

## HIGHER FACTOR THEOREM.

$$\textcircled{1} \quad a) \quad (i) \quad \begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$$f(x) = x^3 - x^2 - 5x - 3$$

So,  $(x+1)$  is a factor

$$\begin{aligned} (ii) \quad & (x+1)(x^2 - 2x - 3) \\ & = (x+1)(x+1)(x-3) \\ & = (x+1)^2(x-3) \end{aligned}$$

$$b) \quad \frac{dy}{dx} = 3x^2 - 2x - 5$$

$$\text{For S.P.} \quad \frac{dy}{dx} = 0$$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0$$

$$3x - 5 = 0$$

$$x = \frac{5}{3}$$

$$x + 1 = 0$$

$$x = -1$$

T.P. at  $(-1, 0)$

②

$$2x^3 - 7x^2 + 4x + 4$$

$$\begin{array}{r|rrrr} 2 & 2 & -7 & 4 & 4 \\ & & 4 & -6 & -4 \\ \hline & 2 & -3 & -2 & 0 \end{array}$$

$$(x-2)(2x^2 - 3x - 2)$$

$$= (x-2)(2x+1)(x-2)$$

③

$$x^3 + kx^2 - 4x - 12$$

$$\begin{array}{r|rrrr} 2 & 1 & k & -4 & -12 \\ & & 2 & 2k+4 & 4k \\ \hline & 1 & k+2 & 2k & 4k-12 \end{array}$$

$$4k - 12 = 0$$

$$4k = 12$$

$$k = 3$$

④

$$x^4 - x$$

$$\begin{array}{c|cccccc} 1 & 1 & 0 & 0 & -1 & 0 \\ & & 1 & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 & 0 & 0 \end{array}$$

$$\begin{aligned} & (x-1)(x^3 + x^2 + x) \\ &= (x-1)x(x^2 + x + 1) \\ &= x(x-1)(x^2 + x + 1) \end{aligned}$$

or.

$$x^4 - x = x(x^3 - 1)$$

$$\begin{array}{c|cccc} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$$x(x-1)(x^2 + x + 1)$$

$$\textcircled{5} \quad f(x) = 2x^4 - x^3 + px^2 + qx + 12$$

$$\begin{array}{r|rrrrr} 2 & 2 & -1 & p & q & 12 \\ & & 4 & 6 & 2p+12 & 4p+2q+24 \\ \hline & 2 & 3 & p+6 & 2p+q+12 & \boxed{4p+2q+36} \end{array}$$

$$4p + 2q + 36 = 114$$

$$4p + 2q = 78$$

$$\begin{array}{r|rrrrr} -1 & 2 & -1 & p & q & 12 \\ & & -2 & 3 & -p-3 & p-q+3 \\ \hline & 2 & -3 & p+3 & -p+q-3 & \boxed{p-q+15} \end{array}$$

$$p - q + 15 = 0$$

$$p - q = -15$$

$$\textcircled{A} \quad 4p + 2q = 78 \quad \textcircled{A} \times 1 \quad 4p + 2q = 78$$

$$\textcircled{B} \quad p - q = -15 \quad \textcircled{B} \times 2 \quad 2p - 2q = -30$$

$$\begin{array}{r} \textcircled{A} + \textcircled{B} \\ 6p \quad = 48 \\ p \quad = 8 \end{array}$$

$$\text{Sub in } \textcircled{B} \text{ for } p=8 \quad 8 - q = -15$$

$$-q = -23$$

$$q = 23$$

$$(p=8, q=23)$$