H

National
Qualifications
SPECIMEN ONLY

SQ30/H/01

Mathematics
Paper 1
(Non-Calculator)

Date — Not applicable

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

SQ30H01

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

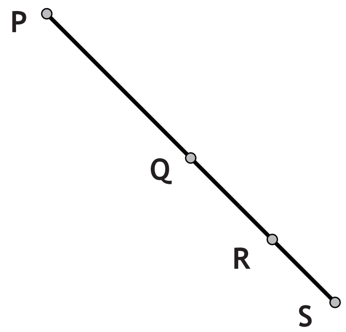
Attempt ALL questions **MARKS**

Total marks — 60

1. Find $\int \frac{3x^3+1}{2x^2}\,dx,\ x \neq 0$. **4**

2. Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$. **5**

3. In the diagram, P has coordinates $(-6, 3, 9)$,



$\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$.

Find the coordinates of S. **5**

4. Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p, where $p \in \mathbb{R}$. **4**

5. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. **2**

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. **2**

6. (a) Find an equivalent expression for $\sin(x + 60)°$. **1**

(b) Hence, or otherwise, determine the exact value of $\sin 105°$. **3**

7. (a) Show that $(x + 1)$ is a factor of $x^3 - 13x - 12$. **3**

(b) Factorise $x^3 - 13x - 12$ fully. **2**

MARKS

8. $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that

$f(x) = 1 - \frac{1}{2}x$ and $g(x) = 8x^2 - 3$.

(a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$. **3**

(b) Express $h(x)$ in the form $a(x+p)^2 + q$. **3**

(c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y = h(x)$. **1**

(d) Sketch the graph of $y = h(x) + 3$, showing clearly the coordinates of the turning point and the y-axis intercept. **2**

9. (a) AB is a line parallel to the line with equation $y + 3x = 25$.

A has coordinates $(-1, 10)$.

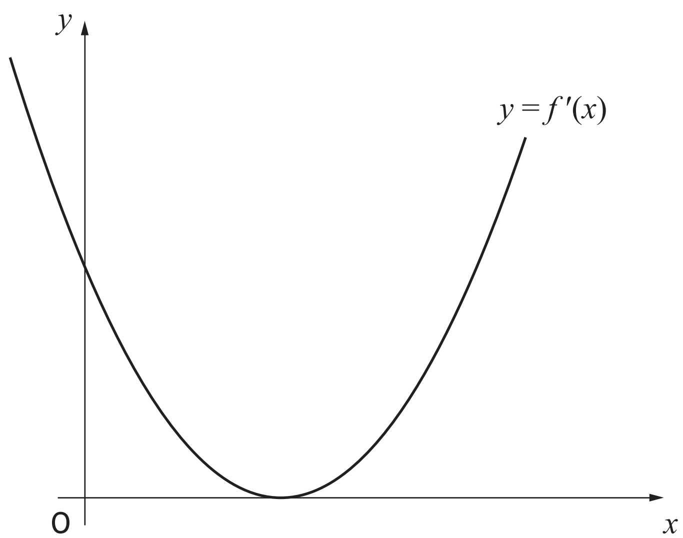
Find the equation of AB. **1**

(b) $3y = x + 11$ is the perpendicular bisector of AB.

Determine the coordinates of B. **5**

10. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

11. The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.



(a) Explain why the function $f(x)$ is never decreasing. **1**

(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$. **2**

MARKS

12. The voltage, $V(t)$, produced by a generator is described by the function $V(t) = 120\sin 100\pi t,\ t > 0$, where t is the time in seconds.

(a) Determine the period of $V(t)$. **2**

(b) Find the first three times for which $V(t) = -60$. **6**

[END OF SPECIMEN QUESTION PAPER]

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Mathematics
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Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored out or erased working which has not been replaced should be marked where still legible. However, if the scored out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in solutions
- Repeated error within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper.

Determine: find a numerical value or values from the information given.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown.

Express: use given information to rewrite an expression in a specified form.

Hence: use the previous answer to proceed.

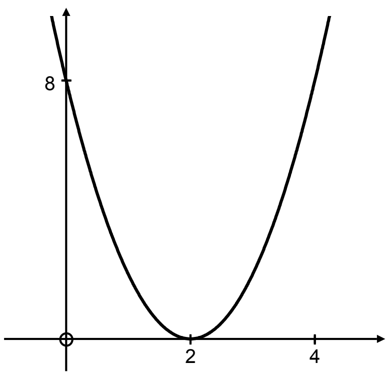
Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

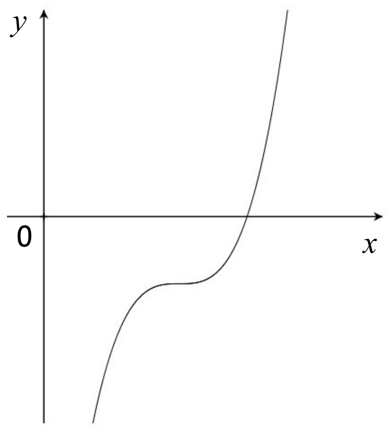
Justify: show good reason(s) for the conclusion(s) reached.

Specific Marking Instructions for each question

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
1			Ans: $\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$	4	
			•[1] preparation for integration		•[1] $\frac{3}{2}x + \frac{1}{2}x^{-2}$
			•[2] correct integration of first term		•[2] $\frac{3}{2} \bullet \frac{x^2}{2} + \ldots$
			•[3] correct integration of second term		•[3] $\ldots + \frac{1}{2} \bullet \frac{x^{-1}}{-1}$
			•[4] includes constant of integration		•[4] $\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$
2			Ans: $(-1,0),(0,4),(3,16)$	5	
			•[1] sets equation of curve equal to equation of line		•[1] $x^3 - 2x^2 + x + 4 = 4x + 4$
			•[2] equates to zero		•[2] $x^3 - 2x^2 - 3x = 0$
			•[3] factorises fully		•[3] $x(x+1)(x-3) = 0$
			•[4] calculates x-coordinates		•[4] $x = 0, x = -1, x = 3$
			•[5] calculates y-coordinates		•[5] $(0,4),(-1,0),(3,16)$
3			Ans: $S(5,25,-2)$	5	
			•[1] find coordinate of Q or component vector $\mathbf{q}$		•[1] $\mathbf{q} = \mathbf{p} + \overrightarrow{PQ} = \begin{pmatrix} 0 \\ 15 \\ 3 \end{pmatrix}$ or $Q(0,15,3)$
			•[2] sets up vector equation for $\mathbf{r}$		•[2] $\mathbf{r} = \mathbf{q} + \overrightarrow{QR} = \begin{pmatrix} 0 \\ 15 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$
			•[3] find coordinate of R or component vector $\mathbf{r}$		•[3] $\mathbf{r} = \begin{pmatrix} 3 \\ 21 \\ 0 \end{pmatrix}$ or $R(3,21,0)$
			•[4] sets up vector equation for $\mathbf{s}$		•[4] $\mathbf{s} = \mathbf{r} + \overrightarrow{RS} = \begin{pmatrix} 3 \\ 21 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$
			•[5] find coordinate of S		•[5] $S(5,25,-2)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
4			Ans: $-4 < p < 12$ •[1] know discriminant < 0 •[2] simplify •[3] factorise LHS •[4] correct range	4	•[1] $b^2 - 4ac < 0$ and $a = 2,\ b = p,\ c = p + 6$ stated or implied by •[2] •[2] $p^2 - 8p - 48 < 0$ •[3] $(p-12)(p+4) < 0$ •[4] $-4 < p < 12$
5	(a)		Ans: $m_{l_2} = -\sqrt{3}$ •[1] rearranging equation to calculate gradient of line l_1 •[2] calculating gradient of l_2	2	•[1] $y = \frac{1}{\sqrt{3}}x \quad m = \frac{1}{\sqrt{3}}$ •[2] $m_{l_2} = -\sqrt{3}$
	(b)		Ans: $\theta = \frac{2\pi}{3}$ or $120°$ •[3] using $m = \tan\theta$ •[4] calculating angle	2	•[3] $\tan\theta = -\sqrt{3}$ •[4] $\theta = \frac{2\pi}{3}$ or $120°$
6	 (a) (b)		Ans: $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$ •[1] **correct** expansion •[2] **any** expression equivalent to $\sin 105°$ •[3] correct exact value equivalents •[4] correct answer	4	•[1] $\sin x° \cos 60° + \cos x° \sin 60°$ •[2] $\sin(45+60)°$ or equivalent •[3] $\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$ •[4] $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$
7	(a)		•[1] know to use $x = -1$ •[2] complete synthetic division •[3] recognition of zero remainder	3	•[1] -1 | 1 0 -13 -12 -1 1 12 1 -1 -12 0 •[2] -1 | 1 0 -13 -12 -1 1 12 **1 -1 -12 0** •[3] $(x+1)$ is a factor as remainder is zero
	(b)		Ans: $(x+1)(x+3)(x-4)$ •[4] identify quotient •[5] factorised fully	2	•[4] $x^2 - x - 12$ •[5] $(x+1)(x+3)(x-4)$
Notes			Alternative methods of showing $(x+1)$ is a factor, such as long division, inspection and evaluating are perfectly acceptable.		

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
8	(a)		Ans: $h(x)=2x^2-8x+5$ •1 correct substitution •2 squaring •3 expanding and simplifying	3	•1 $h(x)=8\left(1-\frac{1}{2}x\right)^2-3$ •2 $1-x+\frac{1}{4}x^2$ •3 $h(x)=2x^2-8x+5$
	(b)		Ans: $2(x-2)^2-3$ •4 identify common factor •5 complete the square •6 process for q	3	•4 $2(x^2-4x...$ **stated or implied by •3** •5 $2(x-2)^2...$ •6 $2(x-2)^2-3$
Notes			Values for p and q must be consistent with the value for a.		
	(c)		Ans: $(2,-3)$ •7 state turning point	1	•7 $(2,-3)$
	(d)		Ans:  •8 correct shape •9 annotation, including y-axis intercept	2	•8 parabola with minimum turning point labelled (positioned consistently with answer to (b)) •9 (0,8)

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
9	(a)		Ans: $y-10=-3(x+1)$ •[1] finding equation of line	1	•[1] $y-10=-3(x+1)$ or equivalent
	(b)		Ans: $\text{B}(3,-2)$ •[2] use of simultaneous equations •[3] solving to find one coordinate of midpoint •[4] finding remaining coordinate of midpoint •[5] using midpoint formula or 'stepping out' •[6] finding coordinates of B	5	•[2] $y=-3x+7$ and $3y=x+11$ •[3] either $x=1$ or $y=4$ •[4] M (1, 4) •[5] either $x=3$ or $y=-2$ •[6] $\text{B}(3,-2)$
10			Ans: $\dfrac{3\sqrt{3}}{2}$ •[1] start to differentiate •[2] complete differentiation •[3] evaluate $f'\left(\dfrac{5\pi}{6}\right)$	3	•[1] $3\times4\sin^2 x$ •[2] $\times\cos x$ •[3] $12\left(\dfrac{1}{2}\right)^2\times\dfrac{-\sqrt{3}}{2}=12\times\dfrac{1}{4}\times\dfrac{-\sqrt{3}}{2}=\dfrac{-3\sqrt{3}}{2}$
11	(a)		•[1] knows derived function represents gradient and that the minimum value of $f'(x)$ is zero	1	•[1] $m=f'(x)\geq0$ stated explicitly
	(b)		•[2] interprets information correctly •[3] completes sketch	2	•[2] stationary point plotted in fourth quadrant •[3] point of inflexion on an increasing graph 

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
12	(a)		Ans: $\frac{1}{50}$ sec or 0·02 sec •[1] knows how to find period •[2] correct answer	**2**	 •[1] $T = \frac{2\pi}{100\pi}$ •[2] $\frac{1}{50}$ or 0·02
	(b)		Ans: $\frac{7}{600}$, $\frac{11}{600}$, and $\frac{19}{600}$ sec •[1] equating function with –60 •[2] rearranging •[3] solve equation for $100\pi t$ •[4] process solutions for t •[5] knowing to use period or demonstrating another solution from the third quadrant •[6] third value for t	**6**	 •[1] $120\sin 100\pi t = -60$ •[2] $\sin 100\pi t = -\frac{1}{2}$ •[3] $100\pi t = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$ •[4] $t = \frac{7}{600}$ and $\frac{11}{600}$ •[5] $T = \frac{1}{50}$ or $100\pi t = 3\pi + \frac{\pi}{6}$ •[6] $\frac{19}{600}$

[END OF SPECIMEN MARKING INSTRUCTIONS]