Part

3

2

logs Exponential

1. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. [SQA]

Marks

Content	Answer	U3 OC3
A28	2	2000 P1 Q9
A28		

•¹ pd: use $\log_a x + \log_a y = \log_a xy$ •² pd: use $\log_a x - \log_a y = \log_a \frac{x}{y}$

Level C

A/B

Calc.

NC

NC

 $\bullet^1 \log_5 100 - \log_5 4$ $\bullet^2 \log_5 25$

• 3 pd: use $\log_a a = 1$

- 3 2
- 2. Solve $\log_{h} x \log_{h} 7 = \log_{h} 3$ for x > 0.

A.
$$x = 21$$

B.
$$x = 10$$

C.
$$x = \frac{7}{3}$$

D.
$$x = \frac{3}{7}$$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
Α	3.3	A/B	0.64	0.59	CN	A28, A32	HSN 175

$$\log_b x - \log_b 7 = \log_b 3$$

$$\log_b \frac{x}{7} = \log_b 3$$

$$\frac{x}{7} = 3$$

$$\log_b \frac{x}{7} = \log_b 3$$

$$\frac{x}{7} = 3$$

$$\chi = 21$$

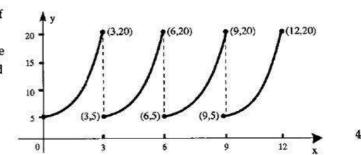
$$\begin{array}{c} \cdot \log_b x - \log_b y = \log_b \frac{x}{y} \\ \cdot \log_b x = \log_b y \Leftrightarrow x = y \end{array}$$

- [SQA] 3. Medical researchers studying the growth of a strain of bacteria observe that the number of bacteria, present after t hours, is given by the formula $N(t) = 40e^{1.5t}$.
 - (a) State the number of bacteria present at the start of the experiment.

(b) How many minutes will the bacteria take to double in number?

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	С	CR	A6		1989 P1 Q20
(b)	1	С	CR	A30		
(b)	3	A/B	CR	A30		ļ

- ·1 40
- e^2 $40e^{1.5t} = 80$
- $0.3 1.5t = \ln 2$
- t = 0.46
- 5 28 minutes
- [SQA] 4. A medical technician obtains this print-out of a wave form generated by an oscilloscope. The technician knows that the equation of the first branch of the graph (for $0 \le x \le 3$) should be of the form $y = ae^{kx}$.



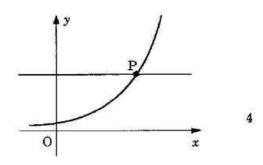
- (a) Find the values of a and k.
- (b) Find the equation of the second branch of the curve (i.e. for $3 \le x \le 6$).

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	С	CR	A2		1993 P1 Q15
(a)	3	A/B	CR	A30		
(b)	1	A/B	CR	A7		

- $^{-1}$ (0,5) $\Rightarrow a = 5$
- e^2 20 = 5 e^{3k}
- $e.g. \ln 20 = \ln 5 + 3k \ln e$
- k = 0.462 (Accept $\frac{1}{3}\ln 4$)
- $\bullet^5 \quad y = 5e^{k(x-3)}$

[SQA] 5. The diagram shows part of the graph with equation $y = 3^x$ and the straight line with equation y = 42. These graphs intersect at P.

Solve algebraically the equation $3^x = 42$, and hence write down, correct to 3 decimal places, the coordinates of P.



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	4	A/B	CR	A30		1994 P1 Q20

•1	use logs	OR	•1	use logs	OR	•1	use exponentials
•2	$\ln 3^x = \ln 42$	1207	•2	$x = \log_3 42$		•2	$\left(e^{1.0986}\right)^x = 42$
•3	$x \ln 3 = \ln 42$		•3	$x = \frac{\ln 42}{\ln 3}$		•3	$1.0986x = \ln 42$
•4	3 - 402		•4	3 · 402		.4	3 · 402

[SQA] 6. Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double, find the value of the constant k.

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CR	A30	k = 0.46	2001 P2 Q9
1			1		1	

- •¹ ic: form exponential equation
- •² ss: express exp. equ. as log equation
- 3 pd: solve log equation

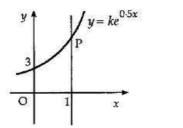
- $\bullet^1 \ 2A_0 = A_0 e^{k \times 1.5}$
- 2 e.g. $1.5k = \ln 2$
- $\bullet^3 k = 0.46$

2

[SQA]

7. The diagram shows part of the graph of $y = ke^{0.5x}$.

- (a) Find the value of k.
- (b) The line with equation x = 1 intersects the graph at P. Find the coordinates of the point P.



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	С	CR	A30, A34		1991 P1 Q4
(b)	2	С	CR	A6		

$$\bullet^1$$
 $3 = ke^0 \Rightarrow k = 3$

•
$$y = 3e^{0.5}$$
 or equivalent

[SQA]

8. The amount A grams of a radioactive substance at time t minutes is given by $A = A_0 e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.

In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(a) Find the value of k.

3

The half-life of a substance is the length of time in which half the substance decays.

(b) Find the half-life of Bismuth.

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	С	CR	A30		1995 P1 Q18
(a)	2	A/B	CR	A30, A34		
(b)	2	A/B	CR	A30		

$$e^{1}$$
 9 = 10 e^{-3k}

$$-3k = \log_e 0.9$$

$$e^{-kt} = 0.5$$

[SQA]

- 9. A mug of tea cools according to the law $T_t = T_0 e^{-kt}$ where T_0 is the initial temperature and T_c is the temperature after t minutes. All temperatures are in °C.
 - (a) A particular mug of tea cooled from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k.

3

(b) By how many degrees will the temperature of this tea fall in the next quarter of an hour?

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	С	CR	A30		1996 P1 Q19
	4	A/B	CR	A30, A34		

 1 75 - 100e $^{-k \times 15}$

• $T_{15} = 75e^{-0.0192 \times 15}$ or $T_{30} = 100e^{-0.0192 \times 30}$

 $\ln 0.75 = -15k$

• fall = 18.75

k = 0.0192

(5)

(4)

[SQA] 10. (a) A tractor tyre is inflated to a pressure of 50 units.

Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places.

(b) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre?

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	С	CR	A30, A34		1991 P2 Q7
(a)	3	A/B	CR	A30, A34		
(b)	1	С	CR	A30		
(b)	3	A/B	CR	A34		

(a)
$$\bullet^1 \quad 10 = 50e^{-24k}$$

$$0.2 = e^{-24k}$$

$$-24k = \ln 0.2$$

$$-24k = -1.609$$

•
$$k = 0.067$$

(b) 6 knowing to find P4

$$P_4 = 50e^{-0.067 \times 4}$$

- ·8 38
- 38 > 30 so can be driven further

[SQA] 11. (a) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years.

(2)

(b) The half-life of any material is the time taken for half of the mass to decay.

Find the half-life of this substance.

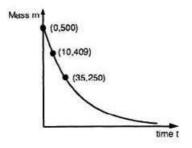
(3)

(c) Illustrate ALL of the above information on a graph.

(3)

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	С	CR	A30, A34		1992 P2 Q4
(b)	1	С	CR	A30		
(b)	2	A/B	CR	A30, A34		
(c)	1	С	CR	A2		
(c)	2	A/B	CR	A2		

- (a) $\bullet^1 \quad m = 500e^{-0.02 \times 10}$
 - 409.37 grams
- (b) \bullet^3 250 = 500e^{-0.02t}
 - •4 $\ln 250 = \ln 500 0.02t \times 1$ or equiv.
 - 5 34.7 years
- (c) •6 any two of the 3 points
 - the remaining point
 - a decreasing curve



[SQA] 12. The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.

Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(a) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures.

(3)

(b) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years?

(3)

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	1	С	CR	A30, A34		1997 P2 Q8
(a)	2	A/B	CR	A30, A34		
(b)	3	A/B	CR	A30		

- (a) $e^1 \qquad \frac{1}{2}y_0 = y_0 e^{5700k}$
 - $\ln \frac{1}{2} = 5700k$
 - k = -0.000122
- (b) $v = v_0 e^{-0.0}$
 - $\frac{y}{y_0} = \dots$
 - ·6 88.5%

- [SQA] 13. The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.
 - (a) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k.

(b) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity.

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	С	CR	A30, A34		1999 P2 Q7
(a)	2	A/B	CR	A30, A34		
(b)	1	С	CR	A30		
(b)	2	A/B	CR	A30		

(a) \bullet^1 90 = 120 e^{-4k}

•2 $e^{-4k} = 0.75$ or $\ln 90 = \ln 120 + \ln e^{-4k}$

•3 $\ln 0.75 = -4k$

 \bullet^4 k = 0.0719

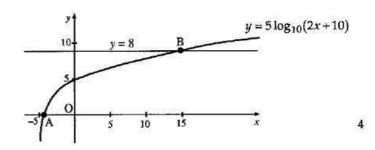
(b) •5 $I_{10} = I_0 e^{-10 \times 0.0719}$ stated or implied by •6

•6 $\frac{l_{10}}{l_0} = 0.487$

•7 51.3% reduction

[SQA] 14. Part of the graph of $y = 5 \log_{10}(2x + 10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line y = 8 at the point B.

Find algebraically the x-coordinates of A and B.



 Part
 Marks
 Level
 Calc.
 Content
 Answer
 U3 OC3

 4
 A/B
 CR
 A31
 1997 P1 Q17

 \bullet^1 $x_A = -4.5$

 2 5log₁₀(2x+10) = 8

 x^3 $2x+10=10^{\frac{8}{5}}$

• x = 14.9

4

3

15. Find the *x*-coordinate of the point where the graph of the curve with equation [SQA] $y = \log_3(x - 2) + 1$ intersects the *x*-axis.

1	Part	Marks	Level	Calc.	Content	Answer	U3 OC3
		2	С	CN	A31		2002 P2 Q7
		1	A/B	CN	A32	$x = 2\frac{1}{3}$	

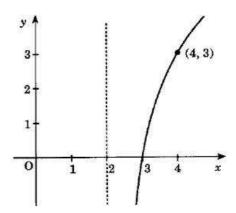
- ss: know to isolate log term
 pd: express log equation as exp. equ.
- •³ pd: process

- $\log_3(x-2) = -1$ $x-2=3^{-1}$
- $\bullet^3 \ \ x = 2\frac{1}{3}$
- 16. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. [SQA]

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	С	NC	A31		1998 P1 Q19
	3	A/B	NC	A28		

- $x = \log_5 12$
- $5^x = 12$
- $\log 5^x = \log 12$

- $\frac{\log 12}{\log 5} = 1.54$
- 17. The diagram shows a sketch of the graph of [SQA] y = f(x) where $f(x) = a \log_2(x - b)$. Find the values of a and b.

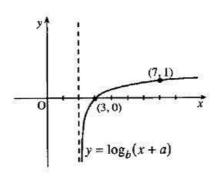


Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CN	A31, A2		1995 P1 Q19

- $3 = a \log_2 2$ stated or implied
- $(4-b)^a = 8$

3

[SQA] 18. The diagram shows part of the graph of $y = \log_b(x + a)$. Determine the values of a and b.



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	A/B	CN	A31, A2		1999 P1 Q15

$$a = -2$$

OR

•
$$1 = \log_b(7+a)$$
 and $0 = \log_b(a+3)$

2
 1 = $\log_{h}(7-2)$

$$a^2$$
 7+a=b and a+3=b⁰

$$b = 5$$

$$a = -2, b = 5$$

[SQA] 19. Two sound intensities P_1 and P_2 are said to differ by n decibels when $n = 10 \log_{10} \left(\frac{P_2}{P_1} \right)$

where P_1 and P_2 are measured in phons and $P_2 > P_1$.

Rustling leaves have a typical sound intensity of 30 phons.

If the sound intensity of a fire alarm siren is 6.5 decibels gretaer than rustling leaves, what is the sound intensity of the fire alrm system, measured in phons?

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	С	CR	A31		1989 P1 Q21
	2	A/B	CR	A31, A34		

•
$$6.5 = 10 \log_{10} \left(\frac{P}{30} \right)$$

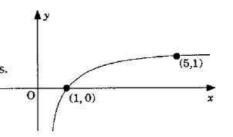
$$P = 30 \times 10^{0.65}$$

2

3

The diagram shows a sketch of part of the graph of $y = \log_5 x$. [SQA]

> (a) Make a copy of the graph of $y = \log_5 x$. On your copy, sketch the graph of $y = \log_5 x + 1$. Find the coordinates of the point where it crosses the x-axis.



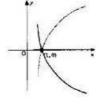
(b) Make a second copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$.

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	С	NC	A32		1994 P1 Q16
(a)	1	A/B	NC	A3		
(b)	2	A/B	NC	A28, A3		

- sketch of new function
- $\log_5 x + 1 = 0$

- $\log_5 \frac{1}{x} = -\log_5 x$
 - reflect in x axis



21. Find x if $4 \log_x 6 - 2 \log_x 4 = 1$. [SQA]

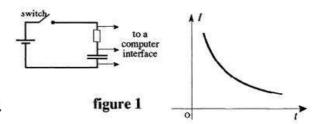
Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	3	С	NC	A32, A28, A31	x = 81	2001 P1 Q8

- pd: use log-to-index rule
 pd: use log-to-division rule
 ic: interpret base for log_x a = 1 and

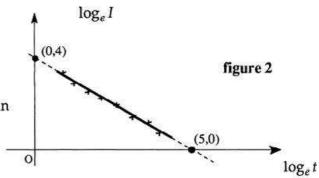
- •¹ $\log_x 6^4 \log_x 4^2$ •² $\log_x \frac{6^4}{4^2}$ •³ all processing leading to x = 81

(3)

[SQA] 22. When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds). This graph is shown in fig. 1.



In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.



- (a) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$.
- (b) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. (4)

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	3	С	CR	G2, G3		1993 P2 Q10
(b)	4	A/B	CR	A33		

(a)
$$\bullet^1$$
 $m = -\frac{4}{5}$ stated or implied

•
2
 $y = mx + 4$ stated or implied

$$\log_e I = -\frac{4}{5} \log_e t + 4$$

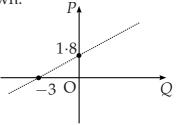
(b)
$$\bullet^4 \log_e t^{-\frac{4}{5}}$$

$$\log_{e} 54.6t^{-\frac{4}{5}}$$

$$I = 54.6t^{-0.8}$$

4

- [SQA] 23. The results of an experiment give rise to the graph shown.
 - (a) Write down the equation of the line in terms of *P* and *Q*.



It is given that $P = \log_e p$ and $Q = \log_e q$.

(b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b.

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	2	A/B	CR	G3	P = 0.6Q + 1.8	2000 P2 Q11
(b)	4	A/B	CR	A33	a = 6.05, b = 0.6	

- •¹ ic: interpret gradient
- \bullet^2 ic: state equ. of line
- •³ ic: interpret straight line
- ss: know how to deal with x of $x \log y$
- •5 ss: know how to express number as log
- •6 ic: interpret sum of two logs

- $\bullet^1 \ m = \frac{1 \cdot 8}{3} = 0 \cdot 6$
- $\bullet^2 P = 0.6Q + 1.8$

Method 1

- $\bullet^3 \log_e p = 0.6 \log_e q + 1.8$
- $\bullet^4 \log_e q^{0.6}$
- \bullet ⁵ $\log_e 6.05$
- $p = 6.05q^{0.6}$

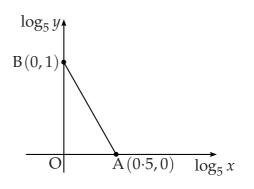
Method 2

$$ln p = ln aq^b$$

- $\bullet^3 \ln p = \ln a + b \ln q$
- •4 $\ln p = 0.6 \ln q + 1.8$ stated or implied by •5 or •6
- 5 ln a = 1.8
- \bullet^6 a = 6.05, b = 0.6

24. The graph illustrates the law $y = kx^n$. [SQA]

> If the straight line passes through A(0.5,0) and B(0,1), find the values of k and n.



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	4	A/B	NC	A33	$y = 5x^{-2}$	2002 P1 Q11

\bullet^1	ic:	interpret graph
•2	ss:	use log laws
•3	ss:	use log laws
\bullet^4	pd:	solve log equation

- $\log_5 y = -2(\log_5 x) + 1$ $\log_5 y = \log_5 x^{-2} + \dots$ $\log_5 5$ $\log_5 x + \log_5 5$

[SQA] 25. (a) The variables x and y are connected by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log y$ and x.

(3)

(6)

(b) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

x	3.1	3.5	4.1	5.2
у	21 876	72 631	439 392	11 913 076

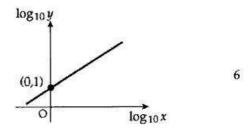
The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b.

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	3	A/B	CR	A33		1998 P2 Q11
(b)	6	A/B	CR	A30		

- (a) $\int_0^1 \log_e y = \log_e ae^{bx}$
 - $\log_e y = \log_e a + \log_e e^{bx}$
 - $\log_e y = \log_e a + bx$
- evidence for strategy being carried out
 will be appearance of two equations at •5 stage
 - •5 e.g. $3.1b + \log a = 9.99$, $5.2b + \log a = 16.29$
 - strategy: know to subtract
 - b = 3
 - $a = e^{0.69}$
 - •9 a = 2

[SQA] 26. As shown in the diagram, a set of experimental results gives a straight line graph when $\log_{10} y$ is plotted against $\log_{10} y$. The straight line passes through (0, 1) and has a gradient of 2.

Express y in terms of x.



Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	С	CN	G3		1990 P1 Q14
	4	A/B	CN	A33, A34		

use y = mx + c

 $\int_{0}^{4} \log_{10} y = \log_{10} x^2 + 1$

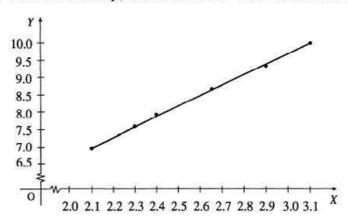
 $^2 \quad \log_{10} y = 2\log_{10} x + 1$

- $\log_{10} y = \log_{10} 10x^2$
- $\log_{10} y = 2\log_{10} x + \log_{10} 10$
- $y = 10x^2$

[SQA] 27. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X	Y
$(=\log_{\mathbf{e}}x)$	(=log _e y)
2.10	7.00
2.31	7.60
2.40	7.92
2.65	8.70
2.90	9.38
3.10	10.00



A graph was drawn and is shown above.

(a) Find the equation of the line in the form Y = mX + c.

- (3)
- (b) Hence find the values of the constants a and b in the relationship $y = ax^b$.

(4)

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(a)	3	С	CR	G3		1996 P2 Q9
(b)	4	A/B	CR	A33, A34		

- (a) e.g. m = 3
 - 2 e.g. $8.70 = 3 \times 2.65 + c$ or equiv.
 - 3 e.g. Y = 3X + 0.75
- - 5 $\ln y = \ln 2.01x^{3}$
 - b = 3
 - a = 2.01

Quest **Higher Mathematics**

28. The point P(p,k) lies on the curve with equation $y = \ln x$. [SQA]

The point Q(q, k) lies on the curve with equation $y = \frac{1}{2} \ln x$.

Find a relationship between p and q and hence find q when p = 5.

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	1	С	CN	A32		1993 P1 Q20
	3	A/B	CN	A34	$q=p^2, q=25$	

 $k = \log_e p$ and $k = \frac{1}{2}\log_e q$ $\log p = \frac{1}{2}\log q$ or $p = e^k$ and $q = e^{2k}$ $q = p^2$ or $p = q^{\frac{1}{2}}$

[END OF QUESTIONS]

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