HIGHER MATHS

2015 Exam

Question Paper with Solutions

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2015 Non Calculator Paper

1. Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.

Determine the value of t. **2**

$$\underline{u}.\underline{v} = 8(-3) + 2(t) + (-1)(-6)$$
$$= -24 + 2t + 6$$
$$= -18 + 2t$$

PERPENDICULAR VECTORS $\underline{u}.\underline{v} = 0$

$$-18 + 2t = 0$$
$$2t = 18$$
$$\boxed{t = 9}$$

2. Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$. **4**

$$y = 2x^3 + 3$$
$$\frac{dy}{dx} = 6x^2$$

POI

$$y = 2(-2)^3 + 3$$
$$= -16 + 3$$
$$= -13$$
$$\boxed{(-2,-13)}$$

GRADIENT

$$\frac{dy}{dx} = 6(-2)^2$$
$$= 24$$
$$\boxed{m = 24}$$

TANGENT

$$y - b = m(x - a)$$
$$y + 13 = 24(x + 2)$$
$$y = 24x + 48 - 13$$
$$\boxed{y = 24x + 35}$$

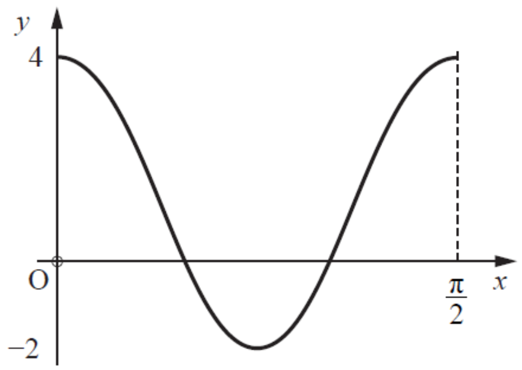
3. Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully. **4**

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & 18 & -24 \\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

REM = 0 SO $(x+3)$ IS A FACTOR

$$x^3 - 3x^2 - 10x + 24$$
$$= (x+3)(x^2 - 6x + 8)$$
$$= (x+3)(x-4)(x-2)$$

4. The diagram shows part of the graph of the function $y = p\cos qx + r$.



Write down the values of p, q and r. **3**

$p = \frac{4-(-2)}{2} = 3$

$q = 4$ (4 graphs in 360° (2π))

$r = +1$

$\boxed{y = 3\cos 4x + 1}$

5. A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$.

(a) Determine an expression for $g^{-1}(x)$. **2**

(b) Write down an expression for $g(g^{-1}(x))$. **1**

(a) $y = 6 - 2x$

$2x + y = 6$

$2x = 6 - y$

$x = \frac{6-y}{2}$

$g^{-1}(x) = \frac{6-x}{2}$

(b) $g(g^{-1}(x)) = x$

6. Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$. **3**

$\log_6 12 + \frac{1}{3}\log_6 27$

$= \log_6 12 + \log_6 27^{1/3}$

$= \log_6 12 + \log_6 3$

$= \log_6 12 \times 3$

$= \log_6 36$

$= 2$

$\boxed{\log_6 12 + \frac{1}{3}\log_6 27 = 2}$

7. A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \dfrac{2}{x\sqrt{x}}\right)$.

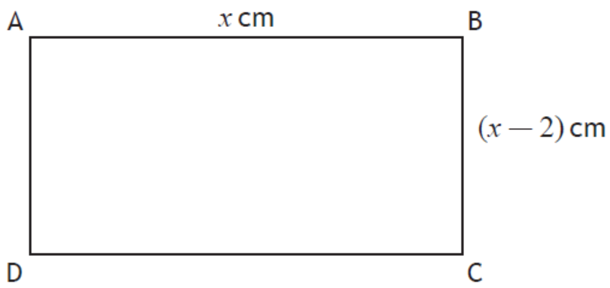
Find $f'(4)$. **4**

$$f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$$
$$= 3x\sqrt{x} - \frac{2\cancel{\sqrt{x}}}{x\cancel{\sqrt{x}}}$$
$$= 3x^1x^{1/2} - \frac{2}{x}$$
$$= 3x^{3/2} - 2x^{-1}$$

$$f(x) = 3x^{3/2} - 2x^{-1}$$
$$f'(x) = \frac{9}{2}x^{1/2} + 2x^{-2}$$
$$= \frac{9}{2}\sqrt{x} + \frac{2}{x^2}$$
$$f'(4) = \frac{9}{2}\sqrt{4} + \frac{2}{(4)^2}$$
$$= 9 + \frac{2}{16}$$
$$= 9\tfrac{1}{8}$$

$$\boxed{f'(4) = 9\tfrac{1}{8}}$$

8. ABCD is a rectangle with sides of lengths x centimetres and $(x - 2)$ centimetres, as shown.



If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of x. **4**

$$A = x(x-2)$$
$$= x^2 - 2x$$
$$x^2 - 2x < 15$$
$$x^2 - 2x - 15 < 0$$
$$(x+3)(x-5) < 0$$

$$-3 < x < 5$$

$\boxed{\text{x MUST BE GREATER THAN 2, SO } 2 < x < 5}$

9. A, B and C are points such that AB is parallel to the line with equation $y + \sqrt{3}x = 0$ and BC makes an angle of $150°$ with the positive direction of the x-axis.

Are the points A, B and C collinear? **3**

$y + \sqrt{3}x = 0$

$y = -\sqrt{3}x$

$\boxed{m_{AB} = -\sqrt{3}}$

$m = \tan\theta$

$\theta = \tan^{-1}(-\sqrt{3})$

$\boxed{\theta = 120°}$

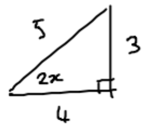
$\tan 60 = \sqrt{3}$

LINE AB MAKES AN ANGLE OF 120° WITH x AXIS SO A, B & C ARE NOT COLLINEAR

10. Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of

(a) $\cos 2x$ **1**

(b) $\cos x$. **2**



(a) $\cos 2x = \frac{A}{H} = \frac{4}{5}$

(b) $\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x - 1 = \frac{4}{5}$

$2\cos^2 x = \frac{9}{5}$

$\cos^2 x = \frac{9}{10}$

$\cos x = \pm\sqrt{\frac{9}{10}}$

BECAUSE $x < \frac{\pi}{4}$

$\cos x = \frac{3}{\sqrt{10}}$

11. T$(-2, -5)$ lies on the circumference of the circle with equation

$$(x+8)^2 + (y+2)^2 = 45.$$

(a) Find the equation of the tangent to the circle passing through T. **4**

(b) This tangent is also a tangent to a parabola with equation $y = -2x^2 + px + 1 - p$, where $p > 3$.

Determine the value of p. **6**

(a) $T(-2,-5)$ CENTRE $(-8,-2)$

$m_{RADIUS} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-(-5)}{-8-(-2)}$

$= \frac{3}{-6}$

$= -\frac{1}{2}$

$m_1 \times m_2 = -1$

$m_\perp = 2$

$y - b = m(x-a)$

$y + 5 = 2(x+2)$

$y + 5 = 2x + 4$

$y = 2x - 1$

(b) POI $2x - 1 = -2x^2 + px + 1 - p$

$2x^2 + 2x - px - 1 - 1 + p = 0$

$2x^2 + (2-p)x + p - 2 = 0$

FOR TANGENT $b^2 - 4ac = 0$

$(2-p)^2 - (4 \times 2 \times (p-2)) = 0$

$4 - 4p + p^2 - 8p + 16 = 0$

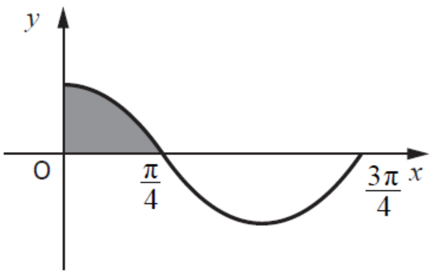
$p^2 - 12p + 20 = 0$

$(p-10)(p-2) = 0$

$p = 10$ $\cancel{p = 2}$

12. The diagram shows part of the graph of $y = a\cos bx$.

The shaded area is $\frac{1}{2}$ unit^2.



What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$? **2**

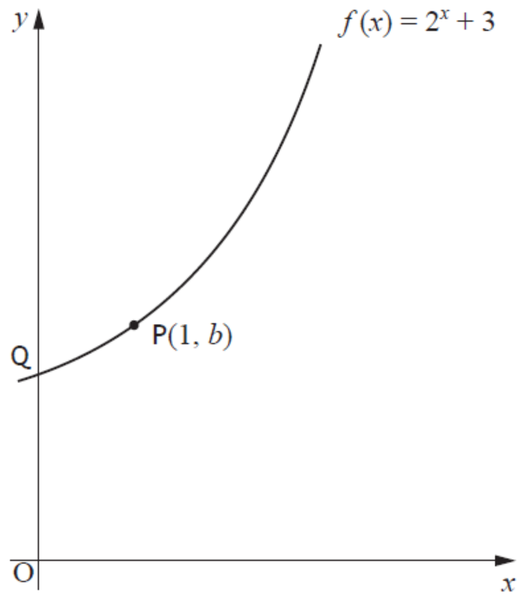
AREA ABOVE AXIS $= \frac{1}{2}$

AREA BELOW AXIS $= -1$

$$\boxed{\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx = -\frac{1}{2}}$$

13. The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? $b = 5$ **1**

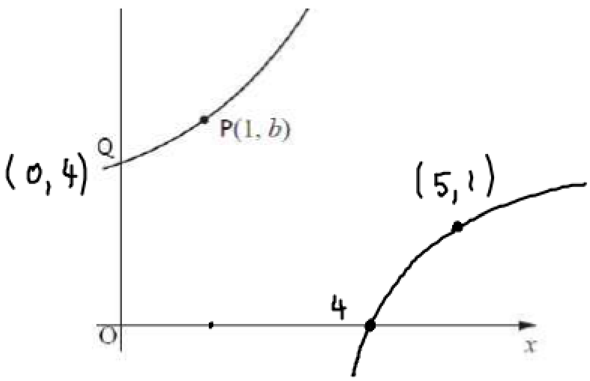
(b) (i) Copy the above diagram.

On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. **1**

(ii) Write down the coordinates of the images of P and Q. **3**

(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x + 1)$. **2**



$(3,11)$ on $-f(x)$ $(3,-11)$

$-f(x+1)$ $(2,-11)$

$-f(x+1)+4$ $(2,-7)$

$\boxed{(2,-7)}$

14. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k? **2**

CENTRE $(6,5)$

$r^2 = 6^2 + 5^2$

$= 36 + 25$

$= 61$

$r = \sqrt{g^2 + f^2 - c}$

So $g^2 + f^2 - c = 61$

$(-6)^2 + (-5)^2 - k = 61$

$61 - k = 61$

$\boxed{k = 0}$

15. The rate of change of the temperature, T °C of a mug of coffee is given by

$$\frac{dT}{dt} = \frac{1}{25}t - k\ , \ 0 \leq t \leq 50$$

- t is the elapsed time, in minutes, after the coffee is poured into the mug
- k is a constant
- initially, the temperature of the coffee is 100 °C
- 10 minutes later the temperature has fallen to 82 °C.

Express T in terms of t. **6**

$T = \int \frac{dT}{dt} = \frac{1}{50}t^2 - kt + c$

when $t = 0$ $T = 100$ $100 = \frac{1}{50}(0)^2 - k(0) + c$

$c = 100$

when $t = 10$ $T = 82$ $82 = \frac{1}{50}(10)^2 - k(10) + 100$

$82 = 2 - 10k + 100$

$10k = 102 - 82$

$10k = 20$

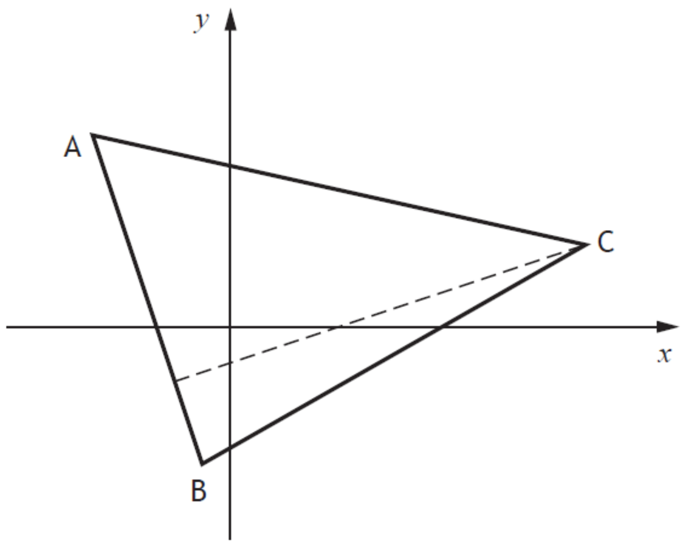
$k = 2$

$\boxed{T = \frac{1}{50}t^2 - 2t + 100}$

2015 Calculator Paper

1. The vertices of triangle ABC are A$(-5, 7)$, B$(-1, -5)$ and C$(13, 3)$ as shown in the diagram.

The broken line represents the altitude from C.



(a) Show that the equation of the altitude from C is $x - 3y = 4$. **4**

(b) Find the equation of the median from B. **3**

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

(a) $M_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-5 - 7}{-1 - (-5)}$

$= \frac{-12}{4}$

$= -3$

$m_1 \times m_2 = -1$

$m_\perp = \frac{1}{3}$

$y - b = m(x - a)$

$y - 3 = \frac{1}{3}(x - 13)$

$3y - 9 = x - 13$

$-x + 3y + 4 = 0$

$\boxed{x - 3y = 4}$

(b) $Q = \text{mid}_{AC} = \left(\frac{(-5)+13}{2}, \frac{7+3}{2}\right)$

$= (4, 5)$

$m_{BQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(-5) - 5}{(-1) - 4}$

$= \frac{-10}{-5}$

$= 2$

$y - b = m(x - a)$

$y + 5 = 2(x + 1)$

$y + 5 = 2x + 2$

$\boxed{y = 2x - 3}$

(c) PoI

$3y = x - 4$

$3y = 6x - 9$

$6x - 9 = x - 4$

$5x = 5$

$x = 1$

$y = 2(1) - 3$

$= -1$

$\boxed{\text{PoI } (1, -1)}$

2. Functions f and g are defined on suitable domains by

$$f(x) = 10 + x \text{ and } g(x) = (1 + x)(3 - x) + 2.$$

(a) Find an expression for $f(g(x))$. **2**

(b) Express $f(g(x))$ in the form $p(x + q)^2 + r$. **3**

(c) Another function h is given by $h(x) = \dfrac{1}{f(g(x))}$.

What values of x cannot be in the domain of h? **2**

(a) $f(g(x)) = f((1+x)(3-x)+2)$

$= 10 + (1+x)(3-x) + 2$

$= 12 + (3 - x + 3x - x^2)$

$\boxed{f(g(x)) = 15 + 2x - x^2}$

(b) $f(g(x)) = -x^2 + 2x + 15$

$= -[x^2 - 2x] + 15$

$= -[(x-1)^2 - 1] + 15$

$= -(x-1)^2 + 1 + 15$

$\boxed{= -(x-1)^2 + 16}$

(c) $-(x-1)^2 + 16 \neq 0$

$-(x-1)^2 \neq -16$

$(x-1)^2 \neq 16$

$(x-1) \neq \pm 4$

$\boxed{x \neq -3 \quad x \neq 5}$

3. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4}t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

(a) $t_2 = \frac{3}{4} \times 13 + 13$

$= \frac{39}{4} + \frac{52}{4}$

$= \frac{91}{4}$ ft $\quad (22\frac{3}{4}$ ft$)$

(b) FROG

LIMIT EXISTS AS $-1 < \frac{1}{3} < 1$

$L = \frac{1}{3}L + 32$

$L - \frac{1}{3}L = 32$

$\frac{2}{3}L = 32$

$2L = 96$

$L = 48$

$\boxed{\text{FROG DOES NOT ESCAPE}}$

TOAD

LIMIT EXISTS AS $-1 < \frac{3}{4} < 1$

$L = \frac{3}{4}L + 13$

$L - \frac{3}{4}L = 13$

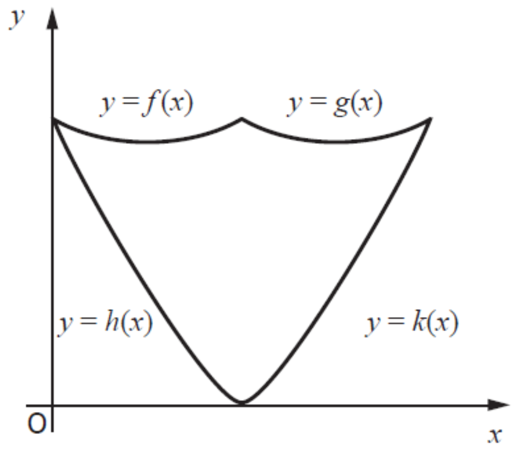
$\frac{1}{4}L = 13$

$L = 52$

$\boxed{\text{TOAD DOES ESCAPE}}$

4. A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.



(a) Find the x-coordinate of the point of intersection of the graphs with equations $y = f(x)$ and $y = g(x)$. **2**

The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis.

The plaque has a vertical line of symmetry.

(b) Calculate the area of the wall plaque. **7**

(a) POI $\quad f(x) = g(x)$

$$\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$$

$$-\frac{1}{2}x + \frac{3}{2}x = 5 - 3$$

$$x = 2$$

(b) AREA $= \int_0^2 f(x) - h(x)\, dx$

$$= \int_0^2 \left(\frac{1}{4}x^2 - \frac{1}{2}x + 3\right) - \left(\frac{3}{8}x - \frac{9}{4}x + 3\right) dx$$

$$= \int_0^2 \left(-\frac{1}{8}x^2 + \frac{7}{4}x\right) dx$$

$$= \left[-\frac{1}{24}x^3 + \frac{7}{8}x^2\right]_0^2$$

$$= \left(-\frac{(2)^3}{24} + \frac{7(2)^2}{8}\right) - 0$$

$$= \left(-\frac{1}{3} + \frac{7}{2}\right) - 0$$

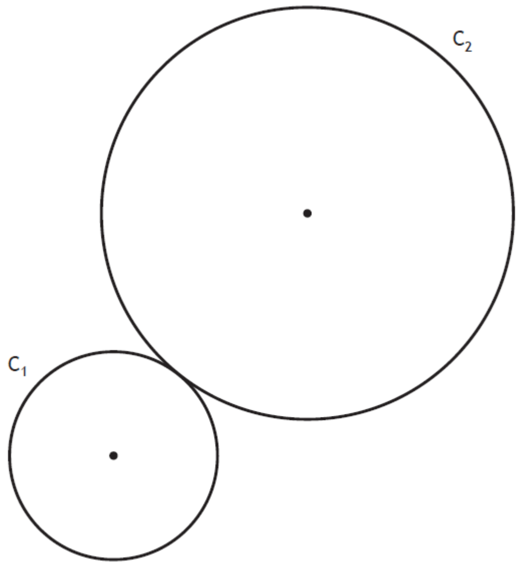
$$= \frac{19}{6}$$

$$\boxed{\text{TOTAL AREA} = 2 \times \frac{19}{6} = \frac{19}{3} \text{ UNITS}^2}$$

5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$.

The centre of circle C_2 is $(9, 11)$.

Circles C_1 and C_2 touch externally.



(a) Determine the radius of C_2. **4**

A third circle, C_3, is drawn such that:

- both C_1 and C_2 touch C_3 internally
- the centres of C_1, C_2 and C_3 are collinear.

(b) Determine the equation of C_3. **4**

(a) C_1 CENTRE $(-3,-5)$ C_2 CENTRE $(9,11)$

RADIUS $= \sqrt{(3)^2+(5)^2-9}$

$= 5$ UNITS

DISTANCE BETWEEN CENTRES $= \sqrt{((-3)-9)^2+((-5)-11)^2}$

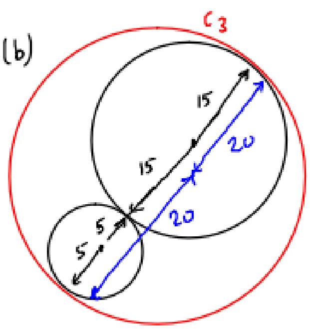
$= \sqrt{144 + 256}$

$= \sqrt{400}$

$= 20$

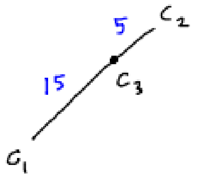
$\boxed{\text{RADIUS } C_2 = 20 - 5 = 15 \text{ UNITS}}$

(b)



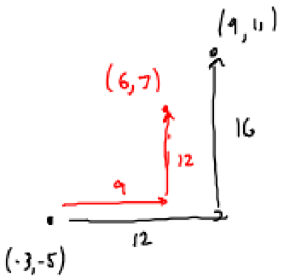
DIAMETER $C_3 = 40$

RADIUS $C_3 = 20$



C_3 SPLITS $C_1 \to C_2$

IN RATIO $3:1$



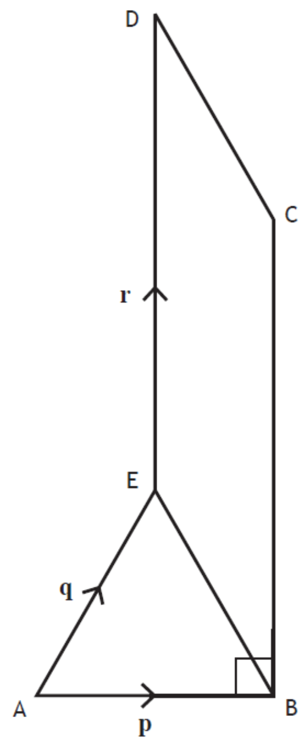
CENTRE $= (6,7)$

RADIUS $= 20$

$\boxed{C_3 \quad (x-6)^2 + (x-7)^2 = 400}$

6. Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC $= 90°$



(a) Evaluate $\mathbf{p}\,.\,(\mathbf{q}+\mathbf{r})$.

(b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

(c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $|\mathbf{r}|$.

(a) $\underline{p}\cdot(\underline{q}+\underline{r})$

$= \underline{p}\cdot\underline{q} + \underline{p}\cdot\underline{r}$

$= |p||q|\cos\theta + |p||r|\cos\theta$

$= 3\times 3\times \cos 60 + 3|r|\cos 90$

$= 9\times\frac{1}{2} + 0$

$= \frac{9}{2}$ $\quad \boxed{\underline{p}\cdot(\underline{q}+\underline{r}) = \frac{9}{2}}$

(b) $\overrightarrow{EC} = -\underline{q} + \underline{p} + \underline{r}$

(c) $\overrightarrow{AE}\,.\,\overrightarrow{EC}$

$= \underline{q}\cdot(-\underline{q}+\underline{p}+\underline{r})$

$= \underline{q}\cdot-\underline{q} + \underline{q}\cdot\underline{p} + \underline{q}\cdot\underline{r}$

$-9 + \frac{9}{2} + \frac{3\sqrt{3}}{2}|\underline{r}| = 9\sqrt{3} - \frac{9}{2}$

$-\frac{9}{2} + \frac{3\sqrt{3}}{2}|\underline{r}| = 9\sqrt{3} - \frac{9}{2}$

$\frac{3\sqrt{3}}{2}|\underline{r}| = 9\sqrt{3}$

$3\sqrt{3}\,|\underline{r}| = 18\sqrt{3}$

$3|\underline{r}| = 18$

$\boxed{|\underline{r}| = 6}$

$\underline{q}\cdot-\underline{q} = |q||-\underline{q}|\cos\theta$

$= 3\times 3\times\cos 180$

$= -9$

$\underline{q}\cdot\underline{p} = \frac{9}{2}$

$\underline{q}\cdot\underline{r} = |\underline{q}||\underline{r}|\cos\theta$

$= 3|\underline{r}|\cos 30$

$= \frac{3\sqrt{3}}{2}|\underline{r}|$

7. (a) Find $\int (3\cos 2x + 1)\,dx$. **2**

(b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$. **2**

(c) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$. **2**

(a)

$$\int (3\cos 2x + 1)\,dx$$

$$\boxed{= \frac{3}{2}\sin 2x + x + c}$$

(b)

$$3\cos 2x + 1$$
$$= 3(\cos^2 x - \sin^2 x) + 1$$
$$= 3\cos^2 x - 3\sin^2 x + 1$$
$$= 3\cos^2 x - 2\sin^2 x + (1 - \sin^2 x)$$
$$= 3\cos^2 x - 2\sin^2 x + \cos^2 x$$
$$= 4\cos^2 x - 2\sin^2 x$$

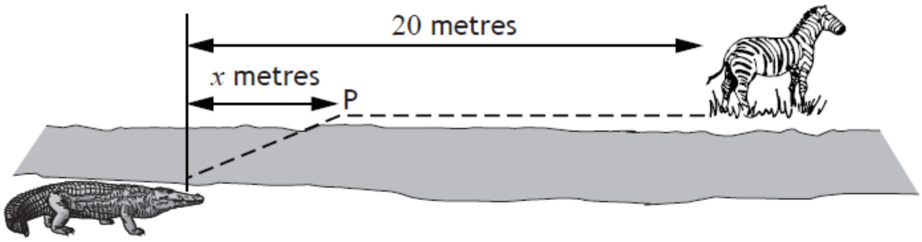
(c)

$$\int (\sin^2 x - 2\cos^2 x)\,dx$$
$$= \int -\frac{1}{2}(4\cos^2 x - 2\sin^2 x)\,dx$$
$$= -\frac{1}{2}\left(\frac{3}{2}\sin 2x + x\right) + c$$
$$= -\frac{3}{4}\sin 2x - \frac{1}{2}x + c$$

8. A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

(a) (i) Calculate the time taken if the crocodile does not travel on land. **1**

(ii) Calculate the time taken if the crocodile swims the shortest distance possible. **1**

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. **8**

(a)(i)

$T(20) = 5\sqrt{36 + (20)^2} + 4(20 - 20)$

$= 5\sqrt{436}$

$= 104.4$ tenths

$\boxed{= 10.4 \text{ seconds}}$

(ii)

$T(0) = 5\sqrt{36 + (0)^2} + 4(20 - 0)$

$= 30 + 80$

$= 110$ tenths

$\boxed{= 11.0 \text{ seconds}}$

(b) $T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$

$= 5(36 + x^2)^{1/2} + 80 - 4x$

$T'(x) = \frac{5}{2}(36 + x^2)^{-1/2}(2x) - 4$

$= \frac{5x}{\sqrt{36 + x^2}} - 4$

For SP $T'(x) = 0$

$\frac{5x}{\sqrt{36 + x^2}} - 4 = 0$

$\frac{5x}{\sqrt{36 + x^2}} = 4$

$\frac{5x}{4} = \sqrt{36 + x^2}$

$\frac{25x^2}{16} = 36 + x^2$

$25x^2 = 576 + 16x^2$

$9x^2 = 576$

$x^2 = 64$

$x = 8$

$T(8) = 5\sqrt{36 + (8)^2} + 4(20 - 8)$

$= 5\sqrt{100} + 4(12)$

$= 50 + 48$

$= 98$ tenths

$\boxed{= 9.8 \text{ seconds}}$

9. The blades of a wind turbine are turning at a steady rate.

The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula

$$h = 36\sin(1\cdot5t) - 15\cos(1\cdot5t) + 65.$$

Express $36\sin(1\cdot5t) - 15\cos(1\cdot5t)$ in the form

$$k\sin(1\cdot5t - a), \text{ where } k > 0 \text{ and } 0 < a < \frac{\pi}{2},$$

and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn. **8**

$$36\sin(1.5t) - 15\cos(1.5t) = k\sin(1.5t - \alpha)$$
$$= k\sin 1.5t\cos\alpha - k\cos 1.5t\sin\alpha$$

$-k\sin\alpha = -15$
$k\sin\alpha = 15$
$k\cos\alpha = 36$

$\tan\alpha = \left(\frac{15}{36}\right)$

$\tan^{-1}\left(\frac{15}{36}\right) = 22.6^\circ$

✓ S	A ✓✓
✓ T	C ✓

$k = \sqrt{36^2 + (-15)^2}$
$= 39$

$36\sin(1.5t) - 15\cos(1.5t) = 39\sin(1.5t - 22.6)^\circ$

$$h = 36\sin(1.5t) - 15\cos(1.5t) + 65$$
$$= 39\sin(1.5t - 22.6)^\circ + 65$$

FOR HEIGHT = 100

$$39\sin(1.5t - 22.6)^\circ + 65 = 100$$
$$39\sin(1.5t - 22.6)^\circ = 35$$
$$\sin(1.5t - 22.6)^\circ = \frac{35}{39}$$

$\sin^{-1}\left(\frac{35}{39}\right) = 63.8^\circ$

✓ S	A ✓
T	C

$$(1.5t - 22.6)^\circ = 63.8^\circ,\ 116.2^\circ$$
$$1.5t = 86.4,\ 138.8^\circ$$
$$t = 57.6,\ 92.5^\circ$$
$$t = 1.005,\ 1.615 \text{ radians}$$