HIGHER MATHS

Trigonometry

Notes with Examples

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Exact Values

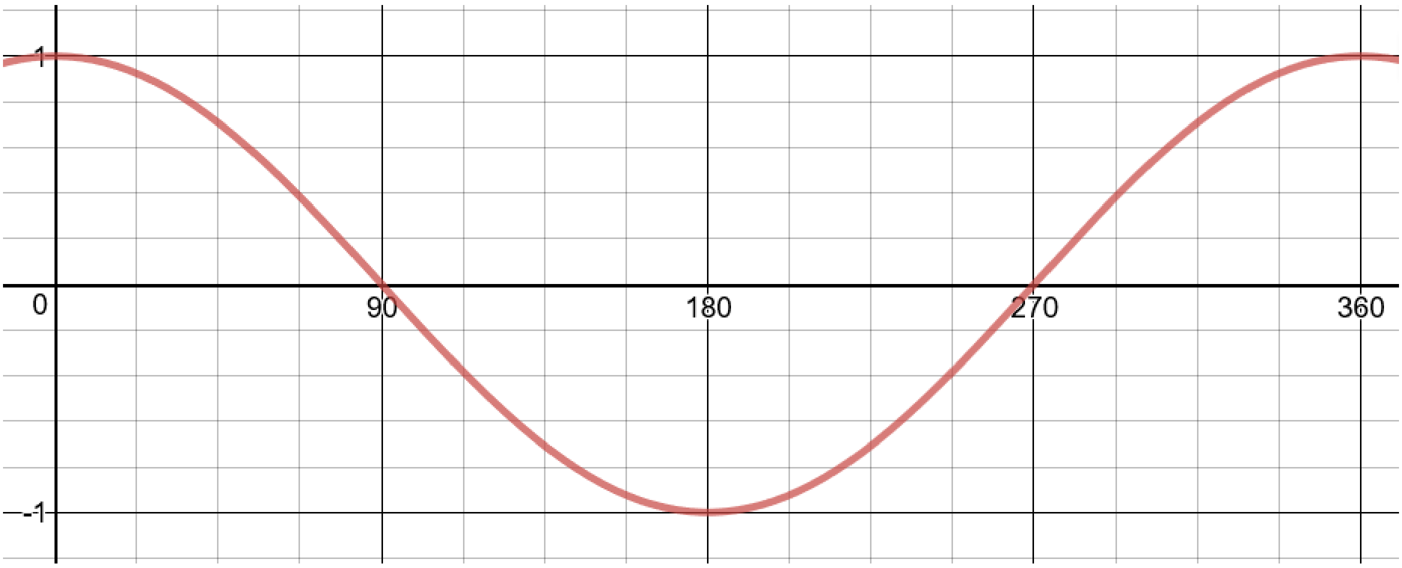
You need to know the exact value of sin, cos and tan of five different angles.

You can either memorize the tables or know how to calculate them.

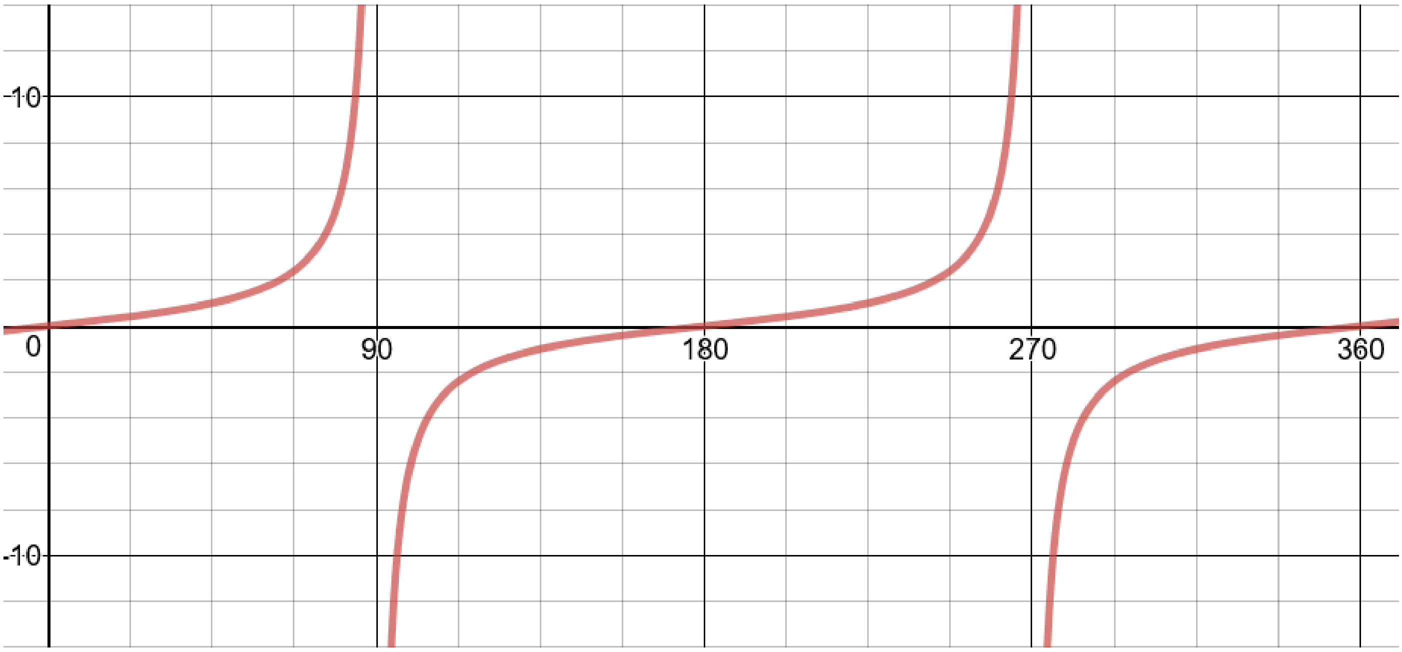
Sin, cos and tan of 0º and 90º

This comes from the graphs of each function

$sin\ 0° \ = \ 0$ $\qquad$ $sin\ 90° \ = \ 1$



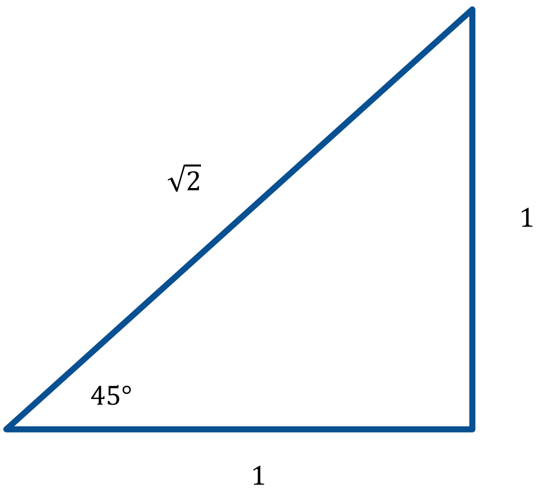
$cos\ 0° \ = \ 1$ $\qquad$ $cos\ 90° \ = \ 0$



$tan\ 0° = 0 \qquad tan\ 90° = \text{undefined}$

Sin, cos and tan of 45°

This comes from a 1, 1,$\sqrt{2}$ triangle



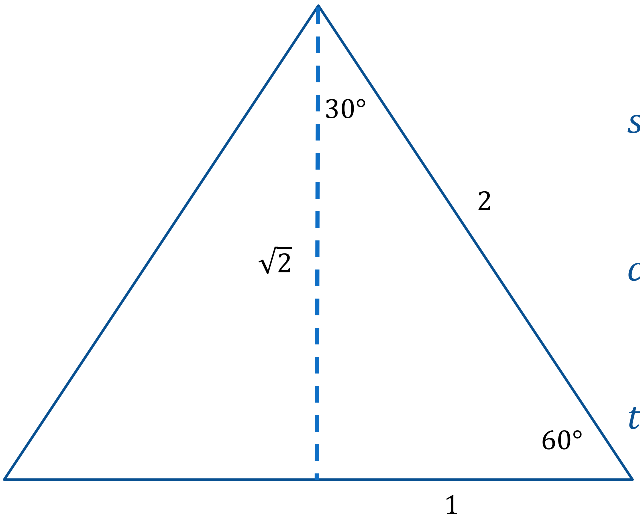
$$sin\ 45° = \frac{O}{H} = \frac{1}{\sqrt{2}}$$

$$cos\ 45° = \frac{A}{H} = \frac{1}{\sqrt{2}}$$

$$tan\ 45° = \frac{O}{A} = \frac{1}{1} = 1$$

Sin, cos and tan of 30º and 60º

This comes from a 1, 2, $\sqrt{3}$ triangle. This starts off as a 2, 2, 2 equilateral triangle.



$$\sin 30° = \frac{O}{H} = \frac{1}{2} \qquad \sin 60° = \frac{O}{H} = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{A}{H} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{A}{H} = \frac{1}{2}$$

$$\tan 30° = \frac{O}{A} = \frac{1}{\sqrt{3}} \qquad \tan 60° = \frac{O}{A} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

This table shows every exact value you need to know.

	0º	**30º**	**45º**	**60º**	**90º**
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Examples

T-01 Calculate the exact value of

(a) sin45º

$= \frac{1}{\sqrt{2}}$

(b) cos300º

$= \cos 60$

$= \frac{1}{2}$

(c) sin(-135)^0

$= \sin 225$

$= -\sin 45$

$= -\frac{1}{\sqrt{2}}$

(d) tan $\frac{7\pi}{4}$

$= -\tan \frac{\pi}{4}$

$= -1$

T-02 Find, in its simplest form, the exact value of

(a) 2 sin300° cos150°

$= 2 \times (-\sin 60) \times (-\cos 30)$

$= 2 \times \left(-\frac{\sqrt{3}}{2}\right) \times \left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{6}{4}$

$= \frac{3}{2}$

(b) sin210° + cos210°

$= (-\sin 30) + (-\cos 30)$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}$

$= \frac{-1-\sqrt{3}}{2}$

T-03 Solve $\sqrt{3}\tan x = 1$ using exact values and giving your answer in radians.

$\sqrt{3}\tan x = 1$

$\tan x = \frac{1}{\sqrt{3}}$

$x = 30, 210$

$= \frac{\pi}{6}, \frac{7\pi}{6}$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= 30°$

S	A ✓
✓T	C

Solving Trigonometric Linear Equations

Solving in degrees

We must know how to solve trig equations in degrees for angles beyond 360°. Plus we must know exact values.

We also need to know how to solve equations with compound angles. A compound angle equation may look like this

$$3sin(x+45)^o = 2 \qquad \text{or} \qquad 4tan2x^o = 3$$

We no longer have $x°$ but the angle is a function of $x°$

Examples

T-04 Solve these equations for x.

(a) $4tanx° - 3 = 0,\ 0° < x < 720°$

$4\tan x - 3 = 0$

$\tan x = \frac{3}{4}$

$\tan^{-1}\left(\frac{3}{4}\right) = 36.9°$

$x = 36.9, 216.9, 396.9, 576.9$

S	A ✓
✓ T	C

(b) $2sinx° + 1 = 0,\ 0° < x < 270°$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = 30$

$x = 210, \cancel{330}$

S	A
✓ T	C ✓

T-05 Solve these equations for x.

(a) $4tan2x° - 1 = 0,\ 0° < x < 360°$

$4\tan 2x - 1 = 0$

$\tan 2x = \frac{1}{4}$

$\tan^{-1}\left(\frac{1}{4}\right) = 14°$

$2x = 14, 194, 374, 554$

$x = 7, 97, 187, 277$

S	A ✓
✓ T	C

(b) $2sin(x+60)° = 1,\ 0° < x < 360°$

$2\sin(x+60) = 1$

$\sin(x+60) = \frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = 30$

$x + 60 = 30, 150, 390$

+360 SO 2 SOLUTIONS ARE IN LIMITS

$x = \cancel{-30}, 90, 330$

✓ S	A ✓
T	C

Solving in radians

We have to be able to solve trig equations in radians as well as degrees. Remember to multiply by π and divide by 180.

We also need to know how to solve equations from a graph or a modelled situation.

Examples

T-06 Solve these equations for x in radians.

(a) $2tan2x - 1 = 0, \ 0 < x < 2\pi$

(b) $2sin3x + 1 = 0, \ 0 < x < 2\pi$

(a) $2\tan 2x - 1 = 0$

$\tan 2x = \frac{1}{2}$

$\tan^{-1}\left(\frac{1}{2}\right) = 27°$

$2x = 27°, 207°, 387°, 567°$

$x = 13.5°, 103.5°, 193.5°, 283.5°$

$x = 0.23, 1.81, 3.38, 4.95$

S | A ✓

✓T | C

(b) $2\sin 3x + 1 = 0$

$\sin 3x = -\frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = 30° = \frac{\pi}{6}$

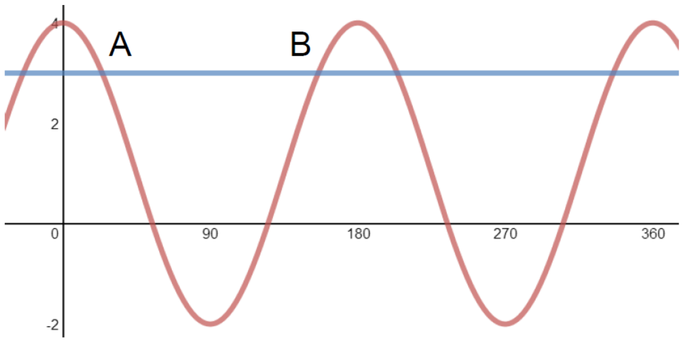
$3x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$

$x = \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}$

S | A

✓T | C ✓

T-07 The diagram shows the graph of a cosine function.



(a) State the equation of the function

(b) The straight line y = 3 cuts the graph at A and B. State the coordinates of A and B.

(a) $y = 3\cos 2x + 1$

(b) $3\cos 2x + 1 = 3$

$3\cos 2x = 2$

$\cos 2x = \frac{2}{3}$

$\cos^{-1}\left(\frac{2}{3}\right) = 48°$

$2x = 48°, 322°, 408°, 682°$

$x = 24°, 161°, 204°, 341°$

S | A ✓

T | C ✓

$\boxed{A\ (24, 3) \quad B\ (161, 3)}$

Modelling real life context

Example

T-08 The depth of water, D metres, in a harbour is given by the formula

$$D = 3 + 1{\cdot}75 \sin 30h^\circ$$

where h is the number of hours after midnight.

(*a*) Calculate the depth of water at 5 am.

(b) Calculate the first six times after midnight Monday that the water will be at 3.6m

(a) $D = 3 + 1.75 \sin 30h$

$= 3 + 1.75 \sin(30 \times 5)$

$= 3 + 1.75 \sin(150)$

$= 3 + 1.75(0.5)$

$= 3.875$ m

(b) $3 + 1.75 \sin 30h = 3.6$

$1.75 \sin 30h = 0.6$

$\sin 30h = \frac{0.6}{1.75}$

$\sin^{-1}\left(\frac{0.6}{1.75}\right)$

$= 20$

✓S	A✓
T	C

$30h = 20, 160, 380, 520, 740, 880$

$h = \frac{20}{30}, \frac{160}{30}, \frac{380}{30}, \frac{520}{30}, \frac{740}{30}, \frac{880}{30}$

$\boxed{\text{TIME} = 00{:}40 \text{ MON}, 05{:}20 \text{ MON}, 12{:}40 \text{ MON}, 17{:}20 \text{ MON}, 00{:}40 \text{ TUE}, 05{:}20 \text{ TUE}}$

Addition Formulae

We looked at compound angles previously and will be working with these angles using the addition formulae.

There are four addition formulae:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

These are given in an assessment and look like:

$$\boxed{\begin{aligned}\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B\\ \cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B\end{aligned}}$$

We need to know how to expand from $\sin(A+B)$ and also recognise the expansion and be able to convert back. This is covered later when we look at the wave function.

Examples

T-09 Rewrite these expressions in terms of a single angle.

(a) $\sin 35°\cos 25° - \cos 35°\sin 25°$

(b) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

(a) $\sin(35-25)$
$= \sin 10$

(b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$
$= \cos\left(\frac{7\pi}{12}\right)$

T-10 Find the exact value of

(a) $sin20°cos10° + cos20°sin10°$

(b) $cos\frac{\pi}{3}\,cos\,\frac{\pi}{12} + sin\,\frac{\pi}{3}\,sin\frac{\pi}{12}$

(c) $cos15°$

(d) $sin\frac{5\pi}{12}$

(a) $\sin 20 \cos 10 + \cos 20 \sin 10$

$= \sin(20+10)$

$= \sin 30$

$= \frac{1}{2}$

(b) $\cos\frac{\pi}{3}\cos\frac{\pi}{12} + \sin\frac{\pi}{3}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{12}\right)$

$= \frac{1}{\sqrt{2}}$

(c) $\cos 15°$

$= \cos(45 - 30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(d) $\sin\left(\frac{5\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$\frac{5\pi}{12} = 75°$

$= 45 + 30$

$= \frac{\pi}{4} + \frac{\pi}{6}$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

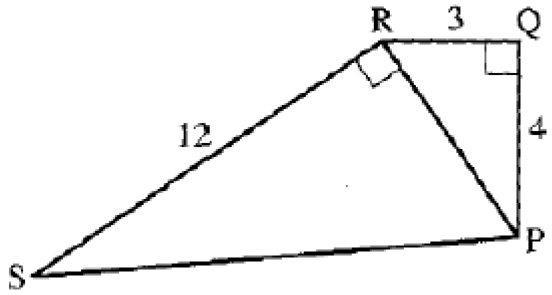
T-11 Expand and simplify $cos(x + 60)°$

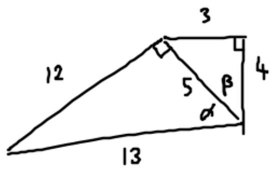
$\cos(x + 60)$

$= \cos x \cos 60 - \sin x \sin 60$

$= \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

T-12 Show that $sinQPS = \frac{63}{65}$





$\sin\alpha = \frac{O}{H} = \frac{12}{13}$ $\quad \sin\beta = \frac{O}{H} = \frac{3}{5}$

$\cos\alpha = \frac{A}{H} = \frac{5}{13}$ $\quad \cos\beta = \frac{A}{H} = \frac{4}{5}$

$\sin QPS = \sin(\alpha+\beta)$

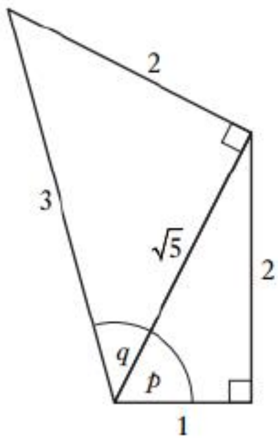
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{12}{13} \times \frac{4}{5} + \frac{5}{13} \times \frac{3}{5}$

$= \frac{48}{65} + \frac{15}{65}$

$= \frac{63}{65}$

T-13 The diagram shows two right-angled triangles with sides and angles as given.



Calculate the value of cos(p + q)

$\sin p = \frac{O}{H} = \frac{2}{\sqrt{5}}$ $\quad \sin q = \frac{O}{H} = \frac{2}{3}$

$\cos p = \frac{A}{H} = \frac{1}{\sqrt{5}}$ $\quad \cos q = \frac{A}{H} = \frac{\sqrt{5}}{3}$

$\cos(p+q)$

$= \cos p\cos q - \sin p\sin q$

$= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{3} - \frac{2}{\sqrt{5}} \times \frac{2}{3}$

$= \frac{\sqrt{5}}{3\sqrt{5}} - \frac{4}{3\sqrt{5}}$

$= \frac{\sqrt{5}-4}{3\sqrt{5}}$

Double Angle Formulae

We looked at compound angles previously and will be working with these angles using the double angle formulae.

There are four double angle formulae:

$$\boxed{\begin{aligned} \sin 2A &= 2sinAcosA \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2\cos^2 A - 1 \\ \cos 2A &= 1 - 2\sin^2 A \end{aligned}}$$

These are given in an assessment.

We need to know how to expand and, in the case of $cos2A$, what expansion to use.

It should be noted that any of these formulae work for any angle that can be expressed as a double angle.

Examples

T-14 Find the exact value of

(a) $2sin15°cos15°$

(b) $1 - 2sin^2\frac{\pi}{4}$

(a) $2\sin 15 \cos 15$

$= \sin(2 \times 15)$

$= \sin 30$

$= \frac{1}{2}$

(b) $1 - 2\sin^2 \frac{\pi}{4}$

$= \cos\left(2 \times \frac{\pi}{4}\right)$

$= \cos \frac{\pi}{2}$

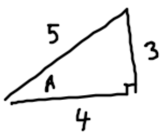
$= 0$

T-15 Find the exact value of

(a) $sin2A$ where A is an acute angle with $tanA = \frac{3}{4}$

(b) $cos2P$ where P is an acute angle with $cosP = \frac{3}{7}$

(a)

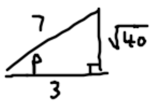


$\sin 2A = 2\sin A \cos A$

$= 2 \times \frac{3}{5} \times \frac{4}{5}$

$= \frac{24}{25}$

(b)



$\cos 2P = 2\cos^2 P - 1$

$= 2\left(\frac{3}{7}\right)^2 - 1$

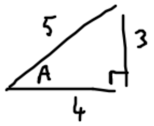
$= \frac{18}{49} - 1 \quad = -\frac{31}{49}$

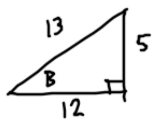
T-16 If A and B are acute angle with $cosA = \frac{4}{5}$ and $sinB = \frac{5}{13}$, find the exact value of:

(a) $sin2A$

(b) $cos2A$

(c) $sin(2A + B)$





$\sin A = \frac{O}{H} = \frac{3}{5}$ $\quad \sin B = \frac{O}{H} = \frac{5}{13}$

$\cos A = \frac{A}{H} = \frac{4}{5}$ $\quad \cos B = \frac{A}{H} = \frac{12}{13}$

(a) $\sin 2A$

$= 2\sin A\cos A$

$= 2 \times \frac{3}{5} \times \frac{4}{5}$

$= \frac{24}{25}$

(b) $\cos 2A$

$= 2\cos^2 A - 1$

$= 2\left(\frac{4}{5}\right)^2 - 1$

$= \frac{32}{25} - \frac{25}{25} = \frac{7}{25}$

(c) $\sin(2A + B)$

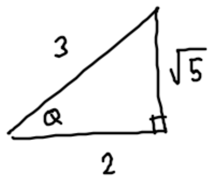
$= \sin 2A \cos B + \cos 2A \sin B$

$= \frac{24}{25} \times \frac{12}{13} - \frac{7}{25} \times \frac{5}{13}$

$= \frac{288}{325} - \frac{35}{325}$

$= \frac{253}{325}$

T-17 If Q is an acute angle with $cosQ = \frac{2}{3}$, show that $sin4Q = -\frac{8\sqrt{5}}{81}$



$\sin Q = \frac{O}{H} = \frac{\sqrt{5}}{3}$

$\cos Q = \frac{A}{H} = \frac{2}{3}$

$\sin 2Q = 2\sin Q\cos Q$

$= 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$

$= \frac{4\sqrt{5}}{9}$

$\cos 2Q = 2\cos^2 Q - 1$

$= 2\left(\frac{2}{3}\right)^2 - 1$

$= \frac{8}{9} - 1$

$= -\frac{1}{9}$

$\sin 4Q = 2\sin 2Q\cos 2Q$

$= 2 \times \frac{4\sqrt{5}}{9} \times -\frac{1}{9}$

$= -\frac{8\sqrt{5}}{81}$

Quadratic Trigonometric Equations

We now need to know how to solve quadratic trigonometric equations. To solve these we need to factorise the equations.

If it helps, think of the quadratic function without the trig part!
$2cos^2x + 3cosx - 2 = 0$ =====> $2x^2 + 3x - 2$ (to help factorise)

Examples

T-18 Solve

(a) $2cos^2x + 3cosx - 2 = 0 \quad 0° < x < 360°$

(b) $2sin^2x - 1 = 0 \quad 0° < x < 360°$

(c) $4sin^2\left(\theta - \frac{\pi}{6}\right) - 3 = 0 \quad 0 < \theta < 2\pi$

(a) $2\cos^2 x + 3\cos x - 2 = 0$

$2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -2$

No solutions

S | A ✓
T | C ✓

$x = 60°, 300°$

(b) $2\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

✓S | A✓
✓T | C✓

$x = 45°, 135°, 225°, 315°$

(c) $4\sin^2\left(\theta - \frac{\pi}{6}\right) - 3 = 0$

$\sin^2\left(\theta - \frac{\pi}{6}\right) = \frac{3}{4}$

$\sin\left(\theta - \frac{\pi}{6}\right) = \pm\frac{\sqrt{3}}{2}$

✓S | A✓
✓T | C✓

$\theta - \frac{\pi}{6} = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\theta = \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Quadratic Trigonometric Equations using Identities

We now need to combine everything we have learned in this topic to solve harder trig equations. Standard equation you need to know how to solve look like:

$a\cos^2 x + b\sin x + c$	or	$a\sin^2 x + b\cos x + c$	quadratic mixed trig
$a\cos 2x + b\sin x + c$	or	$a\cos 2x + b\cos x + c$	$\cos 2x$ expansion
$a\sin 2x + b\sin x + c$	or	$a\sin 2x + b\cos x + c$	$\sin 2x$ expansion

where a, b and c are constants.

It is important that when we expand using the double angle formula that we choose the expansion that will let us factorise the resultant expression.

Remember that $\cos^2 x + \sin^2 x = 1$

Examples

T-19 Solve

(a) $5sin^2x - 2cosx - 2 = 0 \qquad 0° < x < 360°$

(b) $6cos2x + sinx = 5 \qquad 0° < x < 360°$

(a) $5\sin^2 x - 2\cos x - 2 = 0$

$\sin^2 x = 1 - \cos^2 x$

$5(1-\cos^2 x) - 2\cos x - 2 = 0$

$5 - 5\cos^2 x - 2\cos x - 2 = 0$

$-5\cos^2 x - 2\cos x + 3 = 0$

$-(5\cos^2 x + 2\cos x - 3) = 0$

$5x^2 + 2x - 3 = (5x-3)(x+1)$

$-(5\cos x - 3)(\cos x + 1) = 0$

$\cos x = \frac{3}{5}$ $\qquad$ $\cos x = -1$

$\cos^{-1}\left(\frac{3}{5}\right) = 37°$

S	A ✓
T	C ✓

$x = 37°, 323°$ $\qquad$ $x = 270°$

(b) $6\cos 2x + \sin x = 5$

$6\cos 2x + \sin x - 5 = 0$

$\cos 2x = 1 - 2\sin^2 x$

$6(1 - 2\sin^2 x) + \sin x - 5 = 0$

$6 - 12\sin^2 x + \sin x - 5 = 0$

$-12\sin^2 x + \sin x + 1 = 0$

$-(12\sin^2 x - \sin x - 1) = 0$

$12x^2 - x - 1 = (4x+1)(3x-1)$

$-(4\sin x + 1)(3\sin x - 1) = 0$

$\sin x = -\frac{1}{4}$ $\qquad$ $\sin x = \frac{1}{3}$

$\sin^{-1}\left(\frac{1}{4}\right) = 14°$

S	A
✓ T	C ✓

$\sin^{-1}\left(\frac{1}{3}\right) = 19°$

✓ S	A ✓
T	C

$x = 194°, 346°$ $\qquad$ $x = 19°, 161°$

T-20 Solve

(a) $3cos2x - 10sinx = -1 \qquad 0° < x < 360°$

(b) $cos2x + 3cosx + 2 = 0\ 0° < x < 360°$

(a)
$$3\cos 2x - 10\sin x = -1$$
$$3\cos 2x - 10\sin x + 1 = 0$$
$$3(1 - 2\sin^2 x) - 10\sin x + 1 = 0$$
$\cos 2x = 1 - 2\sin^2 x$
$$3 - 6\sin^2 x - 10\sin x + 1 = 0$$
$$-6\sin^2 x - 10\sin x + 4 = 0$$
$$-2(3\sin^2 x + 5\sin x - 2) = 0$$
$$-2(3\sin x - 1)(\sin x + 2) = 0$$
$3x^2 + 5x - 2$
$= (3x-1)(x+2)$

$\sin x = \frac{1}{3}$ $\qquad$ $\sin x = -2$

$x = 19°, 161°$ $\qquad$ NO SOLUTIONS

$\sin^{-1}\left(\frac{1}{3}\right)$
$= 19°$

S ✓	A ✓
T	C

(b)
$$\cos 2x + 3\cos x + 2 = 0$$
$$2\cos^2 x - 1 + 3\cos x + 2 = 0$$
$\cos 2x = 2\cos^2 x - 1$
$$2\cos^2 x + 3\cos x + 1 = 0$$
$$(2\cos x + 1)(\cos x + 1) = 0$$
$2x^2 + 3x + 1$
$(2x+1)(x+1)$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$

$\cos^{-1}\left(\frac{1}{2}\right)$
$= 60°$

$x = 60°, 300°$ $\qquad$ $x = 180°$

S	A ✓
T	C ✓

T-21 Solve

$$2sin2x + 3sinx = 0 \qquad 0° < x < 360°$$

$2\sin 2x + 3\sin x = 0$

$2\sin x\cos x + 3\sin x = 0$

$\sin x(2\cos x + 3) = 0$

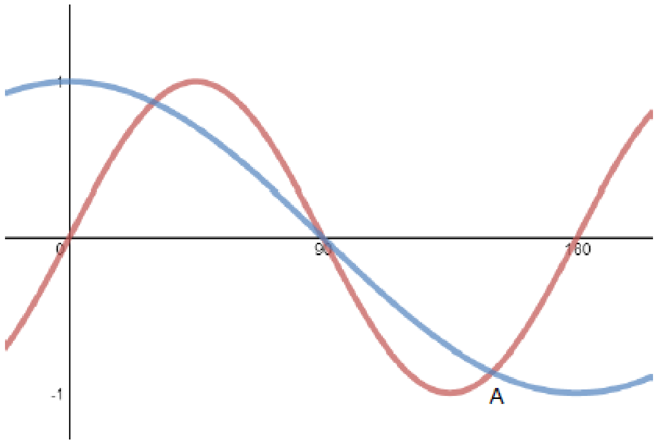
$\sin x = 0$ $\qquad \cos x = -\frac{3}{2}$

$x = \cancel{0°}, 180°, \cancel{360°}$ $\qquad$ NO SOLUTIONS

$0 < x < 360$
SO $0°$ AND $360°$
IS NOT A SOLUTION

T-22 The diagram shows the graphs of $y = sin2x$ and $y = cosx$.

Find the coordinates of A.



POINT OF INTERSECTION:

$\sin 2x = \cos x$

$\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\qquad \sin x = \frac{1}{2}$ $\qquad \sin^{-1}\frac{1}{2}$

$x = 90°$ $\qquad x = 30°, 150°$ $\qquad = 30°$

S | A
T | C

THREE POINTS OF INTERSECTION WHEN $x = 30, 90, 150$

POINT A $\quad x = 150 \quad y = \cos 150$

$= -\cos 30$

$= -\frac{\sqrt{3}}{2}$

$\boxed{A\left(150, -\frac{\sqrt{3}}{2}\right)}$

all things trigonometric

DEGREES & RADIANS:

π radians $= 180°$

to convert

degrees $\xrightarrow{\times \pi \div 180}$ radians

radians $\xrightarrow{\times 180 \div \pi}$ degrees

EXAMPLE

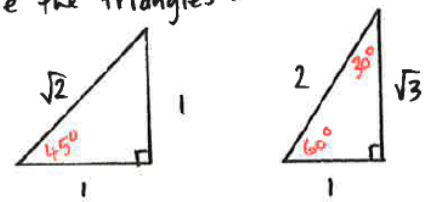
$45° = \frac{45\pi}{180} = \frac{\pi}{4}$

$\frac{\pi}{4} = \frac{180\pi}{4\pi} = 45°$

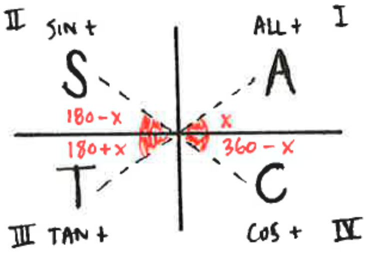
EXACT VALUES:

	$0°$	$30°$ ($\frac{\pi}{6}$)	$45°$ ($\frac{\pi}{4}$)	$60°$ ($\frac{\pi}{3}$)	$90°$ ($\frac{\pi}{2}$)
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	und.

use the triangles:



TRIG EQUATIONS & CAST DIAGRAM:



to solve

1. rearrange to trig function =
2. find first quadrant angle by ignoring sign
3. use CAST diagram & sign to identify quads
4. solve and state answers

$\sqrt{2}\sin x + 1 = 0$

$\sqrt{2}\sin x = -1$

$\sin x = \frac{-1}{\sqrt{2}}$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

sin (−)

SOL III, IV $x = 180 + 45, 360 - 45$

$= 225°, 315°$

EQUATIONS WITH x AND 2x:

equations with mixed angles ($x°$ and $2x°$) cant be solved without replacing the 2x using:

GIVEN IN FORMULAE SHEET

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

choose the right substitution then factorise

COMPOUND ANGLE FORMULAE : these are given in a formulae sheet, just know how to use them

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$ same sign

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$ opposite sign

TRIG IDENTITIES : Remember these?

$$\sin^2 x + \cos^2 x = 1 \qquad \frac{\sin x}{\cos x} = \tan x$$

For trig graphs see "FUNCTIONS & GRAPHS"