

Wave questions

[SQA] 1. Solve the equation $2 \sin x^\circ - 3 \cos x^\circ = 2.5$ in the interval $0 \leq x < 360$.

8

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	8	A/B	CR	T16	$100.2^\circ, 192.4^\circ$	1999 P2 Q9

<ul style="list-style-type: none"> •¹ strategy: e.g. $k \sin(x - a)$ <i>stated or implied by</i> •⁶ •² $k \sin x \cos a - k \cos x \sin a$ <i>stated explicitly</i> •³ $k \cos a = 2$ and $k \sin a = 3$ <i>stated explicitly</i> •⁴ $k = \sqrt{13}$ •⁵ $a = 56.3$ •⁶ $\sin(x - 56.3) = \frac{2.5}{\sqrt{13}}$ •⁷ $x - 56.3 = 43.9, 136.1$ •⁸ 100.2° and 192.4° <p>OR</p> <ul style="list-style-type: none"> •⁷ $x - 56.3 = 43.9, x = 100.2^\circ$ •⁸ 192.4° 	<p><i>136.1 stated or implied by the appearance of 192.4 in</i> •⁸</p>	$k \cos(x - a)$ $k \cos x \cos a + k \sin x \sin a$ $k \cos a = -3, k \sin a = 2$ $k = \sqrt{13}, \tan a = -\frac{2}{3}$ $a = 146.3$ $\cos(x - 146.3) = 0.693$ $x - 146.3 = 46.1, 313.9$ $x = 192.4, 460.2$ $x = 192.4, 100.2$	$k \sin(x + a)$ $k \sin x \cos a + k \cos x \sin a$ $k \cos a = 2, k \sin a = -3$ $k = \sqrt{13}, \tan a = -\frac{3}{2}$ $a = 303.7$ $\sin(x + 303.7) = 0.693$ $x + 303.7 = 43.9, 136.1$ $x = -259.8, -167.6$ $x = 100.2, 192.4$	$k \cos(x + a)$ $k \cos x \cos a - k \sin x \sin a$ $k \cos a = -3, k \sin a = -2$ $k = \sqrt{13}, \tan a = \frac{2}{3}$ $a = 213.7$ $\cos(x + 213.7) = 0.693$ $x + 213.7 = 46.1, 313.9$ $x = -167.6, 100.2$ $x = 192.4, 100.2$
---	---	--	--	---

2. (a) $12 \cos x^\circ - 5 \sin x^\circ$ can be expressed in the form $k \cos(x + a)^\circ$, where $k > 0$ and $0 \leq a < 360$.

Calculate the values of k and a .

4

- (b) (i) Hence state the maximum and minimum values of $12 \cos x^\circ - 5 \sin x^\circ$.
 (ii) Determine the values of x , in the interval $0 \leq x < 360$, at which these maximum and minimum values occur.

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CN	T13	$k = 13, a = 22.6$	2010 P2 Q2
(bi)	1	C	CN	T14	max 13, min -13	
(bii)	2	C	CN	T14	max at 337.4, min at 157.4	

<ul style="list-style-type: none"> •¹ ss: use addition formula •² ic: compare coefficients •³ pd: process k •⁴ pd: process a •⁵ ss: state maximum and minimum •⁶ ic: find x corresponding to max. value •⁷ pd: find x corresponding to min. value 	<ul style="list-style-type: none"> •¹ $k \cos x^\circ \cos a^\circ - k \sin x^\circ \sin a^\circ$ •² $k \cos a^\circ = 12$ and $k \sin a^\circ = 5$ •³ 13 (do not accept $\sqrt{169}$) •⁴ 22.6 (accept any answer which rounds to 23) •⁵ 13, -13 •⁶ maximum at 337.4 and no others •⁷ minimum at 157.4 and no others
---	--

- [SQA] 3. (a) Express $\sin x^\circ - 3 \cos x^\circ$ in the form $k \sin(x - a)^\circ$ where $k > 0$ and $0 \leq a < 360$. Find the values of k and a .

4

- (b) Find the maximum value of $5 + \sin x^\circ - 3 \cos x^\circ$ and state a value of x for which this maximum occurs.

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1992 P1 Q7
(b)	1	C	CR	T14		
(b)	1	A/B	CR	T14		

<ul style="list-style-type: none"> •¹ $k \cos a = 1$ •² $k \sin a = 3$ •³ $k = \sqrt{10}$ •⁴ $a = 71.6$ 	<ul style="list-style-type: none"> •⁵ maximum = $5 + \sqrt{10}$ •⁶ angle = 161.6°
---	--

[SQA] 4.

- (a) Show that $2 \cos(x^\circ + 30^\circ) - \sin x^\circ$ can be written as $\sqrt{3} \cos x^\circ - 2 \sin x^\circ$. 3
- (b) Express $\sqrt{3} \cos x^\circ - 2 \sin x^\circ$ in the form $k \cos(x^\circ + \alpha^\circ)$ where $k > 0$ and $0 \leq \alpha \leq 360$ and find the values of k and α . 4
- (c) Hence, or otherwise, solve the equation $2 \cos(x^\circ + 30^\circ) = \sin x^\circ + 1$, $0 \leq x \leq 360$. 3

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	3	C	CR	T8, T3		1990 P2 Q5
(b)	4	C	CR	T13		
(c)	3	A/B	CR	T16		

<p>(a)</p> <ul style="list-style-type: none"> •¹ $\cos(x + 30)^\circ = \cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$ •² $\frac{\sqrt{3}}{2} \cos x^\circ - \frac{1}{2} \sin x^\circ$ •³ $2 \times \left(\frac{\sqrt{3}}{2} \cos x^\circ - \frac{1}{2} \sin x^\circ \right) - \sin x^\circ$ <p>(b)</p> <ul style="list-style-type: none"> •⁴ $k \cos x^\circ \cos \alpha^\circ - k \sin x^\circ \sin \alpha^\circ$ •⁵ $k \sin \alpha^\circ = \sqrt{3}$ and $k \sin \alpha^\circ = 1$ •⁶ $k = \sqrt{7} \rightarrow OG = 426$ •⁷ $\alpha = 49.1$ <p>(c)</p> <ul style="list-style-type: none"> •⁸ $\sqrt{7} \cos(x + 49.1)^\circ = 1$ •⁹ $x = 18.7^\circ$ •¹⁰ $x = 243.1^\circ$

- [SQA] 5. (a) Express $3\sin x^\circ - \cos x^\circ$ in the form $k\sin(x - \alpha)^\circ$, where $k > 0$ and $0 \leq \alpha \leq 90$. (4)
- (b) Hence find algebraically the values of x between 0 and 180 for which $3\sin x^\circ - \cos x^\circ = \sqrt{5}$. (4)
- (c) Find the range of values of x between 0 and 180 for which $3\sin x^\circ - \cos x^\circ \leq \sqrt{5}$. (2)

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1994 P2 Q5
(b)	4	C	CR	T16		
(c)	2	A/B	CR	T16		

(a)	• ¹	$k(\sin x \cos \alpha - \cos x \sin \alpha)$ or equivalent
	• ²	$k \cos \alpha = 3$ and $k \sin \alpha = 1$
	• ³	$k = \sqrt{10}$
	• ⁴	$\alpha = 18.4$
(b)	• ⁵	$\sqrt{10} \sin(x - 18.4)^\circ = \sqrt{5}$
	• ⁶	$\sin(x - 18.4)^\circ = \frac{1}{\sqrt{2}}$ or equivalent
	• ⁷	63.4
	• ⁸	153.4
(c)	• ⁹	strategy stated or implied
	• ¹⁰	$x \leq 63.4$ and $x \geq 153.4$

- [SQA] 6. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$.

6

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	6	A/B	CN	T14	max value $\sqrt{2}$ when $x = \frac{7\pi}{4}$	2000 P1 Q10
				<ul style="list-style-type: none"> •¹ ss: use e.g. $k \cos(x + a)$ •² ic: expand chosen rule •³ pd: compare coefficients •⁴ pd: process •⁵ pd: process •⁶ ic: interpret trig expression 	<ul style="list-style-type: none"> •¹ e.g. use $k \cos(x + a)$ •² $k \cos x \cos a - k \sin x \sin a$ •³ $k \cos a = 1$ and $k \sin a = 1$ •⁴ $k = \sqrt{2}$ •⁵ $\tan a = 1, a = \frac{\pi}{4}$ (<i>45° is bad form</i>) •⁶ max. value = $\sqrt{2}$ when $x = \frac{7\pi}{4}$ (<i>do not accept 45°</i>) 	

[END OF QUESTIONS]