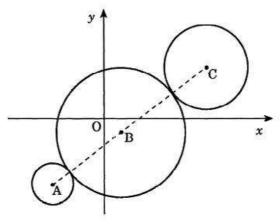
(8)

Vectors Supported Study

[SQA] 1. When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.



The equations of the circumferences of the outer circles are

$$(x+12)^2 + (y+15)^2 = 25$$
 and $(x-24)^2 + (y-12)^2 = 100$.

Find the equation of the central circle.

 Part
 Marks
 Level
 Calc.
 Content
 Answer
 U3 OC1

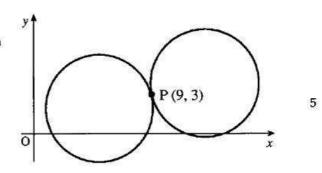
 8
 C
 CN
 G9, G10, G25
 1995 P2 Q8

- •¹ (-12,-15) and (24,12)
- radii are 5 and 10
- 3 AC = 45
- •4 radius = 15
- B divides AC in ratio 4:5
- •6 $\overrightarrow{OB} = \frac{1}{9} \left[4 \overrightarrow{OC} + 5 \overrightarrow{OA} \right]$ stated or implied
- $\vec{OB} = \frac{1}{9} \left[4 \begin{pmatrix} 24 \\ 12 \end{pmatrix} + 5 \begin{pmatrix} -12 \\ -15 \end{pmatrix} \right]$
- •8 $(x-4)^2 + (y+3)^2 = 15^2$

[SQA] 2. Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation

 $x^2 + y^2 - 10x - 4y + 12 = 0$

Find the equation of the other circle.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	С	CN	G9, G25		1997 P1 Q12

- use P as midpoint of C₁C₂
- $C_1 = (5,2)$
- $C_2 = (13,4)$
- 4 radius = $\sqrt{17}$
- $(x-13)^2 + (y-4)^2 = 17$

(3)

(3)

3. The shape shown in the diagram is composed [SQA] of 3 semicircles with centres A, B and C which lie on a straight line.

C E (2, 4) D (-1, 2)

DE is a diameter of one of the semicircles. The coordinates of D and E are (-1, 2) and (2, 4).

(a) Find the equation of the circle with centre A and diameter DE.

The circle with centre B and diameter EF has equation $x^2 + y^2 - 16x - 16y + 76 = 0$.

- (b) (i) Write down the coordinates of B.
 - (ii) Determine the coordinates of F and C.
- (c) In the diagram the perimeter of the shape is represented by the thick black line. (3) Show that the perimeter is $5\pi\sqrt{13}$ units.

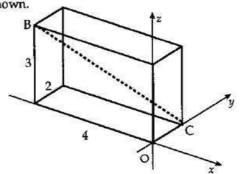
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	С	CN	G10		1998 P2 Q6
(b)	3	С	CN	G9, G25		
(c)	3	A/B	CN	CGD		ļ

- - $r^2 = \frac{9}{4} + 1$ or $d^2 = 13$
 - $(x-\frac{1}{2})^2 + (y-3)^2 = \frac{13}{4}$ or $x^2 + y^2 - x - 6y + 6 = 0$

- (c) •⁷ $\frac{1}{2}\pi DF + \frac{1}{2}\pi DE + \frac{1}{2}\pi EF$ •⁸ $\frac{1}{2}\pi DF = \frac{5}{2}\pi\sqrt{13}$ OR $\frac{1}{2}\pi EF = 2\pi\sqrt{13}$ •⁹ $\frac{5}{2}\pi\sqrt{13} + \frac{1}{2}\pi\sqrt{13} + 2\pi\sqrt{13}$

[SQA] 4. A cuboid crystal is placed relative to the coordinate axes as shown.

- (a) Write down BC in component form.
- (b) Calculate BC.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	С	CN	G16		1990 P1 Q5
(b)	1	С	CN	G16		

$$\overrightarrow{BC} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$$

$$\stackrel{2}{\checkmark} \sqrt{29}$$

[SQA] 5. A is the point (-3,2,4) and B is (-1,3,2). Find

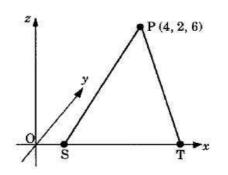
- (a) the components of vector AB;
- (b) the length of AB.

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	ò	2	

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	С	CN	G16		1993 P1 Q1
(b)	2	С	CN	G16		

• 1
$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$
• 2 $\sqrt{(-3+1)^2 + (2-3)^2 + (4-2)^2}$
• 3 3

The diagram shows a point P with coordinates (4, 2, 6) [SQA] and two points S and T which lie on the x-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	A/B	CN	G16		1994 P1 Q18

(x,0,0) or equiv.

 $PQ = \sqrt{40}$ OR

OR

 $(x-4)^2 + 4 + 36 = 49$ or equiv.

d = 3

$$x = 1, 7$$

(1,0,0), (7,0,0)

(1,0,0), (7,0,0)

7. Vectors p, q and r are defined by [SQA]

$$p = i + j - k$$
, $q = i + 4k$ and $r = 4i - 3j$.

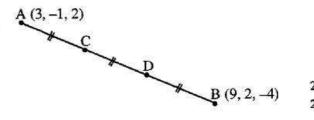
- (a) Express p-q+2r in component form.
- (b) Calculate p.r
- (c) Find |r|.

2
1

1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	С	CN	G16		1998 P1 Q3
(b)	1	С	CN	G26		
(c)	1	С	CN	G16		

[SQA] 8. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, -1, 2) and (9, 2, -4).



- (a) Find the components of \overrightarrow{AB} and \overrightarrow{AC} .
- (b) Find the coordinates of C and D.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	С	CN	G16		1998 P1 Q5
(b)	2	С	CN	G16		

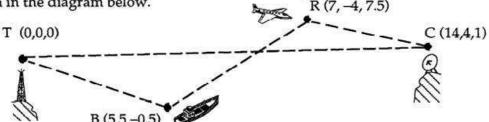
$$\bullet^1 \qquad \overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$$

•
3
 $C = (5,0,0)$

$$\bullet^2 \qquad \overrightarrow{AC} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

(3)

[SQA] 9. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, -0.5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, -4, 7.5).

- (a) Find the distance from the bridge of the ship to the reflector on the aircraft.
- (b) Three minutes earlier the aircraft was at the point M(-2, 4, 8.5). Find the speed of the aircraft in kilometres per hour.(2)
- (c) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. (3)
- (d) Calculate the size of angle TCR. (5)

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	С	CR	G16		1992 P2 Q2
(b)	2	С	CR	G16		
(c)	3	С	CR	G27		
(d)	5	С	CR	G28		

(a) •1 Strategy: use vectors or 3-D distance formula

•2
$$\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$$
 or $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(b) •4
$$|\overrightarrow{MR}| = \sqrt{115.25}$$
 or equivalent

•5 answer

•7
$$\overrightarrow{TC}.\overrightarrow{BR} = 0$$

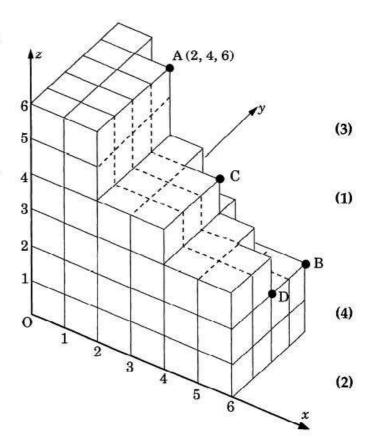
•8 communication: 0 ⇔ perpendicularity

$$\cos T \hat{C} R = \frac{\overrightarrow{TC} \cdot \overrightarrow{RC}}{|TC||RC|}$$
 or equiv.

•10
$$\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$$
 and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

• 12
$$\overrightarrow{TC}$$
. \overrightarrow{RC} = 82

- [SQA] 10. With coordinate axes as shown, the point A is (2,4,6).
 - (a) Write down the coordinates of B,C and D.
 - (b) Show that C is the midpoint of AD.
 - (c) By using the components of the vectors OA and OB, calculate the size of angle AOB, where O is the origin.
 - (d) Hence calculate the size of angle OAB.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	С	CR	G16		1994 P2 Q3
(b)	1	С	CR	G25		
(c)	4	С	CR	G28		
(d)	2	С	CR	CGD		

- (a) •1 One of B, C or D
 - Remaining two of B, C and D
 - •3 B(6,4,2), C(4,3,4), D(6,2,2)
- (b) $\bullet^4 \left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$
- (c) •5 $\cos A\hat{O}B = \frac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{|OA| |OB|}$ or $\frac{OA^2 + OB^2 AB^2}{2 \times OA \times OB}$ or equivalents
 - •6 $\overrightarrow{OA} \cdot \overrightarrow{OB} = 40 \text{ or } AB^2 = 32$
 - $OA = \sqrt{56} = OB$
 - ·8 44

- (d) \bullet strategy: e.g. use isosceles Δ
 - •10 68°

3

2

[SQA] 11. The vectors p, q and r are defined as follows:

$$p = 3i - 3j + 2k$$
, $q = 4i - j + k$, $r = 4i - 2j + 3k$.

- (a) Find 2p q + r in terms of i, j and k.
- (b) Find the value of |2p q + r|.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	С	CN	G18		1989 P1 Q3
(b)	2	С	CN	G16		

$$6i - 7i + 6k$$

$$\sqrt{6^2 + (-7)^2 + 6^2}$$

[SQA] 12. The vector ai + bj + k is perpendicular to both the vectors i - j + k and -2i + j + k.

Find the values of a and b.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18	a = 2, b = 3	1990 P1 Q12

$$a-b+1=0$$
 or $-2a+b+1=0$

$$a = 2$$
 and $b = 3$

[SQA] 13. Calculate the length of the vector $2i - 3j + \sqrt{3}k$.

I	Part	Marks	Level	Calc.	Content	Answer	U3 OC1
ı		2	С	CN	G18	4	1995 P1 Q1

•
$$\sqrt{2^2 + (-3)^2 + (\sqrt{3})^2}$$
 stated or implied by •²

3

3

[SQA] 14. The position vectors of the points P and Q are p = -i + 3j + 4k and q = 7i - j + 5k respectively.

- (a) Express PQ in component form.
- (b) Find the length of PQ.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	С	CN	G18, G16		1997 P1 Q4
(b)	1	С	CN	G16		

$$\begin{array}{ccc}
\bullet^{1} & q-p=8i-4j+k & \bullet^{2} & \overrightarrow{PQ} = \begin{pmatrix} 8\\ -4\\ 1 \end{pmatrix} \\
\text{or } p = \begin{pmatrix} -1\\ 3\\ 4 \end{pmatrix}, q = \begin{pmatrix} 7\\ -1\\ 5 \end{pmatrix} & \bullet^{3} & 9
\end{array}$$

[SQA] 15. The vectors a, b and c are defined as follows:

$$a = 2i - k$$
, $b = i + 2j + k$, $c = -j + k$.

- (a) Evaluate a.b + a.c.
- (b) From your answer to part (a), make a deduction about the vector b + c.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	С	CN	G18, G26		1993 P1 Q12
(b)	2	A/B	CN	G27		

$$\bullet^{1} \quad a = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, c = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\bullet^{4} \quad a.b + a.c = a.(b + c)$$

$$\bullet^{2} \quad a.b = 1$$

$$\bullet^{3} \quad a.c = -1$$

[SQA] 16. Show that the vectors a = 2i + 3j - k and b = 3i - j + 3k are perpendicular.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	С	CN	G18, G27	$a.b = \cdots = 0$	1991 P1 O3

• strat:
$$a.b = \dots$$
• $a.b = 0 \Rightarrow$ perpendicularity explicitly stated
• $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix} = 6 - 3 - 3 = 0$

[SQA] 17. (a) Show that the points L(-5, 6, -5), M(7, -2, -1) and N(10, -4, 0) are collinear.

4 1

3

4

(b) Find the ration in which M divides LN.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	4	С	CN	G23		1991 P1 Q7
(b)	1	С	CN	G25		

•1
$$\overrightarrow{LM} = \begin{pmatrix} 12 \\ -8 \\ 4 \end{pmatrix}$$
 or equivalent combinations for (a)

•3 $\overrightarrow{LM} = 4MN$

•4 vectors are parallel and have common point so L, M, N are collinear

•5 4:1

[SQA] 18. Relative to the top of a hill, three gliders have positions given by R(-1, -8, -2), S(2, -5, 4) and T(3, -4, 6).

Prove that R, S and T are collinear.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	С	CN	G23		1994 P1 Q4

•
$$\overrightarrow{ST} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$
 or equivalent **and** $\overrightarrow{RS} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$ or equivalent

- 2 $\overrightarrow{RS} = 3\overrightarrow{ST}$ or equiv.
- •3 RS // ST and S is common.
- [SQA] 19. Show that P(2,2,3), Q(4,4,1) and R(5,5,0) are collinear and find the ratio in which Q divides PR.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	4	С	CN	G23, G25	$\overrightarrow{QR} = \frac{1}{2}\overrightarrow{PQ},$ $PQ: QR = 2:1$	1990 P1 Q4

•
$$\overrightarrow{PQ} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$
 or equivalent

• vectors parallel and have pt in common so pts collinear

•
4
 PQ: QR = 2:1

- [SQA] 20. ABCD is a quadrilateral with vertices A(4,-1,3), B(8,3,-1), C(0,4,4) and D(-4,0,8).
 - (a) Find the coordinates of M, the midpoint of AB.
 - (b) Find the coordinates of the point T, which divides CM in the ratio 2:1.
 - (c) Show that B, T and D are collinear and find the ratio in which T divides BD.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	С	CN	G6, G25		1989 P2 Q2
(b)	3	С	CN	G25		
(c)	4	С	CN	G23, G25		

(a)	•1	(6,1,1)

(c)
$$\bullet$$
 5 e.g. $\overrightarrow{BT} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$

(b)
$$\bullet^2$$
 e.g. $\overrightarrow{CM} = \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$

•6
$$\overrightarrow{TD} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} = 2 \times \overrightarrow{BT}$$

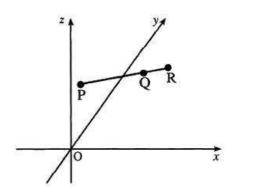
BT:TD = 1:2

$$\bullet^3 \quad \overrightarrow{CT} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$$

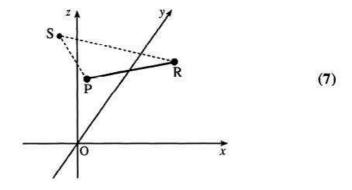
•
4
 $T = (4,2,2)$

(3)

[SQA] 21. Relative to the axes shown and with an appropriate scale, P(-1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



- (a) Find the coordinates of R.
- (b) Roads from P and R are built to meet at the point S (-2, 2, 5).Calculate the size of angle PSR.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	С	CR	G25		1997 P2 Q2
(b)	7	С	CR	G28		

- (a) $\stackrel{1}{\bullet^1} \qquad \stackrel{\rightarrow}{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix} \qquad \stackrel{2}{\bullet^2} \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$
 - 3 R = (7,-1,6)
- (b) $\overrightarrow{SP} \cdot \overrightarrow{SR} = |SP||SR| \cos P\hat{S}R$
 - •5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$
 - $|SF| = \sqrt{11}$
- $|SR| = \sqrt{9}$
- $\vec{SP} \cdot \vec{SR} = 3$
- 10 PSR = $84 \cdot 6^{\circ}$

4

- 22. The diagram shows two vectors a and b, with |a| = 3 and $|b| = 2\sqrt{2}$. [SQA] These vectors are inclined at an angle of 45° to each other.

- (a) Evaluate
- (i) a.a (ii) b.b
- (iii) a.b
- Another vector p is defined by p = 2a + 3b. Evaluate p.p and hence write down |p|.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	С	CN	G26		1999 P1 Q17
(h)	4	A/B	CN	G29. G30		1

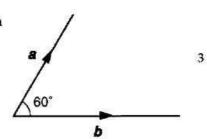
- a.a = 9 and b.b = 8
- (2a+3b).(2a+3b)

- 4a.a + 9b.b + 12a.b
- $\sqrt{180}$
- 23. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? 2 [SQA]

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	2	С	CN	G27	t = 4	2000 P2 Q7

- •¹ ss: know to use scalar product
- u.v = 2t 20 + 3t• $u.v = 0 \Rightarrow t = 4$
- •² ic: interpret scalar product
- The diagram shows representatives of two vectors, a and b, inclined at an 24. [SQA] angle of 60°.

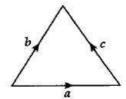
If |a| = 2 and |b| = 3, evaluate $a \cdot (a + b)$



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	С	CN	G29, G26		1992 P1 Q18

- a.a + a.b
- $2 \times 3 \times \cos 60^{\circ}$

[SQA] 25. The sides of this equilateral triangle are 2 units long and represent the vectors a, b and c as shown. Evaluate $a \cdot (a + b + c)$.



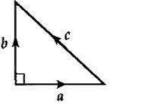
5

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	С	NC	A6		1989 P1 Q9
	4	A/B	NC	G29, G26		

- a.a+a.b+a.c
- $a.a = |a|a|\cos 0$
- \bullet a.b = $|a||b|\cos 60$
- $a.c = |a|c| \cos 120$
- 4
- [SQA] 26. The diagram shows a right-angled isosceles triangle whose sides are represented by the vectors **a**, **b** and **c**.

The two equal sides have length 2 units.

Find the value of $b \cdot (a + b + c)$



5

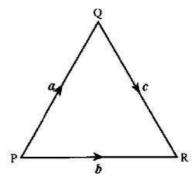
	Part	Marks	Level	Calc.	Content	Answer	U3 OC1
Γ		5	С	CN	G29, G27		1991 P1 Q17

- b.a+b.b+b.c
- 2 1 0
- 3 hh = 4
- 4 14-2.5
- .5 hc=4

[SQA] 27. PQR is an equilateral triangle of side 2 units.

$$\overrightarrow{PQ} = a$$
, $\overrightarrow{PR} = b$ and $\overrightarrow{QR} = c$.

Evaluate a.(b+c) and hence identify two vectors which are perpendicular.



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	С	CN	G26		1997 P1 Q13
	3	A/B	CN	G29, G27		

- \bullet^1 a.b+a.c
- $\bullet^2 \qquad a.b = 2 \times 2 \times \frac{1}{2}$
- $\bullet^3 \quad a.c = 2 \times 2 \times -\frac{1}{2}$
- 0 and a is perpendicular to (b+c)

[END OF QUESTIONS]