\cos x^\circ = -0.9$
 - $\tan x^\circ = 0.678$
 - $\cos x^\circ = 1/4$
 - $\sin x^\circ = -0.707$
 - $\tan x^\circ = -2$
 - $\sin x^\circ = 3/5$
 - $\cos x^\circ = -0.111$
 - $\tan x^\circ = 5/8$
 - $8\sin x^\circ + 5 = 0$
 - $6\cos x^\circ + 3 = 0$
 - $1 - 5\tan x^\circ = 0$
 - $20\sin x^\circ - 17 = 0$
 - $15 - 25\cos x^\circ = 0$
 - $8\tan x^\circ + 7 = 0$
 - $5\sin x^\circ + 3 = 2\sin x^\circ + 5$
 - $7\cos x^\circ - 1 = \cos x^\circ + 4$
 - $10\tan x^\circ + 8 = 3\tan x^\circ + 4$
 - $6\sin x^\circ + 11 = 3\sin x^\circ + 10$

D. The period of a trigonometric function

Exercise 4

1. Determine the **period** and the **maximum** and **minimum** values of these trig functions.



2. Determine the **period** and the **maximum** and **minimum** values of these trig functions.

(a) $y = 5\sin 2x^\circ$

(b) $y = 3\cos 3x^\circ$

(c) $y = 10\tan 4x^\circ$

(d) $y = 2.2\cos 2x^\circ$

(e) $y = 30\sin 6x^\circ$

(f) $y = -5\cos 30x^\circ$

(g) $y = 50\sin 90x^\circ$

(h) $y = -4\cos \frac{1}{2}x^\circ$

(i) $y = 18\sin \frac{1}{4}x^\circ$

(j) $y = 0.9\cos 60x^\circ$

(k) $y = \frac{1}{2}\sin 5x^\circ$

(l) $y = \frac{3}{4}\cos 9x^\circ$

(m) $y = 11\sin 180x^\circ$

(n) $y = 8\sin 1.5x^\circ$

(o) $y = 40\cos 2.5x^\circ$

E. Trigonometric identities

Remember :- $\sin^2 x + \cos^2 x = 1$; and $\tan x = \frac{\sin x}{\cos x}$

Exercise 5

1. Simplify the following using the above 2 identities:

(a) $2\sin^2 x^\circ + 2\cos^2 x^\circ$

(b) $5\cos^2 x^\circ + 5\sin^2 x^\circ$

(c) $\frac{3\sin x^\circ}{\cos x^\circ}$

(d) $\frac{5\sin x^\circ}{2\cos x^\circ}$

2. Write down a simple expression, identical to:

(a) $1 - \sin^2 x^\circ$ (b) $1 - \cos^2 x^\circ$ (c) $\tan x^\circ \cos x^\circ$ (d) $\frac{\sin x^\circ}{\cos x^\circ}$

3. Simplify:

(a) $\frac{1 - \cos^2 x^\circ}{\sin^2 x^\circ}$

(b) $\frac{1 - \sin^2 x^\circ}{2\cos^2 x^\circ}$

(c) $\frac{\sin^2 x^\circ}{\cos^2 x^\circ}$

(d) $\frac{1 - \sin^2 x^\circ}{\cos x^\circ}$

(e) $\frac{1 - \cos^2 x^\circ}{5\sin x^\circ}$

(f) $\tan^2 x^\circ(1 - \sin^2 x^\circ)$

4. Prove the following trigonometric identities:

(a) $3 - 3\sin^2 x^\circ = 3\cos^2 x^\circ$

(b) $5 - 5\cos^2 x^\circ = 5\sin^2 x^\circ$

(c) $\sqrt{1 - \cos^2 x^\circ} = \sin x^\circ$

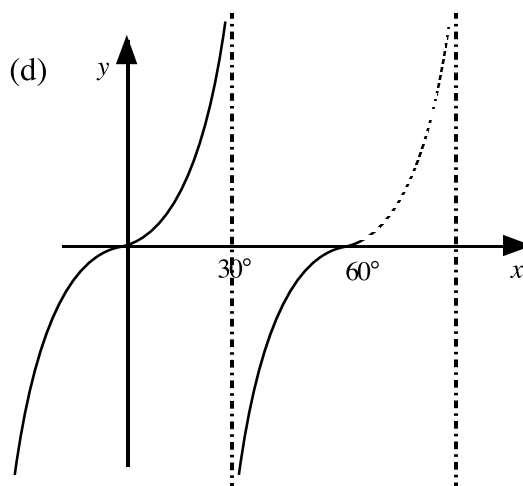
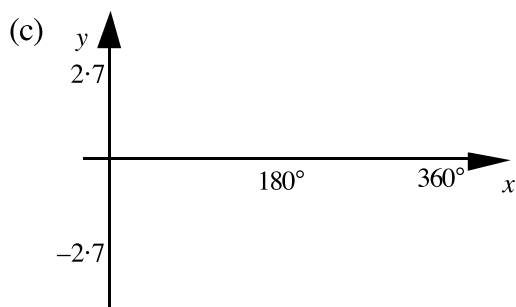
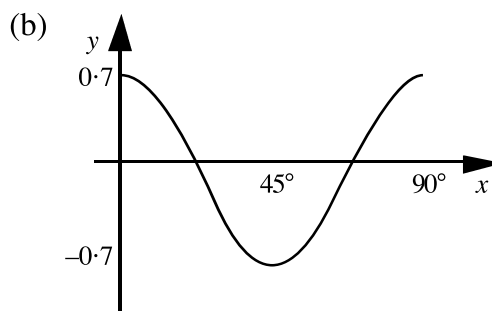
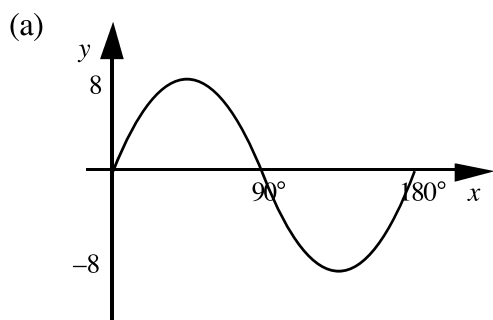
(d) $\tan x^\circ \sqrt{1 - \sin^2 x^\circ} = \sin x^\circ$

(e) $\frac{1 - \cos^2 x^\circ}{1 - \sin^2 x^\circ} = \tan^2 x^\circ$

(f) $\frac{1 - \sin^2 x^\circ}{1 - \cos^2 x^\circ} = \frac{1}{\tan^2 x^\circ}$

Checkup for further trigonometry

1. Make a sketch of the sine, cosine and tangent graphs, indicating all their main features.
2. Write down the equations of the trigonometric functions associated with the following graphs:



3. Make neat sketches of the following, indicating all the main points and features:

(a) $y = 20\sin 4x^\circ$ $0 \leq x \leq 90$

(b) $y = 1.6\cos 2x^\circ$ $0 \leq x \leq 360$

(c) $y = -8\sin 8x^\circ$ $0 \leq x \leq 90$

(d) $y = \tan 2x^\circ$ $0 \leq x \leq 90$

4. Find the two solutions for each of the following in the range $0 \leq x \leq 360$:
(Give your answers correct to 1 decimal place).

(a) $\sin x^\circ = 0.911$

(b) $\cos x^\circ = 0.444$

(c) $\tan x^\circ = 3$

(d) $\cos x^\circ = -0.605$

(e) $\tan x^\circ = -0.8$

(f) $\sin x^\circ = -4/5$

(g) $2\sin x^\circ - 1 = 0$

(h) $8\cos x^\circ + 6 = 0$

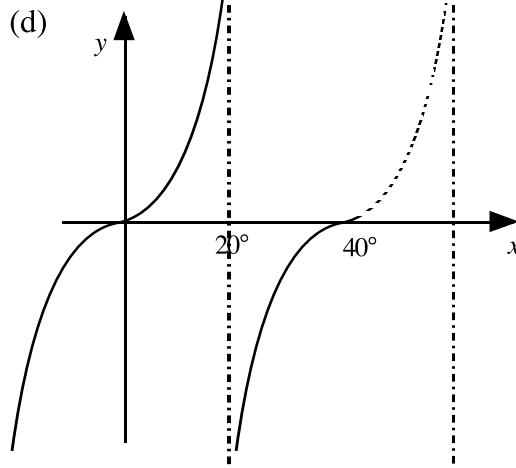
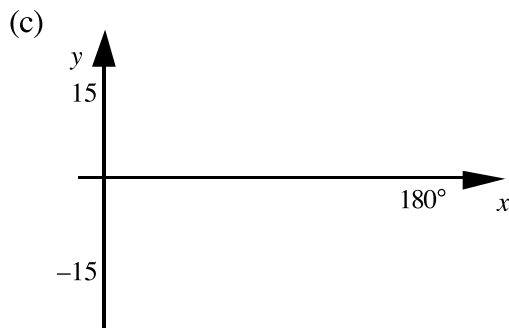
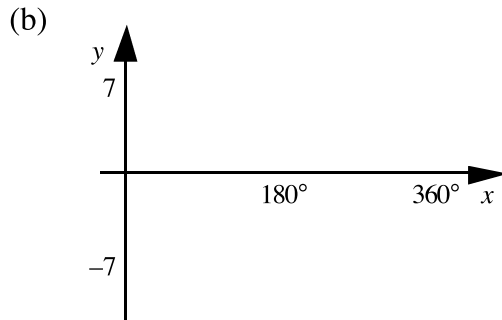
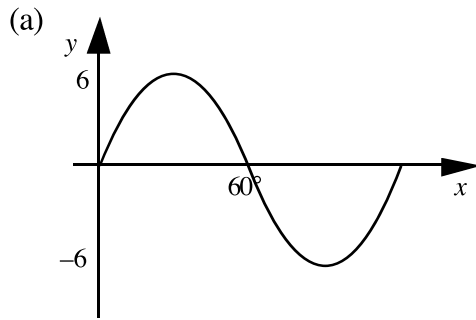
(i) $4\tan x^\circ - 3 = 0$

(j) $3\cos x^\circ - 2 = 0$

(k) $1 + 4\sin x^\circ = 0$

(l) $5\tan x^\circ = 3\tan x^\circ - 2$

5. What are the periods of the following trigonometric graphs and functions?



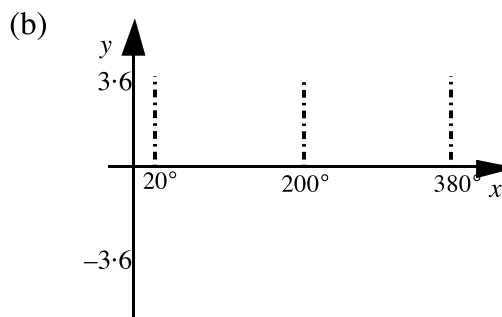
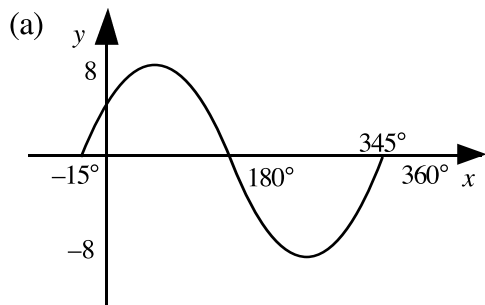
(e) $y = 10\sin 10x^\circ$
 (g) $y = -4\sin 9x^\circ$

(f) $y = 2.3\cos 30x^\circ$
 (h) $y = 5\tan 4x^\circ$

6. (a) Simplify: (i) $6 - 6\sin^2 x^\circ$ (ii) $\frac{\cos x^\circ}{\sin x^\circ}$

(b) Prove these identities :- (i) $\frac{1 - \cos^2 x^\circ}{\sin^2 x^\circ} = 1$ (ii) $(1 - \sin x^\circ)(1 + \sin x^\circ) = \cos^2 x^\circ$

7. Write down the equations of the following trigonometric graphs:



8. Sketch the following trigonometric graphs, indicating their main features:

(a) $y = 18\cos(x + 30)^\circ$ $0 \leq x \leq 360$ (b) $y = 2\sin(x - 10)^\circ$ $0 \leq x \leq 360$

SPECIMEN ASSESSMENT QUESTIONS

1. Factorise fully:

(a) $\frac{12x^2y}{18xy^3}$ (b) $\frac{4a-8}{2}$ (c) $\frac{p^3-pq}{p}$ (d) $\frac{w^2-25}{w-5}$ (e) $\frac{a^2-2a-3}{a^2+2a+1}$

2. Simplify:

(a) $\frac{v^2}{9} \times \frac{36}{v^5}$ (b) $\frac{c}{a^3} \div \frac{2c^2}{a}$ (c) $\frac{5}{p} + \frac{3}{q}$ (d) $\frac{5t}{z} - \frac{t}{2z}$ (e) $\frac{x+1}{2} + \frac{x+3}{3}$

3. Change the subject of each formula to the letter in the bracket:

(a) $p - 2q = r$ [p] (b) $aw + g = h$ [w] (c) $\frac{a}{x} = \frac{x}{9}$ [x] (d) $\frac{1}{4}(m+n) = V$ [m]

4. Express each of these in its simplest form:

(a) $\sqrt{28}$ (b) $3\sqrt{12}$ (c) $\sqrt{\frac{9}{a^4}}$ (d) $5\sqrt{18} - \sqrt{32}$

5. Rationalise the denominators and simplify:

(a) $10/\sqrt{5}$ (b) $\sqrt{12}/\sqrt{2}$ (c) $\frac{p}{\sqrt{q}}$

6. Simplify, giving answers with positive indices:

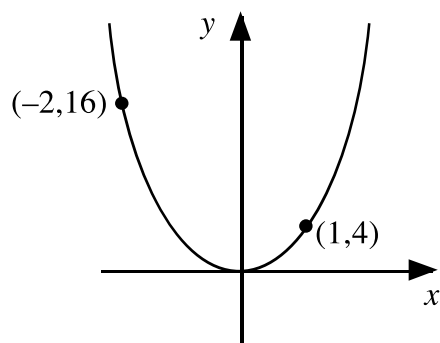
(a) $b^5 \div b^3$ (b) $(w^3)^{-2}$ (c) $\frac{a^2 \times a^4}{a^3}$ (d) $p^{1/2} \times 3p^{-3/2}$ (e) $\frac{v^3(v^2 - v^{-3})}{v^4}$
 (f) $8x^{1/2} \div 4x^{5/2}$

7. Evaluate the following for $v = 4$ and $w = 8$:

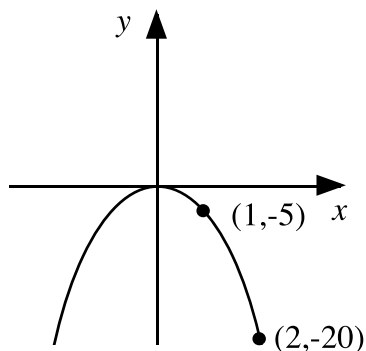
(a) $10v^{1/2}$ (b) $2w^{2/3}$ (c) $v^{-1/2} \div w^{-1/3}$

8. Write down the equation of the quadratic functions corresponding to each of the following parabolas: (each is of the form $y = kx^2$ or $y = \pm(x+a)^2 + b$).

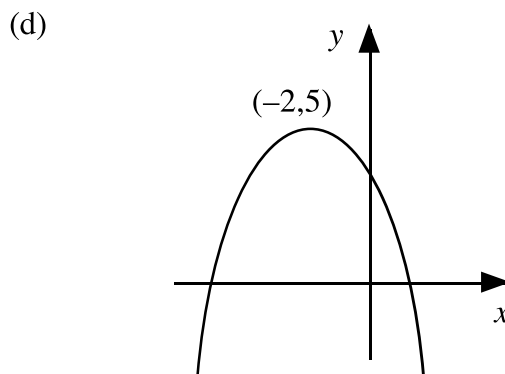
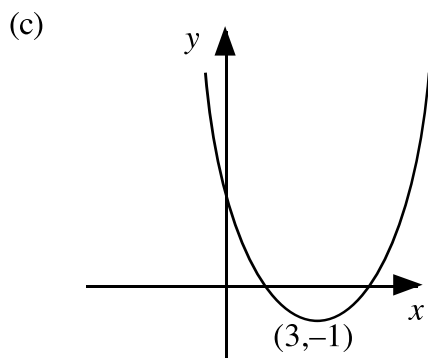
(a)



(b)



cont'd ...



9. Write down the coordinates of the turning point of each of the following, state whether each is a maximum or minimum and give the equation of the axis of symmetry.

(a) $y = (x - 2)^2 + 7$

(b) $y = -(x + 1)^2 - 3$

10. (a) Complete the following table for the function, $y = x^2 - x - 12$.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$y = x^2 - x - 12$	8	-10	-6

(b) Plot the points and sketch the parabola.

(c) Write down the roots of the quadratic equation $x^2 - x - 12 = 0$

11. Solve the following quadratic equations by factorising:

(a) $2x^2 - 6x = 0$

(b) $x^2 - 16 = 0$

(c) $x^2 + 7x + 10 = 0$

(d) $x^2 - 2x - 15 = 0$

(e) $x(x - 2) = 24$

(f) $(x - 1)(x + 2) = 28$

12. Solve the following, correct to 2 decimal places, using the quadratic formula:

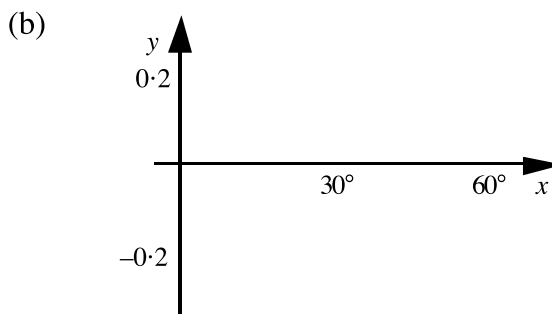
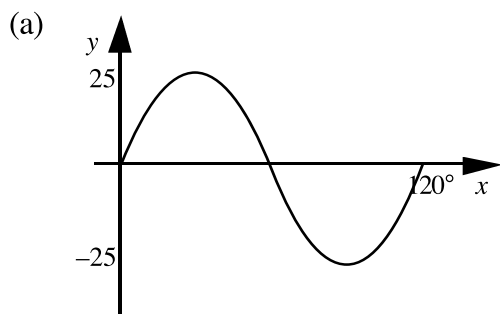
(a) $x^2 + 5x + 1 = 0$

(b) $x^2 - 2x - 7 = 0$

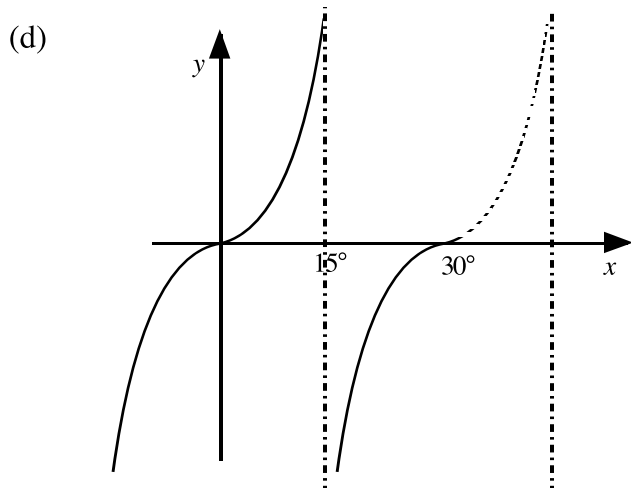
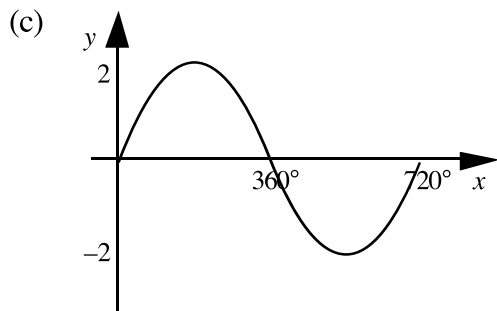
(c) $3x^2 + x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

13. Write down the equations of the trigonometric functions associated with these graphs:



cont'd



14. Sketch the following trigonometric graphs, indicating all the main features and points:

(a) $y = 11 \sin 2x^\circ \quad 0 \leq x \leq 360$

(b) $y = -5 \cos 4x^\circ \quad 0 \leq x \leq 180$

15. Find two solutions for each of the following trig equations in the range $0 \leq x \leq 360$. (Give answers correct to 1 decimal place).

(a) $\sin x^\circ = 0.412$

(b) $8 \cos x^\circ = 5$

(c) $10 \tan x^\circ - 7 = 0$

(d) $\cos x^\circ = -0.234$

(e) $\tan x^\circ = -1$

(f) $3 \sin x^\circ + 1 = 0$

16. What are the **periods** of the following trigonometric functions:

(a) $y = 105 \sin 5x^\circ$

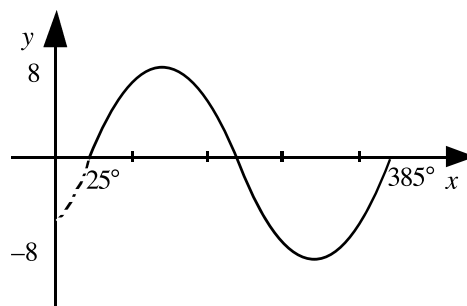
(b) $y = -4 \cos 20x^\circ$

(c) $y = \tan 3x^\circ$

17. (a) Simplify: $\sin^2 x^\circ - 1$

(b) Prove the identity: $\frac{1 - \cos^2 x}{2 - 2 \sin^2 x} = \frac{1}{2} \tan^2 x$

18. Write down the equation of the function given by this trigonometric graph:



19. Sketch the graph of the function, $y = 0.7 \cos(x - 10)^\circ$ showing all the main points and features.

ANSWERS

Algebraic operations

Exercise 1

1. (a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{4}$ (e) $\frac{1}{4}$ (f) $\frac{4}{5}$ (g) $\frac{2}{3}$ (h) $\frac{v}{w}$
(i) $\frac{s}{t}$ (j) $\frac{b}{c}$ (k) $\frac{1}{d}$ (l) $\frac{1}{m}$ (m) $\frac{a}{4}$ (n) $\frac{8}{z}$ (o) x (p) $\frac{1}{d}$
(q) $\frac{5x}{6}$ (r) $\frac{v}{2w}$ (s) $2y$ (t) $\frac{a}{3b}$ (u) $\frac{q}{3}$ (v) $\frac{1}{b}$ (w) y (x) $\frac{3}{7}$
(y) $\frac{5}{2q}$ (z) $\frac{2x}{y}$ (aa) $\frac{1}{(x+1)}$ (ab) $\frac{1}{(x-5)^3}$
2. (a) $\frac{x+4}{2}$ (b) $\frac{x+2}{3}$ (c) $x-4$ (d) $\frac{x-2}{5}$ (e) $\frac{1}{x+3}$
(f) $\frac{1}{x-3}$ (g) $\frac{1}{x+3}$ (h) $\frac{4}{2x-5}$ (i) $x-y$ (j) $q+1$
(k) $v+1$ (l) $1-a$
3. (a) 2 (b) 3 (c) $\frac{1}{3}$ (d) g (e) $\frac{2}{5}$
(f) $\frac{3}{7}$ (g) 4 (h) $\frac{x}{8}$ (i) $x+1$ (j) $y-3$
(k) $a+5$ (l) $\frac{1}{(w-10)}$ (m) $\frac{x-1}{x+1}$ (n) $\frac{3v+1}{v+2}$ (o) $\frac{y+1}{y+3}$
(p) $\frac{2x-3}{x+4}$

Exercise 2

1. (a) $\frac{1}{5}$ (b) $\frac{1}{9}$ (c) $\frac{1}{2}$ (d) 2 (e) 2 (f) 3 (g) $\frac{7}{2}$ (h) $\frac{16}{3}$
(i) $\frac{ac}{bd}$ (j) $\frac{xv}{yw}$ (k) $\frac{m^2}{n^2}$ (l) 1 (m) $\frac{x^2}{10}$ (n) $\frac{x^2}{7}$ (o) $\frac{a^2}{25}$ (p) 1
(q) $\frac{a}{d}$ (r) x (s) a (t) a^2
2. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$ (e) $\frac{3}{2}$ (f) $\frac{1}{4}$ (g) 2 (h) 5
(i) a (j) $\frac{b}{3}$ (k) $\frac{r^2}{3}$ (l) 1 (m) ad (n) $\frac{1}{5w}$ (o) $\frac{1}{a}$ (p) $\frac{dx}{2y}$
(q) a (r) $\frac{2}{5}$
3. (a) $\frac{7}{12}$ (b) $\frac{11}{12}$ (c) $\frac{11}{20}$ (d) $\frac{8}{15}$ (e) $\frac{2}{15}$
(f) $\frac{1}{14}$ (g) $\frac{7}{10}$ (h) $\frac{3}{10}$ (i) $\frac{3}{8}$ (j) $\frac{9}{10}$
(k) $\frac{2x+3a}{6}$ (l) $\frac{2c+5d}{10}$ (m) $\frac{4e-3h}{12}$ (n) $\frac{2m-n}{8}$ (o) $\frac{4x+3k}{6}$
(p) $\frac{5u-4w}{10}$ (q) $\frac{8r+5s}{10}$ (r) $\frac{5a-6d}{15}$ (s) $\frac{4x+9y}{6}$ (t) $\frac{15x+8u}{20}$
4. (a) $\frac{2y+3x}{xy}$ (b) $\frac{5b-2a}{ab}$ (c) $\frac{4d+c}{cd}$ (d) $\frac{q-2p}{pq}$ (e) $\frac{2w+2v}{vw}$

(f) $\frac{h-g}{gh}$	(g) $\frac{7n+k}{kn}$	(h) $\frac{y-8x}{xy}$		
5. (a) $\frac{5x+5}{6}$	(b) $\frac{9x+6}{20}$	(c) $\frac{3x+7}{4}$	(d) $\frac{11x-4}{15}$	(e) $\frac{x+1}{6}$
(f) $\frac{3x+8}{10}$	(g) $\frac{3x+1}{4}$	(h) $\frac{3x+7}{10}$		

Exercise 3A

1. $x = c - 2$	2. $x = c + 4$	3. $x = q - p$	4. $x = q + p$	5. $x = 2a$
6. $x = 7a$	7. $x = ya$	8. $x = mp$	9. $x = rs$	10. $x = 5$
11. $x = a/4$	12. $x = h/g$	13. $x = t/n$	14. $x = 2$	15. $x = \frac{b-1}{2}$
16. $x = \frac{b-c}{2}$	17. $x = \frac{b-c}{a}$	18. $x = \frac{r-q}{p}$	19. $x = \frac{y+w}{v}$	20. $S = D/T$
21. $d = C/\pi$	22. $x = 4$ or -4	23. $x = \sqrt{y}$ or $-\sqrt{y}$	24. $r = \sqrt{(A/\pi)}$	25. $S = D/T$
26. $y = \sqrt{A}$ (or $-\sqrt{A}$)	27. $\pi = P/3r^2$	28. $r = \sqrt{(P/5)}$	29. $h = q + p$	30. $p = h - q$
31. $h = \frac{q+5p}{2}$	32. $p = \frac{2h-q}{5}$	33. $x = \frac{b-c}{a}$		

Exercise 3B

1. (a) $h = g/f$	(b) $h = e - g$	(c) $h = kf$	(d) $h = g - e$
2. (a) $r = \sqrt{Q}$	(b) $r = \sqrt{(N/\pi)}$	(c) $r = \sqrt{(M/2)}$	(d) $r = \sqrt{(P/w)}$
3. (a) $M = A/kl$	(b) $m = B/K$	(c) $m = c/\pi r^2$	(d) $m = 1/3pD$
4. (a) $x = p - q$	(b) $x = s - r$	(c) $x = (s - r)/5$	(d) $x = (r + 3)/7$
(e) $x = (m - 2)/2$	(f) $x = 2m + 5$	(g) $x = 2n - 2$	(h) $x = 2p - q$
5. (a) $P = QR$	(b) $s = 1/t$	(c) $Q = \sqrt{(P/M)}$	(d) $w = v^2z$
(e) $f = e/5d$	(f) $n = K/mT$	(g) $s = \sqrt{(7/9r)}$	(h) $a = \sqrt{(c^2 - b^2)}$
6. (a) $T = V/(a+d)$	(b) $x = r/(p+q)$	(c) $x = c/(a-b)$	(d) $x = r/(m+1)$
(e) $w = (v - vx)/(x+1)$	(f) $r = \frac{(p+1)^2}{4}$		

Exercise 4

1. (a) $2\sqrt{2}$	(b) $2\sqrt{3}$	(c) $3\sqrt{3}$	(d) $2\sqrt{5}$	(e) $5\sqrt{2}$
(f) $2\sqrt{7}$	(g) $3\sqrt{2}$	(h) $2\sqrt{6}$	(i) $10\sqrt{2}$	(j) $5\sqrt{3}$
(k) $3\sqrt{5}$	(l) $6\sqrt{2}$	(m) $10\sqrt{3}$	(n) $7\sqrt{3}$	(o) $3\sqrt{6}$
(p) $14\sqrt{2}$	(q) $20\sqrt{2}$	(r) $12\sqrt{10}$		

2. (a) $8\sqrt{2}$ (b) $\sqrt{5}$ (c) $13\sqrt{10}$ (d) 0 (e) $-2\sqrt{6}$
 (f) $-\sqrt{3}$ (g) 0 (h) $10\sqrt{2} + 10\sqrt{3}$!!
3. (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{5}$ (d) $6\sqrt{3}$ (e) $5\sqrt{5}$
 (f) $\sqrt{7}$ (g) $8\sqrt{2}$ (h) $2\sqrt{2}$

Exercise 5

1. (a) 3 (b) 5 (c) 6 (d) 1 (e) x (f) $\sqrt{6}$ (g) $2\sqrt{5}$ (h) $4\sqrt{a}$
 (i) $\sqrt{2c}$ (j) \sqrt{xy} (k) 4 (l) 8 (m) $3\sqrt{2}$ (n) $10\sqrt{2}$ (o) 6 (p) $6\sqrt{6}$
2. (a) $\sqrt{2} + 2$ (b) $\sqrt{6} + \sqrt{3}$ (c) $5 + \sqrt{5}$ (d) $\sqrt{7} + 7$ (e) $5\sqrt{2} - 2$
 (f) $5\sqrt{2} - 8$ (g) $1 + \sqrt{3}$ (h) $-\sqrt{2}$ (i) 1 (j) 2
 (k) $5 + 2\sqrt{6}$ (l) $8 - 2\sqrt{15}$
3. (a) 6 (b) -2 (c) 6
4. (a) $4\sqrt{3}$ (b) 10 (c) $4\sqrt{15}$
5. (a) 4 cm^2 (b) $2\sqrt{6} \text{ cm}$

Exercise 6

1. (a) $\sqrt{2}/2$ (b) $\sqrt{3}/3$ (c) $\sqrt{5}/5$ (d) $\sqrt{6}/6$ (e) $\sqrt{7}/7$
 (f) $2\sqrt{5}$ (g) $2\sqrt{3}/3$ (h) $3\sqrt{5}/5$ (i) $10\sqrt{2}$ (j) $2\sqrt{3}$
 (k) $2\sqrt{6}$ (l) $3\sqrt{5}/10$ (m) $2\sqrt{2}/5$ (n) $\sqrt{5}/10$ (o) $\sqrt{2}/10$ (p) $\sqrt{2}$
2. (a) $2\sqrt{3}/3$ (b) $\sqrt{10}/2$ (c) $\sqrt{10}/5$ (d) $\sqrt{11}/11$ (e) $\sqrt{15}/5$ (f) $\frac{\sqrt{ab}}{b}$
3. (a) $\sqrt{2} + 1$ (b) $(\sqrt{7} + 1)/6$ (c) $(2 - \sqrt{2})/2$ (d) $\sqrt{5} - 1$ (e) $6 + 3\sqrt{3}$
 (f) $\sqrt{3} + \sqrt{2}$ (g) $\sqrt{5} - \sqrt{3}$ (h) $3\sqrt{5} + 3\sqrt{2}$

Exercise 7

1. (a) 2^7 (b) 3^8 (c) 8^7 (d) 10^{30} (e) a^7 (f) b^8 (g) c^8 (h) d^{10}
 (i) v^{11} (j) x^{12} (k) w^4 (l) z^6 (m) f^{11} (n) g^{20} (o) k^{12} (p) m^{101}
2. (a) 2 (b) 3^2 (c) 8^3 (d) 10^{10} (e) a (f) b^2 (g) c^5 (h) 1
 (i) v (j) x^8 (k) w^2 (l) z (m) f^3 (n) 1 (o) k^{10} (p) m^{99}
 (q) x^4 (r) m (s) a^4 (t) r
3. (a) x^6 (b) y^{15} (c) z^{10} (d) g^{16} (e) a^{21} (f) b^{16} (g) c^{30} (h) d^{21}
4. (a) 2^{10} (b) 7^{15} (c) 6^{20} (d) 8^{15} (e) 2^{49} (f) 3^{12} (g) 9^{10} (h) 2^{25}
5. (a) a^3b^3 (b) c^6d^6 (c) $x^{20}y^{10}$ (d) $4p^2q^2$
6. (a) y^5 (b) t^5 (c) x^3 (d) v^6 (e) $3x^5$ (f) $4x^7$ (g) $12x^4$
 (h) $4x^3$ (i) $3x^2$ (j) $x^5 + x^6$ (k) $x^6 - 4x^3$ (l) u^7v^7 (m) $25y^2$
 (n) m^8n^{16} (o) x^5 (p) u^6
7. (a) $m = 4$ (b) $m = 5$ (c) $m = 3$ (d) $m = 4$

Exercise 8

- (a) 1 (b) 1 (c) 1 (d) 1
- (a) $1/3^2$ (b) $1/5^7$ (c) $1/a^4$ (d) $1/b^9$ (e) $1/x$ (f) $3/y^2$ (g) x/y^3 (h) x^3
(i) t^5 (j) $6c^3$ (k) $1/2y^2$ (l) $1/7x^3$
- (a) 1 (b) $1/7$ (c) $1/27$ (d) $1/64$ (e) 8 (f) $9/4$
- (a) a^2 (b) b^2 (c) c^{-2} (d) 1 (e) e^9 (f) g^{-8} (g) w^2 (h) x^{-6}
(i) y^{-5} (j) z^{16} (k) 1
- (a) $1/4^3$ (b) $1/6^5$ (c) 2^5 (d) $25/4^3$ (e) $81/2^3$ (f) 1 (g) w^6 (h) x
(i) 4 (j) 40 (k) $6/h^2$ (l) $9/s$ (m) $1/2k$ (n) $3/4m^3$
- (a) $x^7 + x^3$ (b) $x^4 - x$ (c) $x + x^{-2}$ (d) $1 + x$ (e) $x^3 - x^{-6}$
(f) $2x^{-5} - 1$ (g) $6x^{-2} - 2x$ (h) $6x^3 - 3$
- (a) 27, 1, $1/9$, 3, $1/27$ (b) 4^{-1} , 4^{-2} , 4^{-3}

Exercise 9

- (a) $4\sqrt{x^3}$ (b) $5\sqrt{m^3}$ (c) $3\sqrt{r^2}$ (d) \sqrt{w} (e) $3\sqrt{n}$ (f) $1/(3\sqrt{r^4})$
- (a) $x^{5/3}$ (b) $b^{4/3}$ (c) $z^{2/3}$ (d) $w^{1/4}$ (e) $x^{-1/3}$ (f) $u^{1/2}$
- (a) 3 (b) 8 (c) 2 (d) 4 (e) 10 (f) 3 (g) 27 (h) 8
(i) 343 (j) $1/4$ (k) $1/3$ (l) $1/125$ (m) 27 (n) $1/27$ (o) 2 (p) $1/32$
- (a) x^2 (b) m^3 (c) c^4 (d) n^{-3} (e) $n^{-1/2}$ (f) g^2 (g) b^{-5}
(h) z^3 (i) $x^{2/5}$ (j) x^2 (k) 13 (l) 6

Exercise 10

- (a) a^2 (b) b (c) 1 (d) d^2 (e) $e^{4/5}$ (f) z (g) w^{-1} (h) 1
- (a) 4 (b) $12b$ (c) $6c$ (d) $2d$ (e) $2e$ (f) $4v$ (g) 1
- (a) $x - 1$ (b) $x^2 + x$ (c) $x^{3/5} + x^{-4/5}$
- (a) 12 (b) 32 (c) 45 (d) $8/9$ (e) $3/2$
- (a) $p^{13/2}$ (b) $p^{11/2}$ (c) $1/p$ (d) $1/p^9$ (e) $p^{3/2}$ (f) $49/p$
- (a) 1 (b) x^6 (c) x (d) $x^{-9/2}$ (e) 1 (f) x
- (a) $2 - x^3 - 1/x^3$ (b) $1/x^2 - 4$ (c) $-4x^{1/2} + 4/x^{1/2} - 15$

Exercise 11

- x^{-1} 2. x^{-2} 3. $7x^{-2}$ 4. $5x^{-3}$ 5. $1/3x^{-1}$ 6. $4/5x^{-1}$ 7. $3/2x^{-2}$
- $x^{-1/2}$ 9. $5/2x^{-1/2}$ 10. $3/5x^3$ 11. $\frac{x-3}{\sqrt{3}}$ 12. $\frac{2x^{-3/2}}{\sqrt{5}}$

Checkup for algebraic operations

- (a) $1/2$ (b) $3a/b$ (c) $1/(x-3)$ (d) $v-w$ (e) $\frac{x+y}{2x+3y}$ (f) $1/(a+8)$
(g) $(v+1)/(v-2)$
- (a) $b^2/2$ (b) a (c) $3z$ (d) a (e) $3c/8$ (f) $(15u-8v)/20$
(g) $(3m+k)/km$ (h) $(3x+1)/10$
- (a) $x=1$ (b) $x=w+a$ (c) $x=(p-m)/a$ (d) $x=pz/w$
(e) $x=\sqrt{(N/2)}$ (f) $x=(T-3)/4$ (g) $x=2w-y$ (h) $x=\sqrt{(5/8M)}$
- (a) $4\sqrt{2}$ (b) $10\sqrt{10}$ (c) $6\sqrt{5}$ (d) $\sqrt{5}$ (e) $2\sqrt{2}$ (f) $11\sqrt{2}$ (g) $3/a$ (h) $3\sqrt{2}$
- (a) 4 (b) -10 (c) 8 6. (a) $\sqrt{5/5}$ (b) $4\sqrt{2}$ (c) $3\sqrt{5}$ (d) $\sqrt{3/3}$
- (a) 5^{14} (b) x^2 (c) $6m^4$ (d) a^6 (e) $16a^6b^2$ (f) a^4 (g) 1 (h) x^5-x^4
- (a) $1/5^2$ (b) a/b^3 (c) y^6 (d) $(x^2)/4$
- (a) \sqrt{b} (b) $1/\sqrt{c^3}$
- (a) $x^{4/3}$ (b) $a^{-3/2}$
- (a) 216 (b) $1/4$ (c) x^4 (d) $1/y^7$ (e) $1/a-1/a^4$ (f) $s/2$ (g) x

Quadratic functions

Exercise 1

- (a) $y=x^2$ (b) $y=2x^2$ (c) $y=1/2x^2$
(d) $y=-2x^2$ (e) $y=-1/2x^2$ (f) $y=-5x^2$
- (a) $y=x^2+1$ (b) $y=x^2+3$ (c) $y=x^2-2$
(d) $y=-x^2+4$ (e) $y=-x^2+1$ (f) $y=-x^2-1$
- (a) $y=(x-1)^2+1$ (b) $y=(x-2)^2+3$ (c) $y=(x-3)^2$
(d) $y=(x+3)^2+2$ (e) $y=(x+4)^2$ (f) $y=(x+5)^2-3$
(g) $y=(x-3)^2+2$ (h) $y=(x-1)^2-4$ (i) $y=(x+3)^2-4$

Exercise 2

- (a) $(2,1)$ (b) $x=2$ (c) $y=(x-2)^2+1$
- (a) (i) $(3,2)$ (ii) $x=3$ (iii) $y=(x-3)^2+2$
(b) (i) $(1,-1)$ (ii) $x=1$ (iii) $y=(x-1)^2-1$
(c) (i) $(-2,1)$ (ii) $x=-2$ (iii) $y=(x+2)^2+1$
(d) (i) $(-3,1)$ (ii) $x=-3$ (iii) $y=(x+3)^2+1$
(e) (i) $(3,-2)$ (ii) $x=3$ (iii) $y=(x-3)^2-2$
(f) (i) $(-1,-3)$ (ii) $x=-1$ (iii) $y=(x+1)^2-3$
- (a) $(4,1); x=4$ (b) $(2,7); x=2$ (c) $(8,3); x=8$
(d) $(-1,2); x=-1$ (e) $(1,-3); x=1$ (f) $(-3,-7); x=-3$
(g) $(5,0); x=5$ (h) $(-2,0); x=-2$ (i) $(0,3); x=0$

4. (a) (2,6) (b) $x = 2$ (c) $y = -(x - 2)^2 + 6$
5. (a) (i) (3,2) (ii) $x = 3$ (iii) $y = -(x - 3)^2 + 2$
 (b) (i) (4,1) (ii) $x = 4$ (iii) $y = -(x - 4)^2 + 1$
 (c) (i) (-1,4) (ii) $x = -1$ (iii) $y = -(x + 1)^2 + 4$
 (d) (i) (-3,1) (ii) $x = -3$ (iii) $y = -(x + 3)^2 + 1$
 (e) (i) (4,0) (ii) $x = 4$ (iii) $y = -(x - 4)^2$
 (f) (i) (-3,-1) (ii) $x = -3$ (iii) $y = -(x + 3)^2 - 1$
6. (a) (2,6) ; $x = 2$ (b) (5,1) ; $x = 5$ (c) (6,-2) ; $x = 6$
 (d) (-1,7) ; $x = -1$ (e) (-4,-5) ; $x = -4$ (f) (-3,0) ; $x = -3$
 (g) (1,7) ; $x = 1$ (h) (8,1) ; $x = 8$ (i) (-5,-2) ; $x = -5$

Exercise 3

1. (a) $x = -1, 3$ (b) $x = 1, 3$ (c) $x = 2, -2$
2. (a) 4, 0, -2, -2, 0, 4 (b) graph (c) $x = -1, 2$
3. (a) 0, 4 (b) 1, -2 (c) 2, 4
 (d) -3, 2 (e) 2, -2 (f) 5, -1

Exercise 4

1. (a) 0, 4 (b) 0, 10 (c) 0, 8
 (d) 0, -6 (e) 0, -1 (f) 0, 1
 (g) 0, 3 (h) 0, -3 (i) 0, $3/2$
2. (a) 2, -2 (b) 3, -3 (c) 5, -5
 (d) 4, -4 (e) 10, -10 (f) 7, -7
 (g) 9, -9 (h) 3, -3 (i) $4/5, -4/5$
3. (a) -1, -2 (b) 2, 3 (c) -1, -5
 (d) 5, 4 (e) -2, -5 (f) 3
 (g) 3, 4 (h) 1, 7 (i) 6, 7
 (j) -5, 2 (k) 4, -1 (l) -4, 2
 (m) 5, -4 (n) -4, 3 (o) -7, 5
 (p) -6, 2 (q) -6, 3 (r) -1, -20
 (s) 1, 8 (t) 12, -2 (u) -8, 3
 (v) 6, -4 (w) 24, -1 (x) 6, 9
4. (a) -3, $-1/2$ (b) -1, $-3/2$ (c) -2, $-1/3$
 (d) 3, $3/2$ (e) -3, $-2/3$ (f) -2, $-1/5$
 (g) 2, $-4/3$ (h) 2, $-1/3$ (i) -1, $1/3$
 (j) 4, $-1/2$ (k) -3, $2/5$ (l) -2, $-5/2$
5. (a) 1, -3 (b) 5, -4 (c) 5, -2
 (d) 6, -1 (e) 7, -10 (f) 7, -8
 (g) 2, -5 (h) 5, -6 (i) 2, -4
 (j) 3, -3 (k) 5, -5 (l) 1, 4
 (m) -1, -3 (n) 2, -5 (o) 2, -4
 (p) 4, $-5/2$ (q) 2, $-5/3$ (r) 1, $-7/5$
 (s) 3, $-11/2$ (t) 2, $-7/3$

Exercise 5

- | | | |
|--------------------|--------------------|--------------------|
| (a) $-0.27, -3.73$ | (b) $-0.38, -2.62$ | (c) $-0.35, -5.65$ |
| (d) $-0.76, -9.24$ | (e) $3.41, 0.59$ | (f) $3.62, 1.38$ |
| (g) $-0.31, -3.19$ | (h) $-0.61, -2.72$ | (i) $1.39, 0.36$ |
- | | | |
|---------------------|---------------------|---------------------|
| (a) $0.732, -2.732$ | (b) $1.791, -2.791$ | (c) $1.372, -4.372$ |
| (d) $3.449, -1.449$ | (e) $3.303, -0.303$ | (f) $0.243, -8.243$ |
| (g) $0.851, -2.351$ | (h) $1.215, -0.549$ | (i) $0.588, -0.213$ |
- | | | |
|--------------------|---------------------|--------------------|
| (a) $0.372, -5.37$ | (b) $3.30, -0.303$ | (c) $0.679, -3.68$ |
| (d) $2.39, -1.39$ | (e) $-0.807, -6.19$ | (f) $3.24, -1.24$ |

Checkup for quadratic equations

- | | | |
|-------------------------|-------------------------|-------------------------|
| (a) $y = x^2$ | (b) $y = 3x^2$ | (c) $y = -2x^2$ |
| (d) $y = (x - 3)^2 + 2$ | (e) $y = (x + 2)^2 + 2$ | (f) $y = (x + 1)^2 - 4$ |
- | | |
|------------------------------------|-------------------------------------|
| (a) minimum at $(3, -1)$; $x = 3$ | (b) minimum at $(-1, 0)$; $x = -1$ |
| (c) maximum at $(2, 1)$; $x = 2$ | |
- | | |
|-------------------------------------|--------------------------------------|
| (a) minimum at $(21, 5)$; $x = 21$ | (b) minimum at $(-2, -1)$; $x = -2$ |
| (c) maximum at $(3, 2)$; $x = 3$ | |
- | | |
|-----------------------------------|-----------------------------------|
| (a) graph and $x = 4$ or $x = -2$ | (b) graph and $x = 1$ or $x = -3$ |
|-----------------------------------|-----------------------------------|
- | | | | | | |
|---------------------------------|-------------|-----------------------|-----------------------|-------------|-------------|
| (a) $0, 7$ | (b) $3, -3$ | (c) $-2, -6$ | (d) $0, -\frac{3}{2}$ | (e) $5, -5$ | (f) $6, -5$ |
| (g) $\frac{3}{2}, -\frac{3}{2}$ | (h) $2, 5$ | (i) $-5, \frac{3}{2}$ | (j) $2, -7$ | (k) $4, -5$ | (l) $6, -2$ |
- | | | |
|--------------------|-------------------|--------------------|
| (a) $-0.55, -5.45$ | (b) $3.73, 0.27$ | (c) $-0.78, -2.55$ |
| (d) $0.37, -5.37$ | (e) $6.61, -0.61$ | (f) $1.85, -1.35$ |

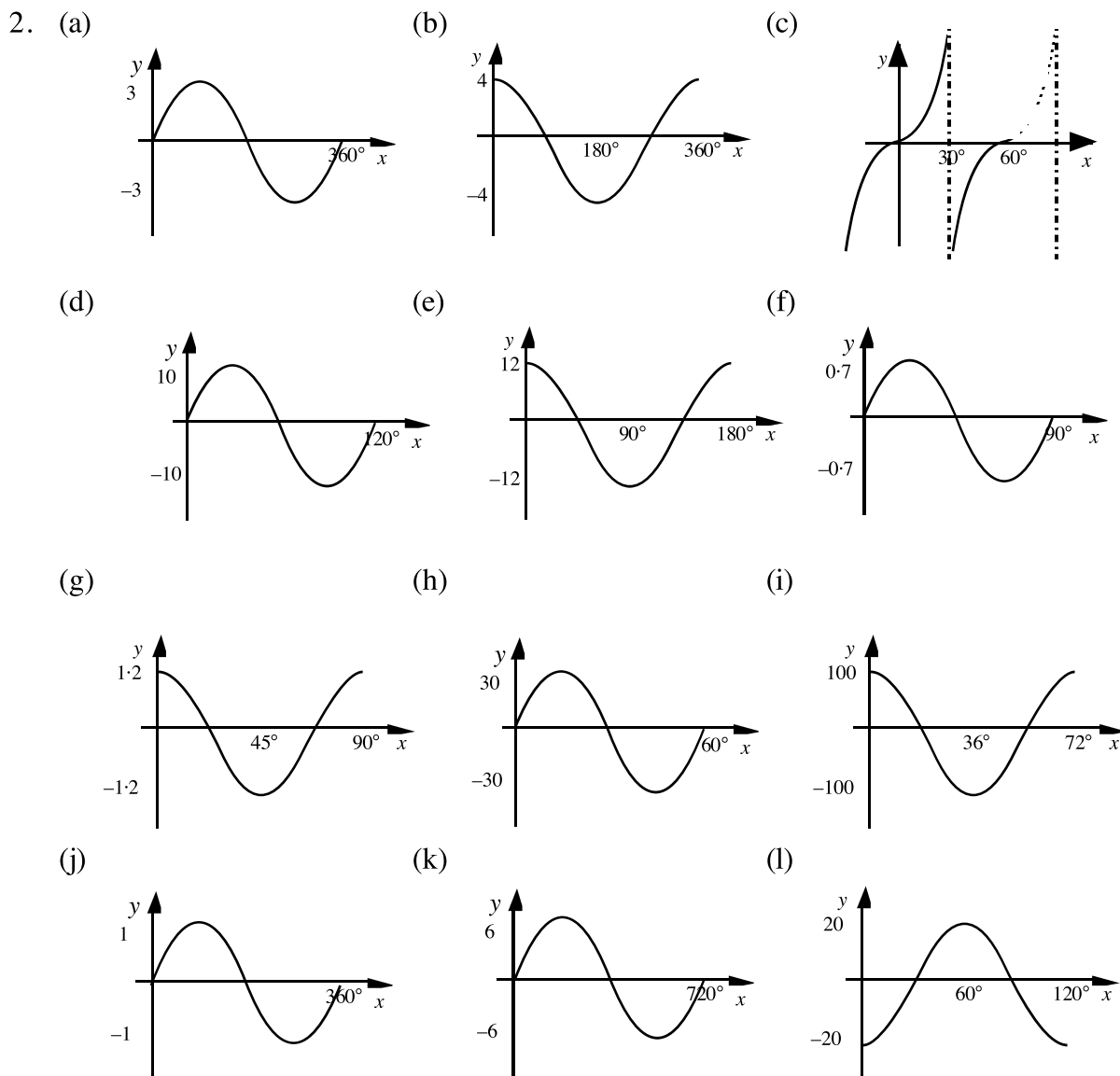
Further trigonometry

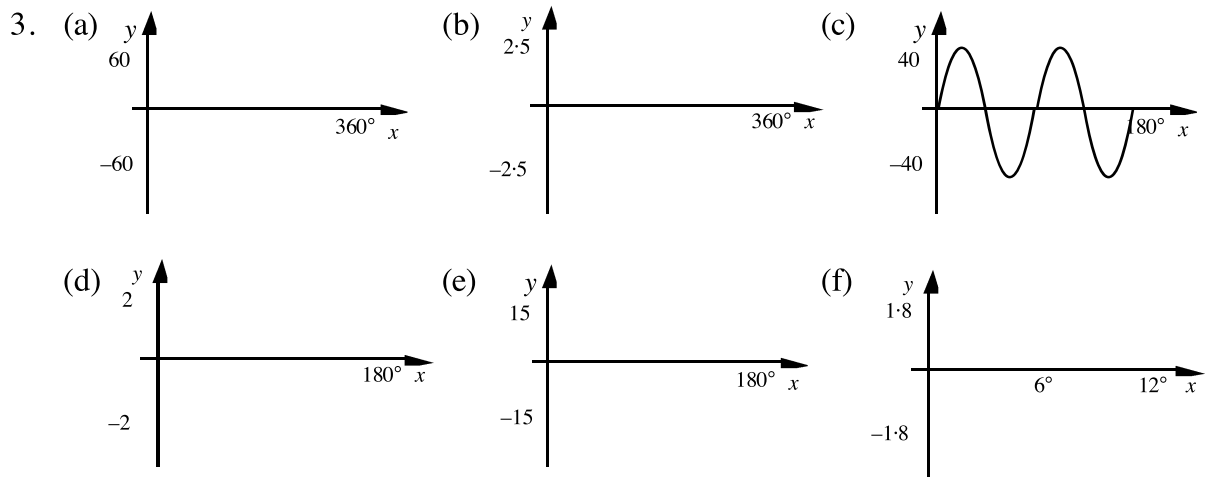
Exercise 1

1. Check Graphs – see graphs at top of page 15 for comparison.

Exercise 2A

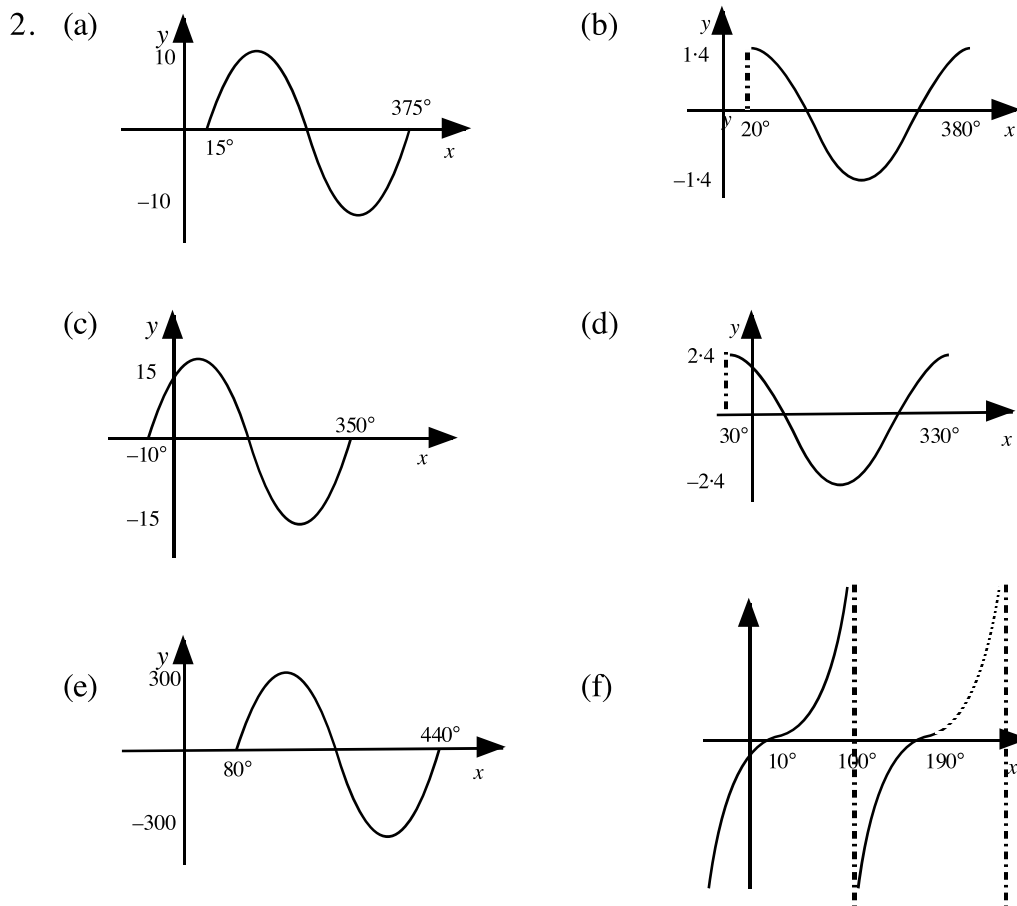
1. (a) $y = 10\sin x^\circ$ (b) $y = 6\cos x^\circ$ (c) $y = 5\sin 2x^\circ$
 (d) $y = 0.8\cos 4x^\circ$ (e) $y = 20\sin 2x^\circ$ (f) $y = 80\cos 2x^\circ$
 (g) $y = 25\sin \frac{1}{2}x^\circ$ (h) $y = -4\sin x^\circ$ (i) $y = -50\cos 3x^\circ$
 (j) $y = \tan 2x^\circ$ (k) $y = 0.2\cos \frac{1}{2}x^\circ$ (l) $y = 15\sin 4x^\circ$





Exercise 2B

1. (a) $y = 5\sin(x + 30)^\circ$
 (c) $y = 5\sin(x - 10)^\circ$
 (e) $y = 4\cos(x - 35)^\circ$
- (b) $y = 8\cos(x + 20)^\circ$
 (d) $y = 0.6\sin(x - 40)^\circ$
 (f) $y = 15\sin(x + 70)^\circ$ or $y = 15\cos(x - 20)^\circ$



Exercise 3

1. (a) 30, 150 (b) 45, 315 (c) 41, 221
 (d) 20, 340 (e) 52, 232 (f) 35, 145
 (g) 82, 98 (h) 81, 261 (i) 67, 293
 (j) 10, 350 (k) 60, 120 (l) 60, 240.
2. (a) 60, 300 (b) 53.1 or 126.9 (c) 35.0, 215.0
 (d) 19.5, 160.5 (e) 33.6, 326.4 (f) 59.0, 239.0.
3. (a) 210, 330 (b) 135, 225 (c) 159, 339
 (d) 107, 253 (e) 135, 315 (f) 240, 300
 (g) 104, 284 (h) 190, 350 (i) 158, 202.
4. (a) 194.5, 345.5 (b) 126.9, 233.1 (c) 161.6, 341.6
 (d) 151.0, 209.0 (e) 228.6, 311.4 (f) 122.0, 302.0.
5. (a) 18.8, 161.2 (b) 154.2, 205.8 (c) 34.1, 214.1
 (d) 75.5, 284.5 (e) 225, 315 (f) 116.6, 296.6
 (g) 36.9, 143.1 (h) 96.4, 263.6 (i) 32.0, 212.0
 (j) 218.7, 321.3 (k) 120, 240 (l) 11.3, 191.3
 (m) 58.2, 121.8 (n) 53.1, 306.9 (o) 138.8, 318.8
 (p) 41.8, 138.2 (q) 33.6, 326.4 (r) 150.3, 330.3
 (s) 199.5, 340.5°.

Exercise 4

1.

Graph	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Period	180	60	90	36	180	120	360
Max/Min	±7	±8	±0.5	±25	±9	±6	±0.1

2.

Question	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Period	180	120	45	180	60	12	4	720
Max/Min	±5	±3	±10	±2.2	±30	±5	±50	±4

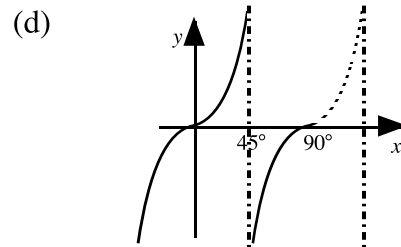
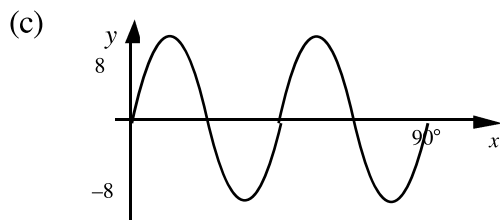
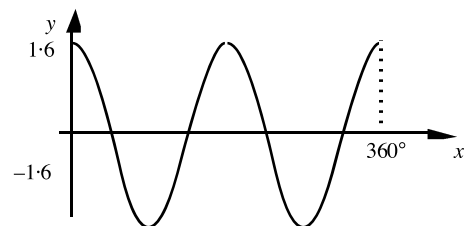
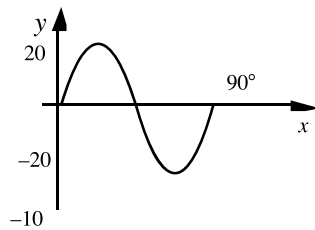
Question	(i)	(j)	(k)	(l)	(m)	(n)	(o)
Period	1440	6	72	40	2	240	144
Max/Min	±18	±0.9	±0.5	±0.75	±11	±8	±40

Exercise 5

- (a) 2 (b) 5 (c) $3\tan x$ (d) $\frac{5}{2}\tan x$
- (a) $\cos^2 x^\circ$ (b) $\sin^2 x^\circ$ (c) $\sin x^\circ$ (d) $\tan x^\circ$
- (a) 1 (b) $\frac{1}{2}$ (c) $\tan^2 x^\circ$ (d) $\cos x^\circ$ (e) $\frac{1}{5}\sin x^\circ$ (f) $\sin^2 x^\circ$
- All proofs.

Checkup for further trigonometry

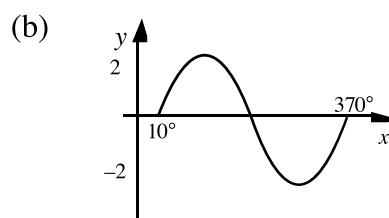
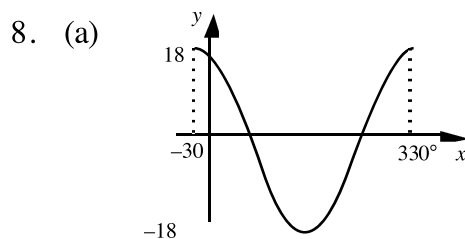
- See sketches on page 15.
- (a) $y = 8\sin 2x^\circ$ (b) $y = 0.7\cos 4x^\circ$ (c) $y = -2.7\sin 3x^\circ$ (d) $y = \tan 3x^\circ$
- (a)



- | | | |
|------------------|------------------|------------------|
| (a) 65.6, 114.4 | (b) 63.6, 296.4 | (c) 71.6, 251.6 |
| (d) 127.2, 232.8 | (e) 141.3, 321.3 | (f) 233.1, 306.9 |
| (g) 30, 150 | (h) 138.6, 221.4 | (i) 36.9, 216.9 |
| (j) 48.2, 311.8 | (k) 194.5, 345.5 | (l) 135, 315. |
- | | | | |
|-----------------|-----------------|----------------|------------------|
| (a) 120° | (b) 180° | (c) 60° | (d) 40° |
| (e) 36° | (f) 12° | (g) 40° | (h) 90° . |

- | | | |
|---------------------|-------------------------|------------|
| (a) (i) $6\cos^2 x$ | (ii) $\frac{1}{\tan x}$ | (b) Proof. |
|---------------------|-------------------------|------------|

- | | |
|-------------------------------|---------------------------------|
| (a) $y = 8\sin(x + 15)^\circ$ | (b) $y = 3.6\cos(x - 20)^\circ$ |
|-------------------------------|---------------------------------|



Specimen assessment questions

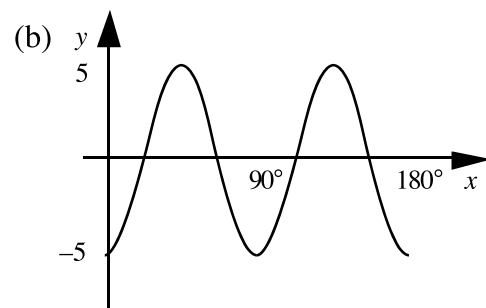
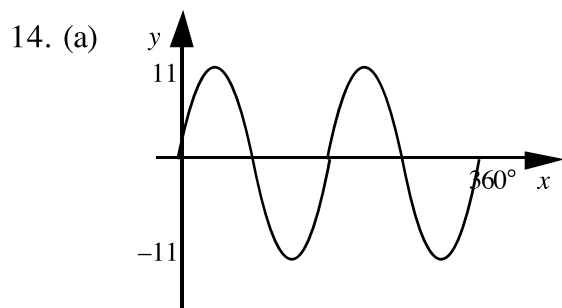
1. (a) $\frac{2x}{3y^2}$ (b) $2a - 4$ (c) $p^2 - q$ (d) $w + 5$ (e) $\frac{a-3}{a+1}$
2. (a) $\frac{4}{v^3}$ (b) $\frac{1}{2a^2c}$ (c) $\frac{5q+3p}{pq}$ (d) $\frac{9t}{2z}$ (e) $\frac{5x+9}{6}$
3. (a) $p = r + 2q$ (b) $w = (h - g)/a$ (c) $3\sqrt{a}$ (d) $4V - n$
4. (a) $2\sqrt{7}$ (b) $6\sqrt{3}$ (c) $3/a^2$ (d) $11\sqrt{2}$
5. (a) $2\sqrt{5}$ (b) $\sqrt{6}$ (c) $(p\sqrt{q})/q$
6. (a) b^2 (b) $1/w^6$ (c) a^3 (d) $3/p$
 (e) $v - 1/v^4$ (f) $2/x^2$
7. (a) 20 (b) 8 (c) 1
8. (a) $y = 4x^2$ (b) $y = -5x^2$ (c) $y = (x - 3)^2 - 1$ (d) $y = -(x + 2)^2 + 5$
9. (a) (2,7); minimum; $x = 2$ (b) (-1,-3); maximum; $x = -1$

10. (a)

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$y = x^2 - x - 12$	8	0	-6	-10	-12	-12	-10	-6	0	8	18

- (b) Graph
 (c) $x = -3$ or 4

11. (a) 0, 3 (b) 4, -4 (c) -2, -5 (d) -3, 5 (e) -4, 6 (f) -6, 5
12. (a) -0.21, -4.79 (b) 3.83, -1.83 (c) 0.43, -0.77
13. (a) $y = 25\sin 3x^\circ$ (b) $y = 0.2\cos 6x^\circ$
 (c) $y = -2\sin \frac{1}{2}x^\circ$ (d) $y = \tan 6x^\circ$



15. (a) $24.3^\circ, 155.7^\circ$ (b) $51.3^\circ, 308.7^\circ$ (c) $35.0^\circ, 215.0^\circ$
 (d) $103.5^\circ, 256.5^\circ$ (e) $135^\circ, 315^\circ$ (f) $199.5^\circ, 340.5^\circ$
16. (a) 72° (b) 18° (c) 60°
17. (a) $-\cos^2 x^\circ$ (b) Proof

18. $y = 8\sin(x - 25)^\circ$

19.

