Vectors Supported Study
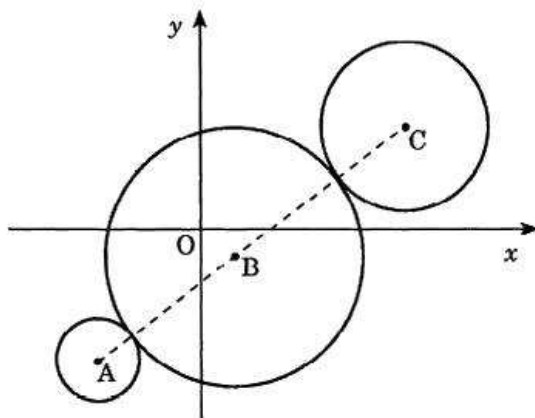
[SQA] 1. When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.

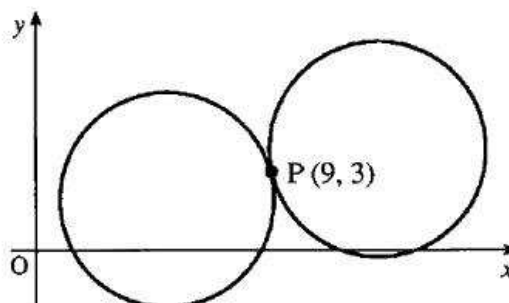The equations of the circumferences of the outer circles are

$(x+12)^2 + (y+15)^2 = 25$ and $(x-24)^2 + (y-12)^2 = 100$.
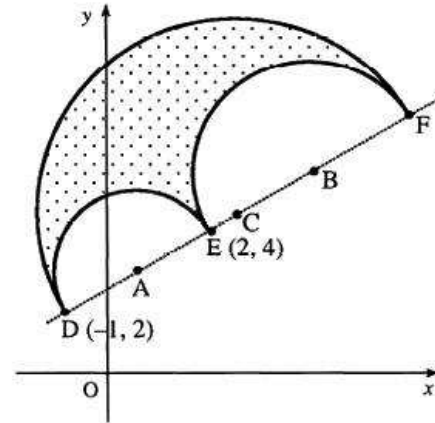
Find the equation of the central circle. (8)

[SQA] 2. Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation

$x^2 + y^2 - 10x - 4y + 12 = 0.$

Find the equation of the other circle.

5

[SQA]   3.   The shape shown in the diagram is composed of 3 semicircles with centres A, B and C which lie on a straight line.

DE is a diameter of one of the semicircles. The coordinates of D and E are (−1, 2) and (2, 4).

(a)   Find the equation of the circle with centre A and diameter DE.   (3)

The circle with centre B and diameter EF has equation $x^2 + y^2 - 16x - 16y + 76 = 0$.

(b)   (i)   Write down the coordinates of B.
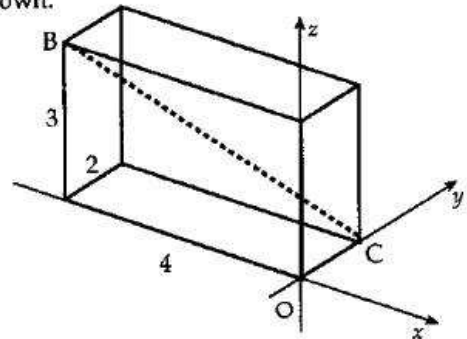   (ii)   Determine the coordinates of F and C.   (3)

(c)   In the diagram the perimeter of the shape is represented by the thick black line. Show that the perimeter is $5\pi\sqrt{13}$ units.   (3)

[SQA]   4.   A cuboid crystal is placed relative to the coordinate axes as shown.

(a)   Write down $\overrightarrow{BC}$ in component form.
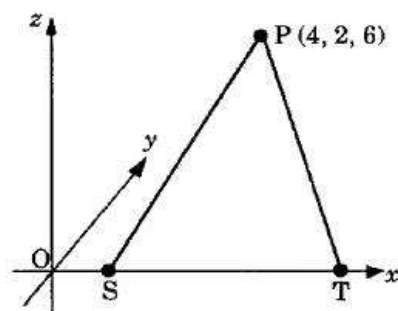
(b)   Calculate $\left|\overrightarrow{BC}\right|$.   2

[SQA]   5.   A is the point (−3,2,4) and B is (−1,3,2). Find

(a)   the components of vector $\overrightarrow{AB}$;   1
(b)   the length of AB.   2

[SQA] 6. The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the *x*-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T.
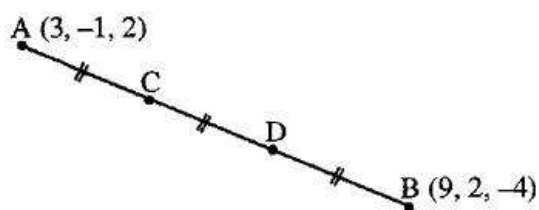
3

[SQA] 7. Vectors *p*, *q* and *r* are defined by
$$p = i + j - k, \quad q = i + 4k \quad \text{and} \quad r = 4i - 3j.$$
(a) Express $p - q + 2r$ in component form.

2

(b) Calculate *p.r*

1

(c) Find $|r|$.

1

[SQA] 8. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).
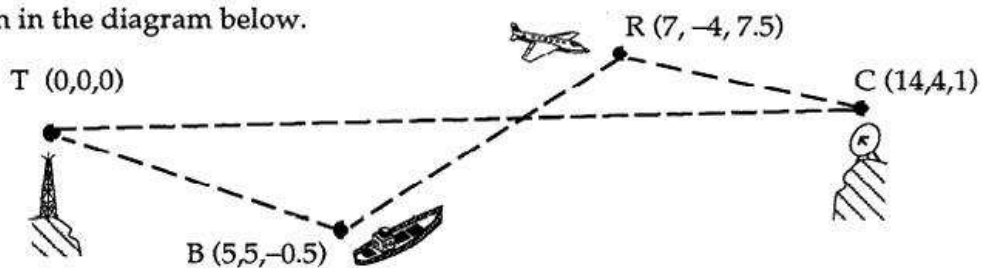
A (3, –1, 2)

C

D

B (9, 2, –4)

(a) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

2

(b) Find the coordinates of C and D.

2

[SQA]   9.   Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2
kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish
are shown in the diagram below.

The top T of the transmitter mast is the origin, the bridge B on the ship is the
point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point
(14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(a) Find the distance from the bridge of the ship to the reflector on the
aircraft.   (3)

(b) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the
speed of the aircraft in kilometres per hour.   (2)

(c) Prove that the direction of the beam TC is perpendicular to the direction
of the beam BR.   (3)
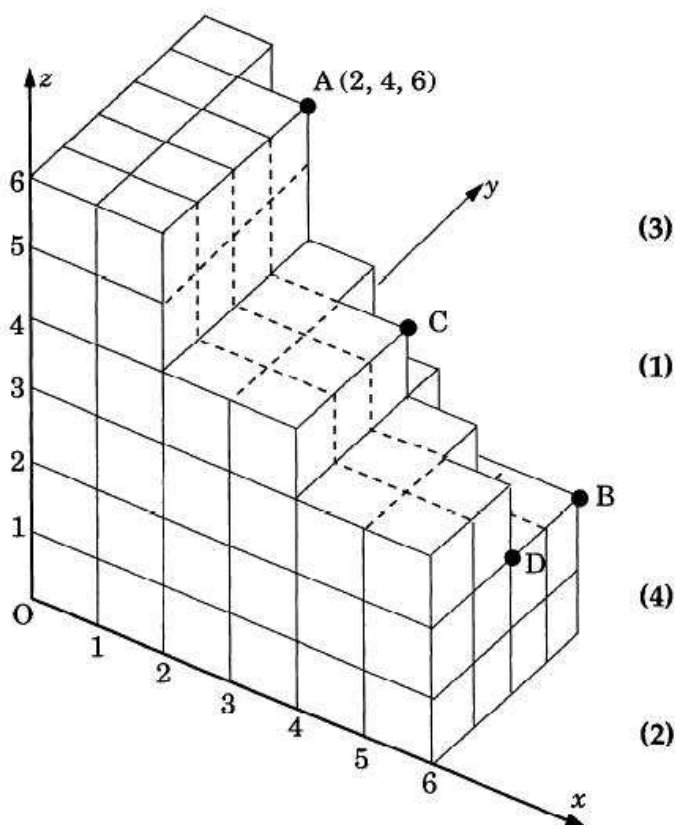
(d) Calculate the size of angle TCR.   (5)

[SQA] 10. With coordinate axes as shown, the point A is (2,4,6).

(a) Write down the coordinates of B,C and D. **(3)**

(b) Show that C is the midpoint of AD. **(1)**

(c) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(d) Hence calculate the size of angle OAB. **(2)**

[SQA] 11. The vectors p, q and r are defined as follows:

$$p = 3i - 3j + 2k,\ q = 4i - j + k,\ r = 4i - 2j + 3k.$$

(a) Find $2p - q + r$ in terms of i, j and k. **1**

(b) Find the value of $|2p - q + r|$. **2**

[SQA] 12. The vector $ai + bj + k$ is perpendicular to both the vectors $i - j + k$ and $-2i + j + k$.

Find the values of a and b. **3**

[SQA] 13. Calculate the length of the vector $2i - 3j + \sqrt{3}k$. **2**

[SQA] 14. The position vectors of the points P and Q are $p = -i + 3j + 4k$ and $q = 7i - j + 5k$ respectively.

(a) Express $\overrightarrow{PQ}$ in component form. **2**

(b) Find the length of PQ. **1**

[SQA] 15. The vectors *a*, *b* and *c* are defined as follows:

$$a = 2i - k, \quad b = i + 2j + k, \quad c = -j + k.$$

    (*a*) Evaluate *a.b* + *a.c* . **3**

    (*b*) From your answer to part (*a*), make a deduction about the vector *b* + *c*. **2**

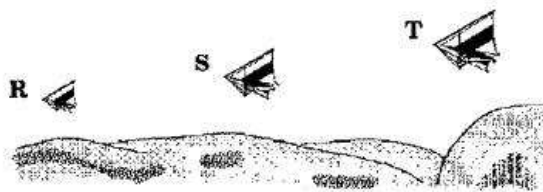[SQA] 16. Show that the vectors $a = 2i + 3j - k$ and $b = 3i - j + 3k$ are perpendicular. **3**

[SQA] 17. (a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear. **4**
       (b) Find the ration in which M divides LN. **1**

[SQA] 18. Relative to the top of a hill, three gliders have positions
given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6).
Prove that R, S and T are collinear. **3**

[SQA] 19. Show that $P(2,2,3)$, $Q(4,4,1)$ and $R(5,5,0)$ are collinear and find the ratio in which Q divides PR. **4**

[SQA] 20. ABCD is a quadrilateral with vertices $A(4,-1,3)$, $B(8,3,-1)$, $C(0,4,4)$ and $D(-4,0,8)$.

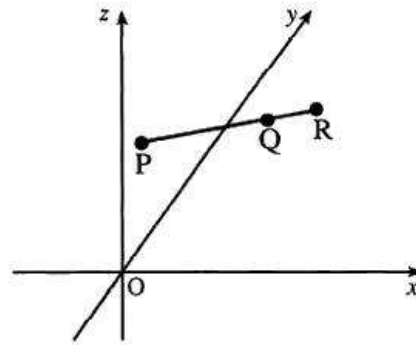   (*a*) Find the coordinates of M, the midpoint of AB. **1**

    (*b*) Find the coordinates of the point T, which divides CM in the ratio $2:1$. **3**

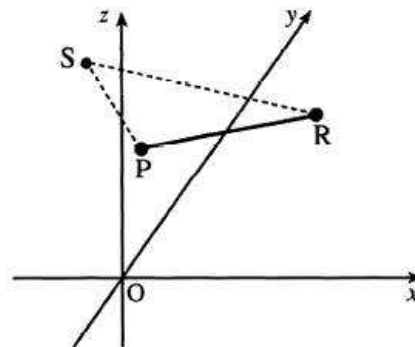   (*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

[SQA] 21. Relative to the axes shown and with an appropriate scale, P(−1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.
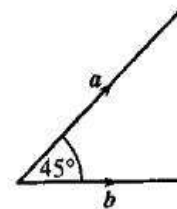
(a) Find the coordinates of R. (3)

(b) Roads from P and R are built to meet at the point S (−2, 2, 5). Calculate the size of angle PSR. (7)

[SQA] 22. The diagram shows two vectors a and b, with $|a| = 3$ and $|b| = 2\sqrt{2}$. These vectors are inclined at an angle of 45° to each other.

(a) Evaluate (i)   a.a
             (ii) b.b
             (iii) a.b 2

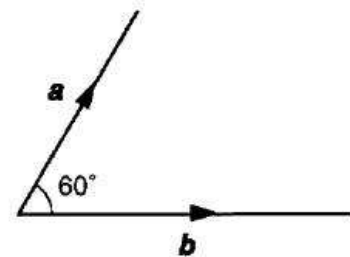(b) Another vector p is defined by $p = 2a + 3b$. Evaluate p.p and hence write down $|p|$. 4

[SQA] 23. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? 2
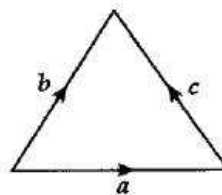
[SQA] 24. The diagram shows representatives of two vectors, a and b, inclined at an angle of 60°.
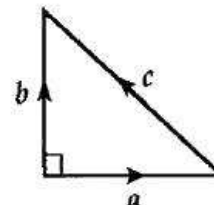If $|a| = 2$ and $|b| = 3$, evaluate $a.(a + b)$ 3

[SQA] 25. The sides of this equilateral triangle are 2 units long
and represent the vectors a, b and c as shown.
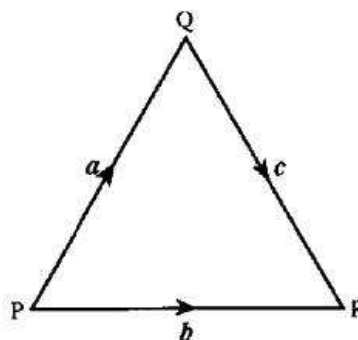Evaluate $a.(a+b+c)$.　　　　　　　　　　　　　　　　　　　　5

[SQA] 26. The diagram shows a right-angled isosceles triangle whose sides are
represented by the vectors a, b and c.
The two equal sides have length 2 units.
Find the value of $b.(a+b+c)$　　　　　　　　　　　　　　　　5

[SQA] 27. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = a$, $\overrightarrow{PR} = b$ and $\overrightarrow{QR} = c$.

Evaluate $a.(b+c)$ and hence identify two vectors
which are perpendicular.　　　　　　　　　　　　　　　　　　4

[END OF QUESTIONS]