

X100/303

NATIONAL
QUALIFICATIONS
2005

FRIDAY, 20 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

1. Find $\int \frac{4x^3 - 1}{x^2} dx$, $x \neq 0$.

4

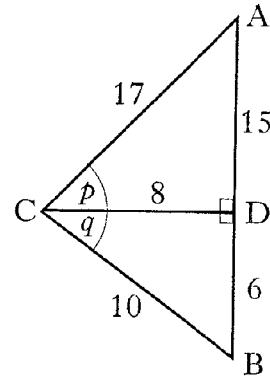
2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.

(a) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}$.

(b) Calculate the exact values of:

(i) $\cos(p + q)$;

(ii) $\tan(p + q)$.

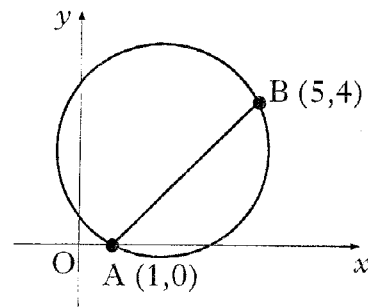


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3. (a) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

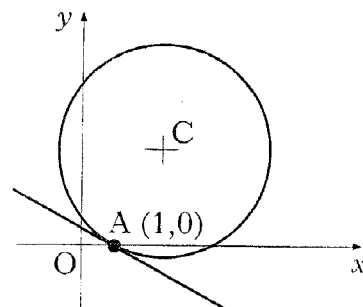
Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$.



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(b) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.

Find the equation of the radius CA.



4

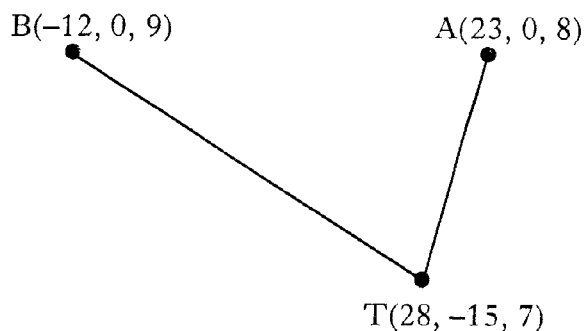
(c) (i) Determine the coordinates of the point C.

(ii) Find the equation of the circle.

4

[Turn over

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.



Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

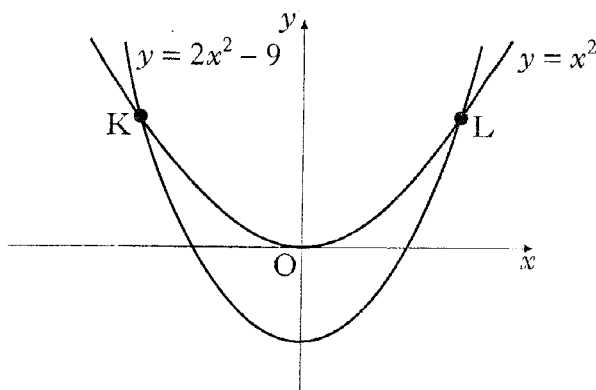
In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

- (a) Express the vectors \vec{TA} and \vec{TB} in component form.
- (b) Calculate the angle between these two beams.

2

5

5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

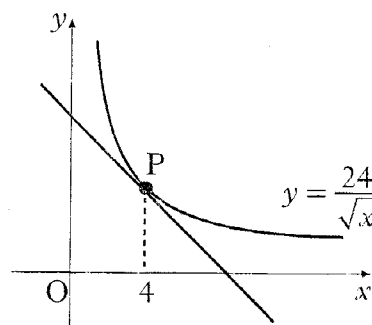


Calculate the area enclosed between the curves.

8

6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$.



6

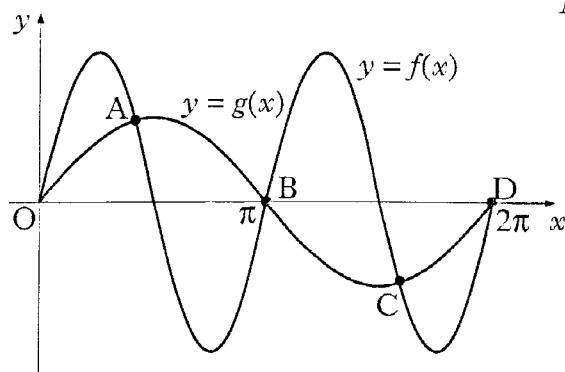
7. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.

4

8. Two functions, f and g , are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \frac{1}{2k}$.



5

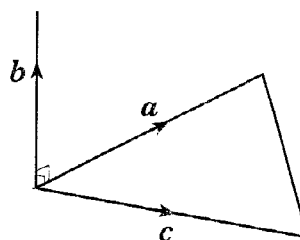
9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$.

- (a) What was its value when launched? 1
- (b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? 4

10. Vectors a and c are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector b is 2 units long and b is perpendicular to both a and c .

Evaluate the scalar product $a \cdot (a + b + c)$.



4

11. (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. 1
- (b) Hence find the range of values of p for which all the roots of the cubic equation are real. 7

[END OF QUESTION PAPER]