



EDUCATION GUIDANCE
Community Services: Education

GUIDANCE on: Mental Agility

Produced:	November 2016
Due for Review:	December 2016
Owner:	Pauline Inglis

Mathematics is an important part of our everyday life. It equips us with the skills we need to interpret and analyse information, simplify and solve problems, assess risk and make informed decisions. The ability to calculate in your head is an important part of mathematics. It is also an essential part of coping with society's demands and managing everyday events. Learning mathematics gives children and young people access to the wider curriculum and the opportunity to pursue further studies and interests.

To face the challenges of the 21st century, each young person needs to have confidence in using mathematical skills, and Scotland needs both specialist mathematicians and a highly numerate population.

Building the Curriculum 1

The Scottish Government has an aim to eradicate innumeracy by the year 2017, and with that view has published the document Making Maths Count which is available on the Scottish Government website: <http://www.gov.scot/Publications/2016/09/3014/downloads#res-1>

One of the main features of effective learning and teaching in mathematics is the developing of mental agility. Across Scotland, schools have identified the teaching of mental maths as a key challenge. Looking at the SSLN results from 2015 clearly highlights that although scores sit above 60% in addition and multiplication, results rapidly fall off within subtraction and division. It also highlights that our secondary pupils are not as strong in their mental skills as their primary counterparts.

<http://www.gov.scot/Topics/Statistics/Browse/School-Education/SSLN>

To this end, the Argyll and Bute Numeracy Forum has developed a Mental Skills Progression Framework for all practitioners. This Framework lays out:

- The key number facts that learners should be able to derive and recall;
- The mental calculation skills that are required;
- The mental methods or strategies they can draw upon to complete these mental calculation skills.

These are shaded to indicate the progression within the level.

From the outset it was decided that high but achievable goals should be set with a view to raising attainment through the use of the strategies which are laid out in the Mental Skills Progression Framework, such as partitioning using near doubles. It is essential that these are practised from Early Level onwards to enable learners to build on prior knowledge. Developing these strategies will support learners in becoming skilled in deciding upon which strategy is the most efficient to use given the calculation they are being asked to solve.

These levels should be seen as guidance and learners should move through them at an appropriate pace and with an appropriate level of challenge. For example, some P7 pupils may well be working from the 3rd Level line of progression in their mental skills, whilst still accessing 2nd Level materials within their written work. It is important however, that this is not viewed as a programme of study but rather as a guide to support practitioners in their own thinking.

The Argyll and Bute Mental Skills Progression Framework focuses on Addition and Subtraction and Multiplication and Division from Early Level through to Fourth Level. The decision to focus on this was made because so many of these basic skills feed into all areas of Mathematics and Numeracy, and without these skills, children are unable to

access many other areas of the mathematics curriculum. Mental agility bridges many of the Maths and Numeracy Experiences and Outcomes. However this document deliberately has not overtly linked each strategy to specific Es and Os to allow practitioners the freedom to make professional judgements about which areas children need to make further progress in.

As children and young people develop their mental skills within mathematics, there needs to be continual reinforcement and revisiting of concepts in order to maintain progression. The Framework provides opportunities for children to develop their skills within more challenging contexts.

Written recording of answers is an excellent tool for thinking, communicating and reflecting and it can help students to think about the mental processes they have used so that they can share their ideas with others. However it is important to remember that learners should not be exposed to standard written methods until they have had appropriate experience of using mental strategies as they can end up relying on written methods rather than developing their mental strategies.

This document is designed to help staff to be more aware of the range of possible strategies that should be taught and used as their pupils progress in mathematics. It allows them to be in a better position to recognise the strategies that learners are using when they calculate mentally; allows them to model a variety of strategies to use. It also means that teachers should be more able to make next step suggestions to learners.

The following teaching principles should play an important role in teaching mental calculations:

- Every day should be a mental mathematics day with regular time committed to teaching mental calculation strategies. A mental maths session should be undertaken every day, lasting for 5-15 minutes, dependant on the age of your learners;
- Provide practice time with frequent opportunities for learners to use one or more facts that they already know to work out more facts. Sumdog is an excellent tool to allow this to happen in the classroom and at home;
- Introduce practical approaches and jottings, using models and images which learners can use to carry out calculations until they are secure in the mental strategy they are learning. It is important that teachers model for the pupils the strategy that they are teaching;
- Engage the learners in discussion with their peers where they can explain their methods and strategies to others to help develop their problem-solving capabilities and critical thinking skills. The use of the app Explain Everything is another excellent tool to allow children to explain the approaches they are using.

This Framework is designed to give schools the flexibility to use their own resources whilst ensuring that there is a robust progression of skills which will support learners in developing their mental agility. It should also work well with the Number Talks resource which is being provided to all schools in Argyll and Bute Council.

We hope that you find this Framework useful to your teaching and that we can successfully use it to give our learners a rich and robust experience in Mental Agility which they can use to raise their attainment in Maths and Numeracy.

Addition and Subtraction – CfE Early Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
<p>Number songs/rhymes/stories e.g. 1,2,3,4,5 once I caught a fish alive, 1,2 buckle my shoe, 10 green bottles</p> <p>Names of numerals to 10</p> <p>Conservation of number e.g. Knowing 3 is 3 regardless of arrangement of concrete materials/objects</p> <p>Value of a set e.g. Counting 3 objects as 3</p> <p>Numbers in environment e.g. Signs around school both inside and outside</p> <p>Understand the language of daily routines e.g. registration, lunches, birthdays</p> <p>Understand signs or instructions e.g. 4 can play, one at a time, two at a time, with a partner</p> <p>Begin to use ordinal numbers in a real life context e.g. 1st, 2nd, 3rd, and last in a set for lining up, dates, sports</p>	<p>Sort and create groups of objects by size, number or other properties</p> <p>Place/identify any given digit on a number line to 10 e.g. before, after, in between</p> <p>Explains that zero means there is none of a particular quantity and is represented by numeral '0'</p> <p>More/less comparison e.g. a set of 5 and a set of 4 – which set has more/less</p> <p>Checks estimates by counting</p>	<p>1-1 correspondence, to at least 20, when counting e.g. matching games to encourage counting aloud</p> <p>When counting objects, understand the number name of the last object counted is the name given to the total number of objects in the group.</p> <p>Order numbers to 10 (forwards and backwards)</p> <p>Use number lines to calculate 1 more/less than</p> <p>Share a group of items and discuss who has more/less</p> <p>Identifies the amount of objects in a group and uses this information to estimate the amount of objects in a larger group.</p> <p>Include the language of before, after and in-between when using ordinal numbers in a real-life context</p>
<p>Use vocabulary such as bigger, smaller and 'the same' to compare groups of items</p> <p>Say number word sequences from zero to at least 30 (forwards and backwards) starting from any given number</p> <p>Identifies missing numbers in a sequence</p> <p>Recognises number names and numerals to at least 20</p> <p>Recognise and use ordinal numbers up to 31st in context</p>	<p>Read and use a variety of number lines (straight, curved dials etc.)</p> <p>Identify the number before, the number after and missing numbers in a sequence</p> <p>Counts on and back in ones to demonstrate understanding of addition and subtraction</p> <p>Counting on and back from a given number, including bridging 10</p> <p>Combine sets of objects together and record this as a number sentence e.g. 1+5=6</p>	<p>Subitise – Recognise a small number of objects without counting. Identifies how many in regular and irregular dot pattern arrays, five frames, ten frames and dice</p> <p>Groupings within 5 e.g. 2 and 3, 4 and 1</p> <p>Groupings with 5 e.g. 5 and 1, 5 and 2</p> <p>Groupings within 10 e.g. 5 and 5, 4 and 6</p> <p>Partitioning a set to show Commutative Law – understand that 3+4 is the same as 4+3</p>

<p>Use a range of strategies to add and subtract mentally to at least 10</p>	<p>Addition facts within 5 e.g. $5-2=3$ (inverse to addition facts) Subtraction skills by taking away from a group of objects and counting how many are left</p> <p>Links number families when explaining mental strategies for subtraction and addition, for example, $8 - 3 = 5$ and $8 - 5 = 3$, $3 + 5 = 8$ $5 + 3 = 8$</p> <p>Solves simple missing number equations for example $3 + \blacksquare = 10$</p> <p>Explain the strategies used to add and subtract mentally in words, with materials, through drawings, on paper</p> <p>Use the signs +, -, =</p>	<p>Partitioning a set to show Associative Law – understand that $6+3+7$ is the same as $9+7$</p> <p>Doubles numbers to a total of at least 20 mentally, for example $9 + 9 = 18$</p> <p>Counts in jumps (skip counts) in 2s, 5s and 10s and begins to use this as a useful strategy to find how many in a larger group</p> <p>Demonstrates skills of estimation in the context of number, money, time and measure using relevant vocabulary for example 'less than, longer than'</p>
--	--	---

Addition and Subtraction – CfE First Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
<p>Number pairs with a total of 10, eg. 3+7, or what to add to a single-digit number to make 10, e.g. 3+ = 10</p> <p>Addition facts for totals to at least 5, e.g. 2+3, 4+3</p> <p>Addition doubles for all numbers to at least 10, e.g. 8+8</p>	<p>Add or subtract a pair of single-digit numbers, e.g. 4+5, 8-3</p> <p>Add or subtract a single-digit number to or from a teen number, e.g. 13+5, 17-3</p> <p>Add or subtract a single-digit to or from 10 and add a multiple of 10 to a single-digit number, e.g. 10+7, 7+30</p> <p>Add near doubles, e.g. 6+7</p> <p>Count on from and back to zero in ones, twos, fives or tens e.g. count back in twos from 8</p>	<p>Reorder numbers when adding e.g. put the larger number first</p> <p>Count on or back in ones, twos or tens</p> <p>Partition small numbers e.g. 8+3 = 8+2+1</p> <p>Partition: double and adjust e.g. 5+6= 5+5+1</p> <p>Use patterns of last digits e.g. 0 and 5 when counting in fives</p>
<p>Addition and subtraction facts for all numbers up to at least 10, eg. 3+4, 8-5</p> <p>Number pairs with totals to 20</p> <p>All pairs of multiples of 10 with totals up to 100, e.g. 30+70 or 60 + = 100</p> <p>What must be added to any two-digit number to make the next multiple of 10, eg. 52 + = 60</p> <p>Addition doubles for all numbers to 20 and multiples of 10 to 50, e.g. 17 + 17, 40+40</p> <p>How whole numbers are constructed.</p> <p>Can understand the importance of zero within the system</p> <p>Can use knowledge to explain the link between a digit, its place and its value</p> <p>Read, write order and recite whole numbers up to at least 1000, starting from any number in the sequence</p>	<p>Add or subtract a pair of single-digit numbers including crossing 10, e.g. 5+8, 12-7</p> <p>Add any single-digit number to/from a multiple of 10, 60+5</p> <p>Subtract any single-digit number to/ from a multiple of 10 e.g. 80-7</p> <p>Add or subtract a single-digit number to/from a two-digit number, including crossing the tens boundary e.g. 23+5, 57-3, then 28+5, 52-7</p> <p>Add or subtract a multiple of 10 to/from any two-digit number e.g. 27+60, 72-50</p> <p>Add 9, 19, 29..... or 11, 21,31.....</p> <p>Add near doubles e.g. 13+14, 39+40</p> <p>Describe patterns in number, for example when using a hundred square</p>	<p>Reorder numbers when adding</p> <p>Partition: bridge through 10 and multiples of 10 when adding and subtracting</p> <p>Partition and combine multiples of tens and ones</p> <p>Use knowledge of pairs making 10 e.g. 60-7, think 10-7=3, so 60 – 7 = 53</p> <p>Partition: count on in tens and ones to find the total</p> <p>Partition: count on or back in tens and ones to find the difference</p> <p>Partition: add a multiple of 10 and adjust by 1</p> <p>Partition: double and adjust</p> <p>Demonstrate understanding of zero as a placeholder in whole numbers to at least 1000</p> <p>Identify the value of each digit in a whole number with up to at least 3 digits</p>

<p>Count forwards and backwards from any whole number up to at least 1000</p> <p>Addition and subtraction facts for all numbers to 20, e.g. $9+8$, $17-9$, drawing on knowledge of inverse operations</p> <p>Sums and differences of multiples of 10, e.g. $50+80$, $120-90$</p> <p>Pairs of two-digit numbers with a total of 100, e.g. $32+68$, or $32 + \blacksquare = 100$</p> <p>Addition doubles for multiples of 10 to 100, e.g. $90 + 90$</p> <p>Recognises, explains and uses the connections between addition and subtraction problems with up to at least 3 digit whole numbers and justify choice of strategy</p> <p>Use correct mathematical vocabulary when discussion the four operations, for example subtract, add, sum of, total</p> <p>Mental agility skills to calculate the total spent in a shopping situation and is able to calculate change</p>	<p>Forwards and backwards in 2s, 5s and 10s</p> <p>Add and subtract groups of small numbers e.g. $5-3+2$</p> <p>Add or subtract a two-digit number to/from a multiple of 10 e.g. $50+38$, $90-27$</p> <p>Add and subtract two-digit numbers which do not bridge 10s or 100s e.g. $34+65$, $68-35$</p> <p>Add near doubles e.g. $18+16$, $60+70$</p> <p>Apply a range of strategies to solve addition and subtraction problems with up to at least 3 digit whole numbers and justify choice of strategy</p> <p>Count forwards and backwards in at least 10s and 100s</p> <p>Add at least 10s and 100s to any whole number up to at least 1000</p>	<p>Apply knowledge of number patterns to start at any number e.g. adding 2 to an even number will always result in an even answer OR adding 10 will change the tens digit, not units digit.</p> <p>Reorder numbers when adding</p> <p>Identify pairs totalling 10 or multiples of 10</p> <p>Partition: add tens and ones separately, then combine</p> <p>Partition: count on in tens and ones to find the total</p> <p>Partition: whole numbers with at least 2 gits into standard component parts to aid mental calculation</p> <p>Partition: count on or back in tens and ones to find the difference</p> <p>Partition: add or subtract 10 or 20 and adjust</p> <p>Partition: double and adjust</p> <p>Partition: count on or back in minutes and hours, bridging through 60 (analogue times)</p> <p>Check answers routinely using inverse operations in mental calculations e.g. $68+32=100$ so $100-32=68$</p>
---	---	--

Addition and Subtraction – CfE Second Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
<p>Sums and differences of pairs of multiples of 10, 100 or 1000</p> <p>Addition doubles of numbers 1 to 100 e.g. 38+38 and the corresponding halves</p> <p>What must be added to any three-digit number to make the next multiple of 100 e.g. 521 + = 600</p> <p>Pairs of fractions and decimal fractions that total 1</p> <p>Simple common fractions</p>	<p>Add or subtract any pair of two digit numbers, including crossing the tens and 100 boundary e.g. 47+58, 91-35</p> <p>Add or subtract a near multiple of 10 e.g. 56+29, 86-38</p> <p>Add near doubles of two-digit numbers e.g. 38-37</p> <p>Add or subtract two-digit or three-digit multiples or 10 e.g. 120-40, 140+150, 370-180</p>	<p>Count on or back in hundreds, tens and ones</p> <p>Partition: add tens and ones separately then recombine</p> <p>Partition: subtract tens and then ones e.g. subtracting 27 by subtracting 20 then 7</p> <p>Subtract by counting up from the smaller to the larger number</p> <p>Partition: add or subtract a multiple of 10 and adjust e.g. 56+29 = 56+30-1 or 86-38 = 86-40+2</p> <p>Partition: double and adjust</p> <p>Use knowledge of place value and related calculations e.g. 140+150 =290 using 14+15=29</p> <p>Partitioning: count on or back in minutes and hours, bridging through 60 (analogue and digital times)</p>
<p>Sums and differences of decimals e.g. 6.5+2.7, 7.8-1.3</p> <p>Doubles and halves of decimals e.g. half of 5.6, double 3.4</p> <p>What must be added to any four-digit number to make the next multiple of 1000 e.g. 4087+ = 5000</p> <p>What must be added to a decimal with units and tenths to make the next whole number eg.7.2 + = 8</p>	<p>Add or subtract a pair of two-digit numbers or three-digit multiples of 10 e.g. 38+86, 620-380, 350+360</p> <p>Add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number e.g. 253+198</p> <p>Find the difference between near multiples of 100 or of 1000 e.g. 607-588, 6070-4087</p> <p>Add or subtract any pairs of decimal fractions each with units and tenths eg.5.7+2.5, 6.3-4.8</p>	<p>Count on or back in hundreds, tens, ones and tenths</p> <p>Partition: add hundreds, tens or ones separately, then combine</p> <p>Subtract by counting up from the smaller to the larger number</p> <p>Partition: double and adjust</p>

		<p>Use knowledge of place value and related calculations e.g. 6.3-4.8 using 63-48 Partition: count on or back in minutes and hours bridging through 60 (analogue and digital times)</p>
<p>Addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place e.g. $650 + \quad = 930$, $\quad - 1.4 = 2.5$</p> <p>What must be added to a decimal with units, tenths and hundredths to make the next whole number e.g. $7.26 + \quad = 8$</p> <p>Round a decimal fraction to three decimal places</p> <p>Applies knowledge of rounding to give an estimate to a calculation appropriate to the context and uses this to check reasonableness of the solutions.</p> <p>Create equivalent fractions</p> <p>Round a decimal fraction to two decimal places</p> <p>Arrange integers in ascending or descending order of size</p>	<p>Add or subtract pairs of decimals with units, tenths or hundredths, e.g. $0.7 + 3.38$</p> <p>Find doubles of decimals each with units and tenths e.g. $1.6 + 1.6$</p> <p>Add near doubles of decimals e.g. $2.5 + 2.6$</p> <p>Add or subtract a decimal with units and tenths that is nearly a whole number e.g. $4.3 + 2.9$, $6.5 - 3.8$</p>	<p>Count on or back in thousands, hundreds, tens ones, tenths and hundredths</p> <p>Use knowledge of place value and related calculations e.g. $680 + 430$, $6.8 + 4.3$, $0.68 + 0.43$ can all be worked out using the related calculation $68 + 43$</p> <p>Use knowledge of place value and of doubles of two-digit whole numbers</p> <p>Partition: double and adjust</p> <p>Partition: add or subtract a whole number and adjust e.g. $4.3 + 2.9 = 4.3 + 3 - 0.1$, $6.5 - 3.8 = 6.5 - 4 + 0.2$</p> <p>Partition: count on or back in minutes and hours bridging through 60 (analogue and digital times, 12 hour and 24 hour clock)</p> <p>Partition: explain the link between a digit, its place and its value for numbers with at least three decimal places</p> <p>Add or subtract 10, 100 and 1000 mentally to and from whole numbers and decimal fractions with at least 3 decimal places</p> <p>Add and subtract multiples of 10 to and from whole numbers and decimal fractions with at least 3 decimal places</p>

Addition and Subtraction – CfE Third Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
Convert between whole and mixed number and fractions Read scales and calculate differences involving integers e.g. temperature	Add and subtract commonly used fractions e.g. $\frac{1}{2} + \frac{1}{4}, \frac{3}{4} - \frac{1}{8},$ Add or subtract any pairs of decimal fractions each with units and tenths e.g. $15.7 + 22.5, 6.32 - 4.18, 7.45 - 3.88$ Calculate and change e.g. £20-£15.75 Add and subtract integers appropriately within a given context e.g. calculate the rise in temperature Or Peter has £12 in his bank. If he spends £15, what will his bank statement now show?	Create the appropriate equivalent fraction (common denominators) then add or subtract Partition: add the whole numbers and the decimal fractions separately, then recombine Partition: subtract the whole numbers and the decimal fractions separately then recombine Partition: add or subtract a whole number and adjust e.g. $7.45 - 3.88 = 7.45 - 4 + 0.12$

Addition and Subtraction – CfE Fourth Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
	Add and subtract mixed numbers e.g. $1\frac{3}{4} - \frac{3}{4} =$	Partition: add the whole numbers and the decimal fractions separately, then recombine Partition: subtract the whole numbers and the decimal fractions separately then recombine

Multiplication and Division – CfE First Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
Counting forwards and backwards in 10s Making equal groups The multiplication sign \times The division sign \div	Count on in equal groups Understand that counting on in 5s is half of counting on in 10s Take a larger number and share equally and understand that there may be some left over when doing this	Partitioning numbers to share equally Grouping a larger number of items into smaller equal groups Link between repeated addition and multiplication Link between repeated subtraction and division
Counting on and back to continue sequences in 10s and 5s Reciting and recalling facts of the 10 and 5 times tables With increasing speed and confidence recite and recall multiplication facts for the 2, 4 and 8 times tables	Look for a pattern and explain what it is e.g. 4, 6, 8, ■■■, 16 Building up and recording the 10 then 5 times tables Explain the links between the 10 and 5 times tables e.g. 5 is one half of 10, 10 is double of 5 Put equal groups into an array Explain the links between the 2, 4 and 8 times tables Understand that when they know one multiplication fact, they also know another e.g. 2×4 is the same as 4×2	Continue a pattern by counting on and backwards Doubling and halving of numbers Counting on and back in 5s and 10s Commutative law: Recognising the importance of an array pattern and understand that 4×3 has the same total as 3×4 but that they look different
With increasing speed and confidence recite and recall multiplication facts for the 3, 6 and 9 times tables Multiply and divide by 10 and 100 Dividing by 2 is the same as finding a half Dividing by 4 is the same as finding a quarter	Explain the links between the 3, 6 and 9 times tables e.g. $3+3+3 = 9$ and $6+3 = 9$ Explain what happens to a number when multiplying or dividing by 10 and by 100 e.g. digits move on place to the left when multiplying by 10, or two places if multiplying by 100 Understand that there may be a remainder when dividing by 10 or 100 e.g. $53 \div 10 = 5$ groups of ten with 3 left over Create new table facts from existing knowledge e.g. $3 \times 4 = 12$, so $30 \times 4 = 120$ and $30 \times 40 = 1200$	Partitioning a set to show Commutative Law – understand that 3×4 is the same as 4×3 Apply knowledge of place value to multiply and divide numbers by 10 and 100 Apply knowledge of fractions and the fraction wall to division strategies e.g. to find Apply a range of strategies to determine multiplication facts for example counting in jumps, doubling, repeated addition and arrays.

	<p>Use different mental methods to work out multiplication calculations e.g. multiplying by 10 to multiply by 9</p>	<p>Apply a range of strategies to determine division facts, for example, repeated subtraction, grouping, arrays and multiplication facts.</p> <p>Commutative Law: Recognise and discuss the links between multiplication and division e.g. $3 \times 9 = 27$, $9 \times 3 = 27$, $27 \div 3 = 9$, $27 \div 9 = 3$</p>
--	---	---

Multiplication and Division – CfE Second Level

Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
<p>With increasing speed and confidence multiplication facts to 10x10 and the corresponding division facts</p> <p>Doubles of numbers 1 to 100 e.g. double 58 and corresponding halves</p> <p>Doubles of multiples of 10 and 100 and corresponding halves</p> <p>Fraction and decimal fraction equivalents of one-half, quarters, tenths and hundredths e.g. $\frac{1}{3}$ is 0.3 and 3-hundredths is 0.03</p> <p>Factor pairs for known multiplication facts</p>	<p>Double any two-digit number e.g. double 39</p> <p>Double any multiple of 10 or 100 e.g. double 340, double 800</p> <p>Halve the corresponding multiples of 10 and 100 e.g. half of 60, half of 400</p> <p>Halve any even number to 200</p> <p>Find unit fractions and simple non-unit fractions of numbers and quantities e.g. $\frac{1}{7}$ of 21, $\frac{3}{8}$ of 24</p> <p>Multiply and divide numbers by 1000 by 10 and then 100 (whole – number answers) e.g. 325×10, 42×100, $120 \div 10$, $600 \div 100$, $850 \div 10$</p> <p>Multiply a multiple of 10 to 100 by a single-digit number e.g. 40×3</p> <p>Multiply numbers to 20 by a single-digit e.g. 17×3</p> <p>Identify the remainder when dividing by 2, 5 or 10</p>	<p>Partition, double and halve the tens and ones separately then recombine</p> <p>Use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder</p> <p>Use knowledge of multiplication facts and place value, e.g. $7 \times 8 = 56$ to find 70×8, 7×80</p> <p>Use partitioning and the distributive law to multiply E.g. $13 \times 4 = (10 + 3) \times 4$ $= (10 \times 4) + (3 \times 4)$ $= 40 + 12 = 52$</p>

	<p>Give the factor pair associated with a multiplication fact e.g. identify that is $2 \times 3 = 6$ then 6 has the factor pair 2 and 3</p>	
<p>Squares to 10×10</p> <p>Division facts corresponding to tables up to 10×10 and the related unit fractions, e.g. $7 \times 9 = 63$ so one-ninth of 63 is 7 and one-seventh of 63 is 9</p> <p>Percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths</p> <p>Factor pairs to 100</p>	<p>Multiply and divide two-digit numbers by 4 or 8 e.g. 26×4, $96 \div 8$</p> <p>Multiply two-digit numbers by 5 or 20 e.g. 32×5, 14×20 Multiply by 25 or 50 e.g. 48×25, 32×50</p> <p>Double three-digit multiples of 10 to 500 and find the corresponding halves e.g. 380×2, $760 \div 2$</p> <p>Find the remainder after dividing a two-digit number by a single-digit number e.g. $27 \div 4 = 6 \text{ r } 3$ Multiply and divide whole numbers and decimal fractions by 10, 100 or 1000 e.g. 4.3×10, 0.75×100, $25 \div 10$, $673 \div 100$, $74 \div 100$</p> <p>Multiply pairs of multiples of 10 and a multiple of 100 by a single-digit number (whole number answers) e.g. $80 \div 4$, $270 \div 3$</p> <p>Find fractions of whole numbers or quantities E.g. $\frac{2}{3}$ of 27, $\frac{1}{5}$ of 70g</p> <p>Find 50%, 25% or 10% of whole numbers or quantities e.g. 25% of 20kg, 10% of £80</p> <p>Find factor pairs for numbers to 100 e.g. 30 has the factor pairs 1 x 30, 2 x 15, 3 x 10 and 5 x 6</p>	<p>Multiply or divide by 4 or 8 by repeated doubling or halving</p> <p>Form an equivalent calculation e.g. to multiply by 5, multiply by 10 then halve; to multiply by 20, double then multiply by 10</p> <p>Use knowledge of doubles/halves and understanding of place value e.g. when multiplying by 50 multiply by 100 and divide by 2 Use knowledge of division facts e.g. when carrying out a division to find a remainder</p> <p>Use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point and zero is used as a place holder</p> <p>Use knowledge of multiplication and division facts and understanding of place value e.g. when calculating with multiples of 10</p> <p>Use knowledge of equivalence between fractions and percentages e.g. to find 50%, 25% and 10%</p> <p>Use knowledge of multiplication and division facts to find factor pairs.</p>
<p>Squares to 12×12</p> <p>Squares to the corresponding multiples of 10</p> <p>Prime numbers less than 100</p> <p>Identify multiples and factors of a range of whole numbers</p>	<p>Multiply pairs of two-digit and single-digit numbers e.g. 28×3</p> <p>Divide a two-digit number by a single-digit number e.g. $68 \div 4$</p> <p>Divide by 25 or 50 e.g. $450 \div 25$, $3200 \div 50$</p> <p>Double decimals with units and tenths and find the corresponding halves e.g. double 7.6, half of 15.2</p> <p>Multiply pairs of multiples of 10 and 100 e.g. 50×30, 600×20</p>	<p>Form an equivalent calculation e.g. to divide by 25, divide by 100 then multiply by 4; to divide by 50, divide by 100 then double</p> <p>Partition: use partitioning and the distributive law to divide 10s and units separately E.g. $92 \div 4 = (80 + 12) \div 4$ $= 20 + 3$ $= 23$</p>

<p>Equivalent fractions, decimal fractions and percentages for hundredths e.g. 35% is equivalent to 0.35 or 35-hundredths</p> <p>More complex but commonly used equivalent fractions, decimal fractions and percentages E.g. $33\frac{1}{3}$ is equivalent to $\frac{1}{3}$ or 0.33 $\frac{2}{3}$ % is equivalent to $\frac{2}{3}$ or 0.67</p> <p>Create equivalent fractions and use this knowledge to put a set of most commonly used fractions in order.</p>	<p>Divide multiples of 100 by multiple of 10 or 100 (whole number answers) e.g. $600 \div 20$, $800 \div 400$, $2100 \div 300$</p> <p>Multiply and divide two-digit decimal fractions such as 0.8×7, $4.8 \div 6$</p> <p>Find 10% or multiples of 10% of whole numbers and quantities e.g. 30% of 50ml, 40% of £30, 70% of 200g</p> <p>Simplify fractions by cancelling</p> <p>Scale up and down using known facts e.g. given that three oranges cost 24p, find the cost of four oranges</p> <p>Multiplies and divides whole numbers and decimal fractions with at least three decimal places mentally by 10, 100 and 1000</p> <p>Solves algebraic equations with one variable, for example $3x + 1 = 10$, $2x - 4 = 14$</p>	<p>Applies knowledge and understanding of multiples and factors when solving relevant problems in number, money and measurement</p> <p>Use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division Recognise how to scale up or down using multiplication and division e.g. if three oranges cost 24p: One orange costs $24 \div 3 = 8p$ Four oranges cost $8 \times 4 = 32p$</p> <p>Multiplies and divides whole numbers and decimal fractions with at least three decimal places by a multiple of 10.</p> <p>Locates and orders numbers less than zero Carry out simple conversion calculations, for example changing 1.45 hours into minutes</p>
---	--	--

Multiplication and Division – CfE Third Level		
Recall:	Mental Calculation Skills:	Mental Methods or Strategies:
<p>Learners should be able to derive and recall:</p> <p>Prime numbers less than 100 and know that 1 is not a prime number</p> <p>Quickly recall number facts including at least the 12th multiplication table and square numbers up to 144</p> <p>Roots of perfect squares up to $\sqrt{144}$ and inverse of commonly used squared numbers e.g. $\sqrt{400}$ E.g. $3.5\% = 0.035 = 35/1000$ Includes x^3 and x to the power of 4</p> <p>Simplify a ration e.g. $8:24 = 1.3$</p> <p>Convert ratio to fraction e.g. the ratio of boys to girls is 2:3 therefore the fraction of boys in class is $\frac{2}{5}$</p> <p>Order numbers which are in standard form</p> <p>Simple decimal hours</p>	<p>Working mentally, with jottings if needed, learners should be able to:</p> <p>Identify numbers with odd and even numbers of factors and no factor pairs other than 1 and itself.</p> <p>Multiply and divide whole numbers and decimal fractions by multiples of ten, hundred or a thousand e.g. 4.3×30, $630 \div 200$</p> <p>Simplify fractions and ratios by recognising common factors and cancelling</p> <p>Calculate percentages of whole numbers and quantities e.g. 11% of 40g, 35% of 500</p> <p>Scale up and down using known facts e.g. at present I get 200 text messages per month on my contract. If the cost of each text message doubles how many texts will I get per month?</p> <p>Convert to/from standard form e.g. $1.255 \times 10^7 = 12550000$ or $7155000 = 7.155 \times 10^6$ or $0.00036 = 3.6 \times 10^{-4}$</p> <p>E.g. 15 minutes = 0.25 etc.</p> <p>Evaluate expression using both +ve and -ve values</p> <p>Calculate the probability of a simple event</p>	<p>Learners should understand when to and be able to apply these strategies:</p> <p>Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors</p> <p>Evaluate simple whole number powers e.g. $2^4=16$</p> <p>Express whole numbers as powers, for example $27=3^3$</p> <p>Form an equivalent calculation e.g. 11% of 40 = 10% of 40 and 1% of 40 and recombine e.g. 35% of 500 = 10% of 500 x 7 ÷ 2</p> <p>Best value comparison</p> <p>In context – order of operations</p>

Multiplication and Division – CfE Fourth Level		
Recall: Learners should be able to derive and recall:	Mental Calculation Skills: Working mentally, with jottings if needed, learners should be able to:	Mental Methods or Strategies: Learners should understand when to and be able to apply these strategies:
Cubed numbers to $5 \times 5 \times 5$ and corresponding roots \times & \div integers Evaluate $\sqrt[3]{x}$	Multiply 2 digit numbers by another 2 digit number	Express whole numbers as powers, for example $27=3^3$