HIGHER MATHS

Straight Line

Notes with Examples

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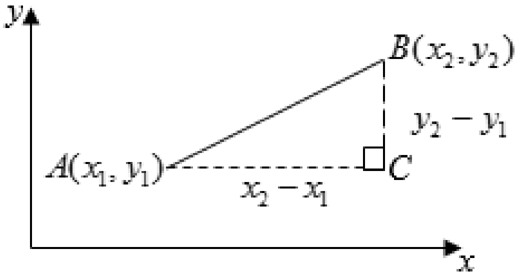
Distance Formula

Given any two distinct coordinate points we can join them by a straight line. Applying Pythagoras' Theorem, we can calculate the distance between the points (or the length of the line).



By Pythagoras' Theorem,

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

Introducing......the distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Examples

SL-01 Calculate the distance between the points

(a) (3, 6) and (2, 1) (b) (4, -7) and (3, 3).

(a) $d = \sqrt{(3-2)^2 + (6-1)^2}$

$= \sqrt{(1)^2 + (5)^2}$

$= \sqrt{26}$

(b) $d = \sqrt{(4-3)^2 + (-7-3)^2}$

$= \sqrt{(1)^2 + (-10)^2}$

$= \sqrt{101}$

SL-02 Triangle PQR has vertices $P(5,9), Q(2,3)$ and $R(8,3)$. Show that the triangle is isosceles.

$d_{PQ} = \sqrt{(5-2)^2 + (9-3)^2}$

$= \sqrt{(3)^2 + (6)^2}$

$= \sqrt{45}$

$d_{QR} = \sqrt{(2-8)^2 + (3-3)^2}$

$= \sqrt{(6)^2 + (0)^2}$

$= 6$

$d_{PR} = \sqrt{(5-8)^2 + (9-3)^2}$

$= \sqrt{(-3)^2 + (6)^2}$

$= \sqrt{45}$

BECAUSE $d_{PQ} = d_{PR} \neq d_{QR}$
TWO SIDES ARE THE SAME LENGTH
SO TRIANGLE IS ISOSCELES

SL-03 $H\,(3,5), R(7,-7)$ and $C(-15,-1)$ are vertices of a triangle.

Show that triangle HRC is right angled and name the right angle.

$$d_{HR} = \sqrt{(3-7)^2 + (5-(-7))^2} = \sqrt{(4)^2 + (12)^2} = \sqrt{160}$$

$$d_{RC} = \sqrt{(7-(-15))^2 + (-7-(-1))^2} = \sqrt{(22)^2 + (-6)^2} = \sqrt{520}$$

$$d_{HC} = \sqrt{(3-(-15))^2 + (5-(-1))^2} = \sqrt{(18)^2 + (6)^2} = \sqrt{360}$$

$$(d_{HR})^2 + (d_{HC})^2 = 160 + 360 = 520$$

$$(d_{rc})^2 = 520$$

$\boxed{\text{520 = 520 SO TRIANGLE IS RIGHT ANGLED. RC IS LONGEST SIDE, SO RIGHT ANGLE IS AT VERTEX H}}$

Midpoint formula

To calculate the midpoint of any line you add the x coordinates of the end points together and divide by two. You do the same for the y coordinates.

$$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Gradient of a Line

Gradient from Equation

The gradient of a line can be taken from the equation of that line. When the line is written in the form of $y = mx + c$ then the gradient is the coefficient of x.

Example

SL-04 For each of the following, find the gradient of the lines

(a) $y = 4x - 2$ (b) $y = 5 - x$ (c) $2y = 5x + 3$

(d) $3y - 6x = 4$ (e) $3x + 4y - 5 = 0$ (f) $y = 5$

(a) $m = 4$ (b) $m = -1$ (c) $m = \frac{5}{2}$

(d) $m = 2$ (e) $m = -\frac{3}{4}$ (f) $m = 0$

Gradient from Two Points

The gradient of a line can be calculated if given two points by using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

SL-05 For each of the following, find the gradient of the lines

(a) $A\ (2,5)\ B\ (4,12)$ (b) $C\ (-1,-4)\ D\ (3,7)$ (c) $E\ (3,-4)\ F(-3,7)$

(a) $\underset{x_1\ y_1}{(2,5)}\underset{x_2\ y_2}{(4,12)}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{12-5}{4-2}$

$= \frac{7}{2}$

(a) $\underset{x_1\ y_1}{(-1,-4)}\underset{x_2\ y_2}{(3,7)}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-(-4)}{3-(-1)}$

$= \frac{11}{4}$

(a) $\underset{x_1\ y_1}{(3,-4)}\underset{x_2\ y_2}{(-3,7)}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

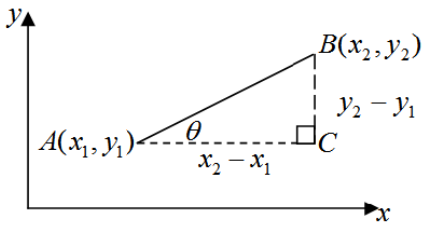
$= \frac{7-(-4)}{-3-3}$

$= -\frac{11}{6}$

Gradient from the Angle the Line Makes with the x-axis

From the triangle on the right we can prove:

$$\begin{aligned} tan\theta &= \frac{opp}{adj} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= m \end{aligned}$$



This results in $m = tan\theta$ “m equals tan theta"

This is always the angle between the line and the positive direction of the x-axis (clockwise).

Examples

SL-06 What angle does a line passing through the points

(a) (9, 9) and (3, 7) (b) (0, 2) and (4, 14)

make with the positive x axis?

(a) $(9,9)(3,7)$
$x_1\, y_1 \quad x_2\, y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-9}{3-9}$

$= \frac{-2}{-6}$

$= \frac{1}{3}$

$m = \tan\theta$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\boxed{\theta = 18.4°}$

( ) $(0,2)(4,14)$
$x_1\, y_1 \quad x_2\, y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{14-2}{4-0}$

$= \frac{12}{4}$

$= 3$

$m = \tan\theta$

$\theta = \tan^{-1}(3)$

$\boxed{\theta = 71.6°}$

SL-07 Find the equation of the line passing through (1,2) at an angle of 135° with the positive x axis.

$m = \tan\theta$

$= \tan 135$

$= -\tan 45$

$\boxed{m = -1}$

$y - b = m(x - a)$

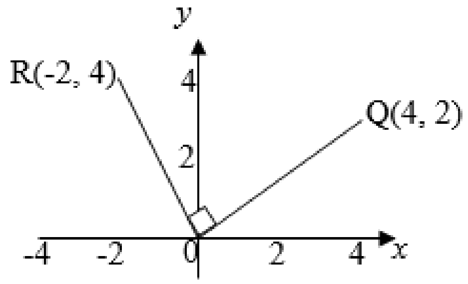
$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

$\boxed{y = -x + 3}$

Perpendicular Lines

If two lines are perpendicular then they are at right angles to each other.



If two lines with gradients m_1 and m_2 are perpendicular then $m_1 \times m_2 = -1$

Conversely,

If $m_1 \times m_2 = -1$ then the lines with gradients m_1 and m_2 are perpendicular.

Examples

SL-08 Given $A(-2, 12)$ and $B(4, -6)$, find the equation of a line perpendicular to AB passing through B.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-12}{4-(-2)} = \frac{-18}{6} = -3$$

$$m_1 \times m_2 = -1$$

$$\boxed{m_\perp = \frac{1}{3}}$$

$$y - b = m(x - a)$$

$$y + 6 = -\frac{1}{3}(x - 4)$$

$$3y + 18 = -x + 4$$

$$\boxed{3y + x + 14 = 0}$$

SL-09 The vertices of triangle PQR are $P(0, 1)$, $Q(-6, 3)$ and $R(2, 7)$. Show that the triangle is right-angled.

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-6-0} = \frac{2}{-6} = -\frac{1}{3}$$

$$m_{QR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{2-(-6)} = \frac{4}{8} = \frac{1}{2}$$

$$m_{PR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{2-0} = \frac{6}{2} = 3$$

$$-\frac{1}{3} \times 3 = -1$$

SINCE $m_{PQ} \times m_{PR} = -1$
PQ & PR ARE PERPENDICULAR
SO TRIANGLE IS RIGHT ANGLED

Equation of a Straight Line

We know the equation of a straight line can be written as

$$y = mx + c \qquad \text{and}$$

$$y - b = m(x - a)$$

There is a third way of writing the equation of a straight line, called the general form. This looks like

$$Ax + By + C = 0$$

We need to know all three forms to allow us to calculate points of intersection, perpendicular gradients and other properties.

Examples

SL-10 Write $y = \frac{2}{3}x - 5$ in the form $Ax + By + C = 0$

$$y = \frac{2}{3}x - 5$$

$$3y = 2x - 15$$

$$-2x + 3y + 15 = 0$$

SL-11 Find the gradient of the line that is perpendicular to $4x + 5y + 13 = 0$

$$4x + 5y + 13 = 0$$

$$5y = -4x - 13$$

$$y = -\frac{4}{5}x - \frac{13}{5}$$

$$m = -\frac{4}{5} \qquad m_1 \times m_2 = -1$$

$$\boxed{m_\perp = \frac{5}{4}}$$

SL-12 Find the equation of the line that is perpendicular to $x - 5y + 20 = 0$ passing through the point (12,-4).

$$x - 5y + 20 = 0$$

$$x + 20 = 5y$$

$$y = \frac{1}{5}x + 4$$

$$m_1 \times m_2 = -1$$

$$\boxed{m_\perp = -5}$$

$$y + 4 = -5(x - 12)$$

$$y + 4 = -5x + 60$$

$$\boxed{y = -5x + 56}$$

Point of Intersection

Where two lines (straight lines, curves, circles) cross over each other, the point is called a point of intersection. At this point, the x value of both lines must be the same, as must the y values. We have seen this before in simultaneous equations!

Examples

SL-13 Find the point of intersection of $y = 2x - 1$ and $y = -3x + 9$

PoI:

$$2x - 1 = -3x + 9$$
$$5x = 10$$
$$x = 2$$

$$y = 2(2) - 1$$
$$= 5$$

$\boxed{\text{PoI } (2,5)}$

SL-14 Find the point of intersection of the line $3x + 5y + 5 = 0$ and the line $y = 2x + 12$.

$$3x + 5y + 5 = 0$$
$$5y = -3x - 5$$

$$y = 2x + 12$$
$$5y = 10x + 60$$

PoI

$$10x + 60 = -3x - 5$$
$$13x = -65$$
$$x = -5$$

$$y = 2(-5) + 12$$
$$= 2$$

$\boxed{\text{PoI } (-5,2)}$

SL-15 Find the equation of the line that is perpendicular to $x - 5y + 20 = 0$ passing through the point (12,-4) and state the point of intersection.

$$x - 5y + 20 = 0$$
$$x + 20 = 5y$$
$$y = \frac{1}{5}x + 4$$

$$y + 4 = -5(x - 12)$$
$$y + 4 = -5x + 60$$
$$\boxed{y = -5x + 56}$$

$$m_1 \times m_2 = -1$$
$$\boxed{m_\perp = -5}$$

PoI:

$$\frac{1}{5}x + 4 = -5x + 56$$
$$x + 20 = -25x + 280$$
$$26x = 260$$
$$x = 10$$

$$y = \frac{1}{5}(10) + 4$$
$$= 6$$

$\boxed{\text{PoI } (10,6)}$

Collinearity

If three (or more) points lie on the same lines they are said to be collinear.
To prove A, B and C are collinear we:

* Calculate m_{AB} and m_{BC}
* If m_{AB} and m_{BC} are equal then AB and BC are parallel
* If AB and BC are parallel AND share a common point (B) then A, B and C must lie on the same straight line.

When proving collinearity we must state:

"Because $m_{AB} = m_{BC}$ then AB and BC are parallel and share a common point (B) so A, B and C are collinear."

Examples

SL-16 Prove that points A (1, 2) B (3, 6) and C (-4, -8) are collinear and find the ratio that A splits BC.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-6}{-4-3} = \frac{-14}{-7} = 2$$

$\boxed{m_{AB} = m_{BC} \text{ SO AB \& BC ARE PARALLEL, SHARE COMMON POINT B, SO COLLINEAR}}$

$$d_{AB} = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$$

$$d_{BC} = \sqrt{(3-(-4))^2 + (6-(-8))^2} = \sqrt{(7)^2 + (14)^2} = \sqrt{245} = 7\sqrt{5}$$

$$\boxed{\begin{array}{c} AB : BC \\ 2\sqrt{5} : 7\sqrt{5} \\ 2 : 7 \end{array}}$$

Does a Point Lie on a Line?

The equation of a straight line is a relationship between x and y.

To prove a point lies on (or above or below) a line, substitute the x value of the point into the equation of the line, and find the related y value.

Examples

SL-17 Prove that point A $(2, 3)$ lies on the line $3x + 2y - 12 = 0$.

$$3(2) + 2(3) - 12$$
$$= 6 + 6 - 12$$
$$= 0$$

ANSWER = 0 SO POINT LIES ON LINE

SL-18 Do points $F\ (4, 0)$ and $G(-2, -2)$ lie above or below the line $-2x + 5y + 3 = 0$?

POINT F

$$-2(4) + 5(0) + 3$$
$$= -8 + 0 + 3$$
$$= -5$$

SO POINT F LIES BELOW LINE

POINT G

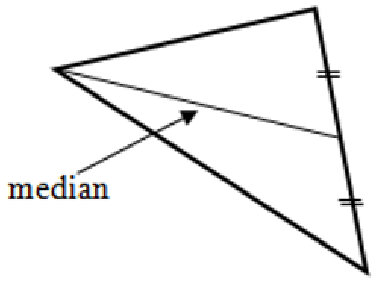
$$-2(-2) + 5(-2) + 3$$
$$= 4 - 10 + 3$$
$$= -3$$

SO POINT G LIES BELOW LINE

Triangles

Medians

You should be familiar with the median of a data set being the middle piece of data. In a triangle, the median is a line joining one vertex to the midpoint of the opposite side. A triangle has three medians.



To find the equation of a median we:
- calculate the midpoint of the opposite side
- calculate the gradient of the line between the midpoint and opposite corner
- Calculate the equation of the line using the gradient and the coordinates of the midpoint.

Medians are concurrent, meaning all three medians meet at one single point called the centroid.

Examples

SL-19 Triangle QHF has vertices Q(-3, 5), H(-5, -2) and F(-7, 4). Find the equation of the median from Q and the equation of the median which cuts QH.

MEDIAN FROM Q

$$\text{MID}_{FH} = \left(\frac{-5+(-7)}{2}, \frac{-2+4}{2}\right) = (-6, 1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{-6-(-3)} = \frac{-4}{-3} = \frac{4}{3}$$

$$y - b = m(x - a)$$
$$y - 5 = \frac{4}{3}(x+3)$$
$$3y - 15 = 4x + 12$$
$$\boxed{-4x + 3y - 27 = 0}$$

MEDIAN CUTS QH:

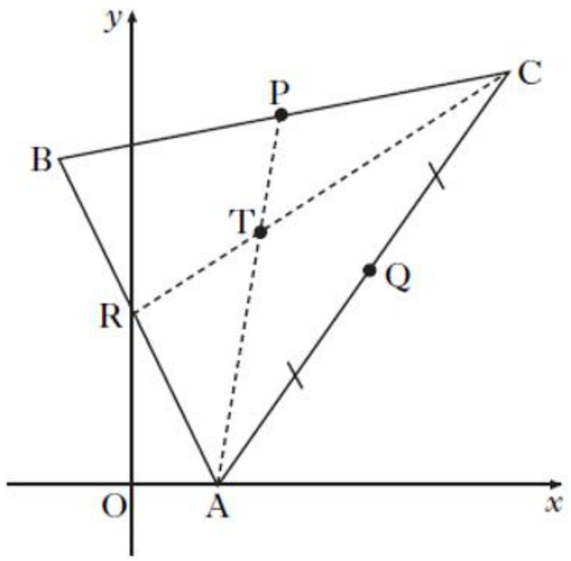
$$\text{MID}_{QH} = \left(\frac{-3+(-5)}{2}, \frac{5+(-2)}{2}\right) = \left(-4, \frac{3}{2}\right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - \frac{3}{2}}{-7-(-4)} = \frac{\frac{5}{2}}{-3} = -\frac{5}{6}$$

$$y - b = m(x - a)$$
$$y - 4 = -\frac{5}{6}(x+7)$$
$$6y - 24 = -5x - 35$$
$$\boxed{5x + 6y + 11 = 0}$$

Triangle ABC has vertices A(4, 0), B(–4, 16) and C(18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).



(*a*) Find the equation of median BQ. **3**

(*b*) Verify that T lies on BQ. **1**

(*c*) Find the ratio in which T divides BQ. **2**

(a) $Q = \left(\frac{4+18}{2}, \frac{0+20}{2}\right)$

$= (11, 10)$

$m_{BQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{16-10}{-4-11}$

$= -\frac{6}{15}$

$= -\frac{2}{5}$

$y - b = m(x - a)$

$y - 16 = -\frac{2}{5}(x+4)$

$5y - 80 = -2x - 8$

$\boxed{2x + 5y - 72 = 0}$

(b) $2(6) + 5(12) - 72$

$= 12 + 60 - 72$

$= 0$

SO T LIES ON LINE BQ

(c) $d_{BT} = \sqrt{((-4)-6)^2 + (16-12)^2}$

$= \sqrt{(-10)^2 + (4)^2}$

$= \sqrt{116}$

$= 2\sqrt{29}$

$d_{TQ} = \sqrt{(11-6)^2 + (10-12)^2}$

$= \sqrt{(5)^2 + (-2)^2}$

$= \sqrt{29}$

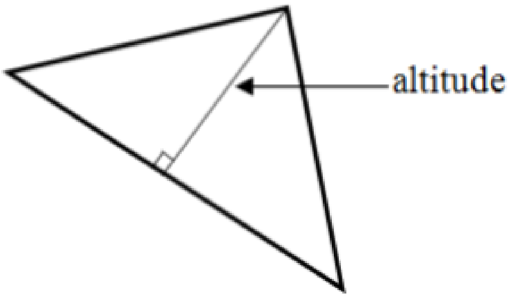
BT : TQ

$2\sqrt{29} : \sqrt{29}$

2 : 1

Altitudes

The word altitude refers to the perpendicular height of an object. In triangles, an altitude is a straight line from one vertex perpendicular to the opposite side. A triangle has three altitudes.



To find the equation of an altitude we:

- Calculate the gradient of the opposite side
- Calculate the perpendicular gradient
- Calculate the equation of the line using the gradient and the coordinates of the vertex.

Altitudes are concurrent, meaning all three altitudes meet at one single point called the orthocentre.

Examples

SL-21 For vertices $K(4,-7)$, $H(5,-6)$ and $C(3,1)$, find the equation of the altitude from C.

$$m_{KH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-7)}{5-4} = \frac{-1}{1} = -1$$

$$m_1 \times m_2 = -1$$

$$m_\perp = 1$$

$$y - b = m(x-a)$$

$$y - 1 = 1(x-3)$$

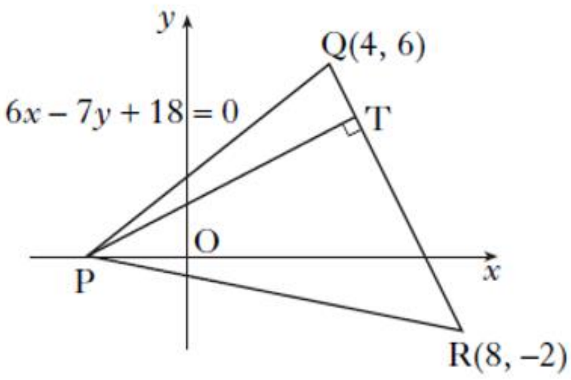
$$y - 1 = x - 3$$

$$\boxed{y = x - 2}$$

Triangle PQR has vertex P on the x-axis, as shown in the diagram.

Q and R are the points (4, 6) and (8, –2) respectively.

The equation of PQ is $6x - 7y + 18 = 0$.



(*a*) State the coordinates of P. **1**

(*b*) Find the equation of the altitude of the triangle from P. **3**

(*c*) The altitude from P meets the line QR at T. Find the coordinates of T. **4**

(a) P LIES ON x AXIS SO $y = 0$

$$6x - 7(0) + 18 = 0$$
$$6x + 18 = 0$$
$$6x = -18$$
$$x = -3$$
$$\boxed{P(-3, 0)}$$

(b) $m_{QR} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2-6}{8-4}$$
$$= \frac{-8}{4}$$
$$= -2$$

$$m_1 \times m_2 = -1$$
$$m_\perp = \tfrac{1}{2}$$
$$y - b = m(x - a)$$
$$y - 0 = \tfrac{1}{2}(x + 3)$$
$$\boxed{y = \tfrac{1}{2}x + \tfrac{3}{2}}$$

(c) LINE QR

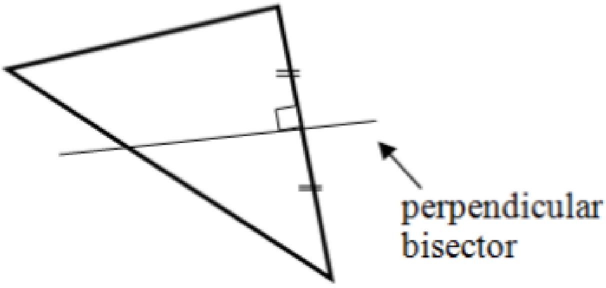
$$y - b = m(x - a)$$
$$y - 6 = -2(x - 4)$$
$$y - 6 = -2x + 8$$
$$y = -2x + 14$$

POI

$$\tfrac{1}{2}x + \tfrac{3}{2} = -2x + 14$$
$$x + 3 = -4x + 28$$
$$5x = 25$$
$$x = 5$$
$$y = -2(5) + 14$$
$$= 4$$
$$\boxed{T(5, 4)}$$

Perpendicular Bisectors

A perpendicular bisector of a line is another line which cuts it in half at right angles. Note that within a triangle, a perpendicular bisector will not necessarily pass through a vertex. A triangle has three perpendicular bisectors.



To find the equation of a perpendicular bisector we:

- Calculate the midpoint of the side
- Calculate the gradient of the side
- Calculate the perpendicular gradient
- Calculate the equation of the line using the perpendicular gradient and the coordinates of the midpoint.

Perpendicular bisectors are concurrent, meaning all three meet at one single point called the circumcentre.

Examples

SL-23 For triangle GJK with vertices $G(-2,-8)$, $J\ (0,5)$ and $K\ (1,2)$ find the equation of the perpendicular bisector of side KG.

MID KG : (POINT A)

$$\left(\frac{-2+1}{2}, \frac{-8+2}{2}\right) = \left(-\frac{1}{2}, -3\right)$$

$$m_{KG} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-8)}{1-(-2)} = \frac{10}{3}$$

$$m_1 \times m_2 = -1$$
$$m_\perp = -\frac{3}{10}$$

$$y - b = m(x - a)$$
$$y + 3 = -\frac{3}{10}\left(x + \frac{1}{2}\right)$$
$$10y + 30 = -3x - \frac{3}{2}$$
$$20y + 60 = -6x - 3$$
$$\boxed{6x + 20y + 63 = 0}$$

(*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9). 4

(*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2). 2

(*c*) Find the point of intersection of ℓ_1 and ℓ_2. 3

(*d*) Hence find the shortest distance between PQ and ℓ_2. 2

(a) MID PQ (POINT A) $= \left(\frac{3+(-1)}{2}, \frac{(-3)+9}{2}\right)$

$= (1, 3)$

$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3-9}{3-(-1)}$

$= \frac{-12}{4}$

$= -3$

$m_1 \times m_2 = -1$

$m_\perp = \frac{1}{3}$

ℓ_1 :

$y - b = m(x - a)$

$y - 3 = \frac{1}{3}(x - 1)$

$3y - 9 = x - 1$

$\boxed{-x + 3y - 8 = 0}$

(b) $m = -3$

ℓ_2 :

$y - b = m(x - a)$

$y + 2 = -3(x - 1)$

$y + 2 = -3x + 3$

$\boxed{3x + y - 1 = 0}$

(c) $-x + 3y - 8 = 0$

$3y = x + 8$

$3x + y - 1 = 0$

$y = -3x + 1$

$3y = -9x + 3$

PoI

$x + 8 = -9x + 3$

$10x = -5$

$x = -\frac{1}{2}$

$y = -3\left(-\frac{1}{2}\right) + 1$

$= \frac{5}{2}$

$\boxed{\text{PoI } \left(-\frac{1}{2}, \frac{5}{2}\right)}$

(d) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(-\frac{1}{2} - 1\right)^2 + \left(\frac{5}{2} - 3\right)^2}$

$= \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{1}{4}}$

$= \frac{\sqrt{10}}{2}$.

Summary

Straight Line

EQUATIONS:

$y = mx + c$

$y - b = m(x - a)$

$Ax + By + C = 0$

Where $m = \frac{y_2 - y_1}{x_2 - x_1}$ (gradient)

Point (a, b) lies on the line

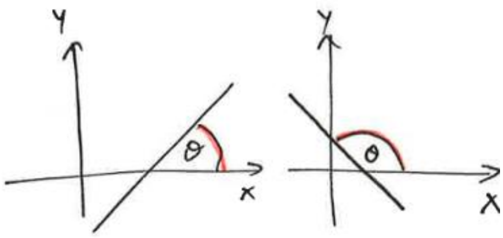
NOTE: NOT (A, B)!

GRADIENT:

two coords on the line

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (x_1, y_1) (x_2, y_2)

$m = \tan\theta$



NOTE: θ ALWAYS ON RIGHT OF LINE

PARALLEL & PERPENDICULAR LINES:

Parallel lines $m_1 = m_2$

Perpendicular lines $m_1 \times m_2 = -1$

TIP: TO FIND $m_\perp$ (m PERP) FLIP m_1 UPSIDE DOWN & CHANGE SIGN

$m_1 = \frac{2}{3}$ $m_\perp = -\frac{3}{2}$; $m_1 = 4$ $m_\perp = -\frac{1}{4}$

DOES A POINT LIE ON A LINE?

METHOD 1:

$y = mx + c$

1. Sub in x value from point
2. If y is equal to the y value of the point then it lies on line

METHOD 2:

$Ax + By + C = 0$

1. Sub in x and y values from point
2. If equation equals zero then point lies on the line.

POINTS OF INTERSECTION:

PoI is where two lines cross.

To find PoI:

1. Rearrange both equations to $y =$
2. Equate one to the other
3. Solve for x
4. Solve for y

COLLINEARITY (DO THREE POINTS LIE ON THE SAME STRAIGHT LINE?)

Collinear if $m_{AB} = m_{BC}$

Must state:

"$m_{AB} = m_{BC}$ so lines are parallel. Share a common point, B, so A, B & C are collinear"

MIDPOINT OF A LINE:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

REMEMBER THESE

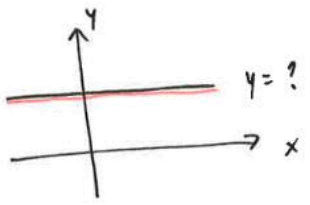
LENGTH OF A LINE (DISTANCE BETWEEN TWO POINTS):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

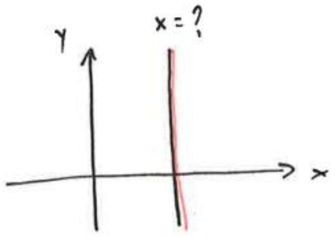
(FROM PYTHAGORAS THEOREM)

this is the distance formula!

HORIZONTAL & VERTICAL LINES:

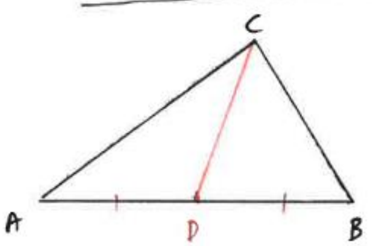


$m = 0$



m = undefined

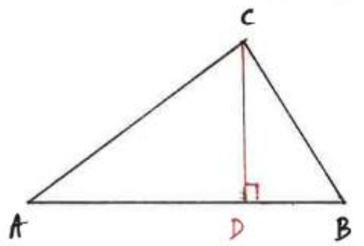
TRIANGLES (MEDIAN, ALTITUDE & PERPENDICULAR BISECTOR):



Median from C or

Median of AB

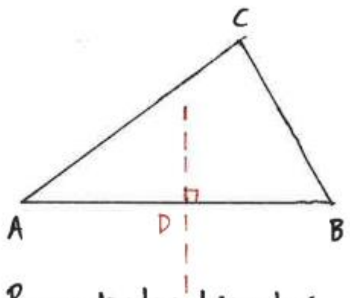
1. Find D using midpoint formula
2. Calculate m_{CD} from C & D
3. Sub m_{CD} & point C into $y - b = m(x - a)$



Altitude from C or

Altitude of AB

1. Find m_{AB}
2. Find m_{CD} from $m_1 \times m_2 = -1$
3. Sub m_{CD} & point C into $y - b = m(x - a)$



Perpendicular bisector of AB

1. Find D using midpoint formula
2. Find m_{AB}
3. Find $m_{\perp}$ from $m_1 \times m_2 = -1$
4. Sub in $m_{\perp}$ & point D into $y - b = m(x - a)$