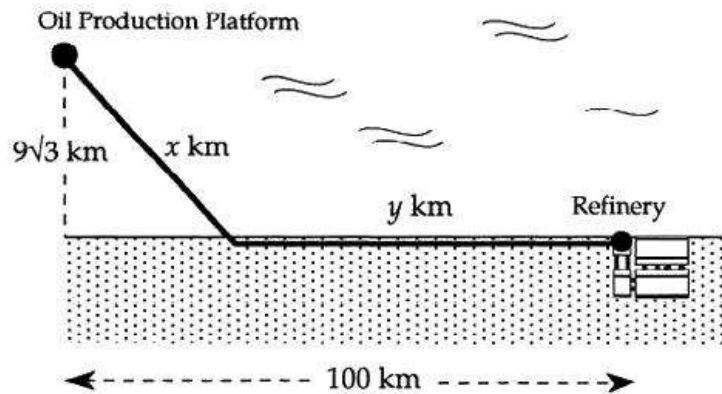


## Unit 3 Further Diff Int

- [SQA] 1. An oil production platform,  $9\sqrt{3}$  km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is  $x$  km and the length of pipeline on land is  $y$  km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

- (a) Show that the total cost of this pipeline is £ $C(x)$  million where

$$C(x) = 2x + 100 - (x^2 - 243)^{\frac{1}{2}}. \quad (3)$$

- (b) Show that  $x = 18$  gives a minimum cost for this pipeline. Find this minimum cost and the corresponding total length of the pipeline. (7)

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	1	C	NC	A6		1993 P2 Q11
(a)	2	A/B	NC	A6		
(b)	1	C	NC	C11, C21		
(b)	6	A/B	NC	C11, C21		

(a) •<sup>1</sup>  $C = 2x + y$

•<sup>2</sup>  $\sqrt{x^2 - (9\sqrt{3})^2}$

•<sup>3</sup> for completing proof

(b) •<sup>4</sup> knowing to differentiate

•<sup>5</sup>  $\frac{1}{2}(x^2 - 243)^{-\frac{1}{2}}$

•<sup>6</sup>  $\times 2x$

•<sup>7</sup>  $C'(18) = 0$

•<sup>8</sup> justification of minimum e.g. nature table

•<sup>9</sup>  $C = 127$

•<sup>10</sup>  $x + y = 109$

	18 <sup>-</sup>	18	18 <sup>+</sup>
$C'(x)$	-	0	+
	minimum		

- [SQA] 2. Find the equation of the tangent to the curve  $y = 2 \sin(x - \frac{\pi}{6})$  at the point where  $x = \frac{\pi}{3}$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	CN	C5, C20	$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	2002 P2 Q6
<ul style="list-style-type: none"> <li>•<sup>1</sup> pd: find derivative</li> <li>•<sup>2</sup> ss: know derivative at <math>x = \dots</math> represents grad.</li> <li>•<sup>3</sup> pd: find corresponding <math>y</math>-coordinate</li> <li>•<sup>4</sup> ic: state equation of tangent</li> </ul>				<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 2 \cos(x - \frac{\pi}{6})</math></li> <li>•<sup>2</sup> <math>m = \sqrt{3}</math></li> <li>•<sup>3</sup> <math>y_{x=\frac{\pi}{3}} = 1</math></li> <li>•<sup>4</sup> <math>y - 1 = \sqrt{3}(x - \frac{\pi}{3})</math></li> </ul>		

- [SQA] 3. A point moves in a straight line such that its acceleration  $a$  is given by  $a = 2(4 - t)^{\frac{1}{2}}$ ,  $0 \leq t \leq 4$ . If it starts at rest, find an expression for the velocity  $v$  where  $a = \frac{dv}{dt}$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	NC	C18, C22	$V = -\frac{4}{3}(4 - t)^{\frac{3}{2}} + \frac{32}{3}$	2002 P2 Q8
<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to integrate acceleration</li> <li>•<sup>2</sup> pd: integrate</li> <li>•<sup>3</sup> ic: use initial conditions with const. of int.</li> <li>•<sup>4</sup> pd: process solution</li> </ul>				<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>V = \int (2(4 - t)^{\frac{1}{2}}) dt</math> stated or implied by •<sup>2</sup></li> <li>•<sup>2</sup> <math>2 \times \frac{1}{-\frac{3}{2}}(4 - t)^{\frac{3}{2}}</math></li> <li>•<sup>3</sup> <math>0 = 2 \times \frac{1}{-\frac{3}{2}}(4 - 0)^{\frac{3}{2}} + c</math></li> <li>•<sup>4</sup> <math>c = 10\frac{2}{3}</math></li> </ul>		

- [SQA] 4. The graph of  $y = f(x)$  passes through the point  $(\frac{\pi}{9}, 1)$ .  
If  $f'(x) = \sin(3x)$  express  $y$  in terms of  $x$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C18, C23	$y = -\frac{1}{3} \cos(3x) + \frac{7}{6}$	2000 P1 Q8
<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to integrate</li> <li>•<sup>2</sup> pd: integrate</li> <li>•<sup>3</sup> ic: interpret <math>(\frac{\pi}{9}, 1)</math></li> <li>•<sup>4</sup> pd: process</li> </ul>				<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y = \int \sin(3x) dx</math> stated or implied by •<sup>2</sup></li> <li>•<sup>2</sup> <math>-\frac{1}{3} \cos(3x)</math></li> <li>•<sup>3</sup> <math>1 = -\frac{1}{3} \cos(\frac{3\pi}{9}) + c</math> or equiv.</li> <li>•<sup>4</sup> <math>c = \frac{7}{6}</math></li> </ul>		

[SQA] 5. A curve for which  $\frac{dy}{dx} = 3 \sin(2x)$  passes through the point  $(\frac{5\pi}{12}, \sqrt{3})$ .

Find  $y$  in terms of  $x$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	CN	C18, C23	$y = -\frac{3}{2} \cos(2x) + \frac{1}{4}\sqrt{3}$	2001 P2 Q10

- <sup>1</sup> pd: integrate trig function
- <sup>2</sup> pd: integrate composite function
- <sup>3</sup> ss: use given point to find "c"
- <sup>4</sup> pd: evaluate "c"

- <sup>1</sup>  $\int 3 \sin(2x) dx$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $-\frac{3}{2} \cos(2x)$
- <sup>3</sup>  $\sqrt{3} = -\frac{3}{2} \cos(2 \times \frac{5}{12}\pi) + c$
- <sup>4</sup>  $c = \frac{1}{4}\sqrt{3} (\approx 0.4)$

6. Differentiate  $3 \cos(2x - \frac{\pi}{6})$  with respect to  $x$ .

- A.  $-3 \sin(2x)$   
 B.  $-3 \sin(2x - \frac{\pi}{6})$   
 C.  $-6 \sin(2x - \frac{\pi}{6})$   
 D.  $6 \sin(2x - \frac{\pi}{6})$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0.68	0.23	NC	C20	HSN 096

$$\frac{d}{dx} \left( 3 \cos \left( 2x - \frac{\pi}{6} \right) \right) = -3 \times 2 \sin \left( 2x - \frac{\pi}{6} \right)$$

$$= -6 \sin \left( 2x - \frac{\pi}{6} \right)$$

Option C

[SQA] 7. Differentiate  $\sin 2x + \frac{2}{\sqrt{x}}$  with respect to  $x$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C3		1989 P1 Q10
	2	A/B	NC	C20		

- <sup>1</sup>  $2x^{-\frac{1}{2}}$
- <sup>2</sup>  $\cos 2x$
- <sup>3</sup>  $\times 2$
- <sup>4</sup>  $-x^{-\frac{3}{2}}$

[SQA] 8. Given that  $f(x) = (5x - 4)^{\frac{1}{2}}$ , evaluate  $f'(4)$ .

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		

<ul style="list-style-type: none"> <li>•<sup>1</sup> pd: differentiate power</li> <li>•<sup>2</sup> pd: differentiate 2nd function</li> <li>•<sup>3</sup> pd: evaluate <math>f'(x)</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}(5x - 4)^{-\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\times 5</math></li> <li>•<sup>3</sup> <math>f'(4) = \frac{5}{8}</math></li> </ul>
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[SQA] 9. Given  $f(x) = \cos^2 x - \sin^2 x$ , find  $f'(x)$ .

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21		1999 P1 Q19
	2	A/B	NC	C21, C20		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(x) = \cos 2x</math></li> <li>•<sup>2</sup> <math>-\sin 2x</math></li> <li>•<sup>3</sup> <math>\times 2</math></li> </ul>	<p>For <math>\frac{d}{dx}(\cos^2 x)</math>    <b>OR</b>    For <math>\frac{d}{dx}(-\sin^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2 \cos x</math>                      •<sup>1</sup> <math>-2 \sin x</math></li> <li>•<sup>2</sup> <math>\times -\sin x</math>                      •<sup>2</sup> <math>\times \cos x</math></li> </ul> <p>For <math>\frac{d}{dx}(-\sin^2 x)</math>                      For <math>\frac{d}{dx}(\cos^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>-2 \sin x \times \cos x</math>                      •<sup>3</sup> <math>2 \cos x \times -\sin x</math></li> </ul>
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[SQA] 10. Given that  $f(x) = 5(7 - 2x)^3$ , find the value of  $f'(4)$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C21		1991 P1 Q13

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(7 - 2x)^2</math></li> <li>•<sup>2</sup> <math>\times 15</math></li> <li>•<sup>3</sup> <math>\times -2</math></li> <li>•<sup>4</sup> <math>-30</math></li> </ul>
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[SQA] 11. Differentiate  $2x^{\frac{3}{2}} + \sin^2 x$  with respect to  $x$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21		1992 P1 Q11
	3	A/B	NC	C21		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>3x^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>(\sin x)^2</math> stated or implied by •<sup>3</sup></li> <li>•<sup>3</sup> <math>2 \sin x</math></li> <li>•<sup>4</sup> <math>\times \cos x</math></li> </ul>
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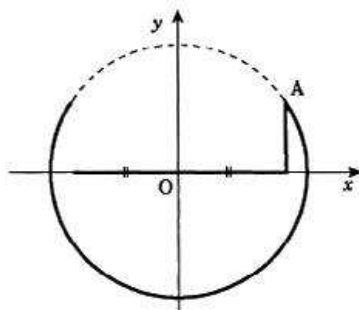
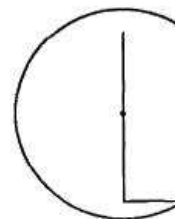
[SQA] 12. Find the derivative, with respect to  $x$ , of  $\frac{1}{x^3} + \cos 3x$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C21		1994 P1 Q10

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>x^{-3}</math> stated or implied by •<sup>2</sup></li> <li>•<sup>2</sup> <math>-3x^{-4}</math></li> <li>•<sup>3</sup> <math>-\sin 3x</math></li> <li>•<sup>4</sup> <math>\times 3</math></li> </ul>
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- [SQA] 13. Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.



The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is  $x^2 + y^2 = 20$ .

- (a) If A has co-ordinates  $(x, y)$ , show that the total length  $T$  of the arms of the 'L' is given by  $T = 2x + \sqrt{20 - x^2}$ . (1)
- (b) Show that for a stationary value of  $T$ ,  $x$  satisfies the equation  $x = 2\sqrt{20 - x^2}$ . (5)
- (c) By squaring both sides, solve this equation. Hence find the greatest length of the arms of the 'L'. (3)

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	1	A/B	CN	CGD		1995 P2 Q11
(b)	1	C	CN	C21		
(b)	4	A/B	CN	C21		
(c)	1	C	CN	C11		
(c)	2	A/B	CN	C11		

(a) •<sup>1</sup>  $T = x + x + y$  and  $y^2 = 20 - x^2$

(b) •<sup>2</sup> appearance of  $\frac{dT}{dx} = 2 + \dots$

•<sup>3</sup>  $\frac{1}{2}(20 - x^2)^{-\frac{1}{2}}$

•<sup>4</sup>  $\times -2x$

•<sup>5</sup>  $\frac{dT}{dx} = 0$  stated or implied

•<sup>6</sup> completing proof

(c) •<sup>7</sup>  $x^2 = 4(20 - x^2)$

•<sup>8</sup>  $x = 4$  (accept  $x = \pm 4$ )

•<sup>9</sup> justifying  $x = 4$  gives  $T_{\max} = 10$

[SQA] 14. If  $f(x) = \cos^2 x - \frac{2}{3x^2}$ , find  $f'(x)$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C21, C1		1990 P1 Q19
	2	A/B	NC	C21, C1		

- <sup>1</sup>  $-\frac{2}{3}x^{-2}$
- <sup>2</sup>  $2\cos x$
- <sup>3</sup>  $\times(-\sin x)$
- <sup>4</sup>  $\frac{4}{3}x^{-3}$

[SQA] 15. Differentiate  $4\sqrt{x} + 3\cos 2x$  with respect to  $x$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C21, C1		1993 P1 Q9
	2	A/B	NC	C21, C1		

- <sup>1</sup>  $4x^{\frac{1}{2}}$
- <sup>2</sup>  $2x^{-\frac{1}{2}}$
- <sup>3</sup>  $-\sin 2x$
- <sup>4</sup>  $\times 2$

[SQA] 16. Differentiate  $\sin^3 x$  with respect to  $x$ .

Hence find  $\int \sin^2 x \cos x dx$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21, C19		1994 P1 Q17
	3	A/B	NC	C21, C19		

- <sup>1</sup> using  $(\sin x)^3$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $3\sin^2 x$
- <sup>3</sup>  $\times \cos x$
- <sup>4</sup>  $\frac{1}{3}\sin^3 x$

[SQA] 17. Find  $\frac{dy}{dx}$  given that  $y = \sqrt{1 + \cos x}$ .

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	3	A/B	NC	C21, C20		1996 P1 Q13
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(1 + \cos x)^{\frac{1}{2}}</math> stated or implied by •<sup>2</sup></li> <li>•<sup>2</sup> <math>\frac{1}{2}(1 + \cos x)^{-\frac{1}{2}}</math></li> <li>•<sup>3</sup> <math>-\sin x</math></li> </ul>						

[SQA] 18. Given  $f(x) = (\sin x + 1)^2$ , find the exact value of  $f'(\frac{\pi}{6})$ .

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2			
	3	A/B	NC	C21, C20, T2		1998 P1 Q16			
<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2(\sin x + 1)</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul> </td> <td style="width: 10%; border: none; text-align: center; vertical-align: middle;">Alternative</td> <td style="width: 40%; border: none;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> expand and differentiate <math>2 \sin x + 1</math></li> <li>•<sup>2</sup> differentiate <math>\sin^2 x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul> </td> </tr> </table>							<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2(\sin x + 1)</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul>	Alternative	<ul style="list-style-type: none"> <li>•<sup>1</sup> expand and differentiate <math>2 \sin x + 1</math></li> <li>•<sup>2</sup> differentiate <math>\sin^2 x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul>
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2(\sin x + 1)</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul>	Alternative	<ul style="list-style-type: none"> <li>•<sup>1</sup> expand and differentiate <math>2 \sin x + 1</math></li> <li>•<sup>2</sup> differentiate <math>\sin^2 x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul>							

[SQA] 19. Find  $\int \sqrt{1 + 3x} dx$  and hence find the exact value of  $\int_0^1 \sqrt{1 + 3x} dx$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C22		1993 P1 Q16
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(1 + 3x)^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\frac{1}{\frac{3}{2}}(1 + 3x)^{\frac{3}{2}}</math></li> <li>•<sup>3</sup> <math>+ 3</math></li> <li>•<sup>4</sup> <math>\frac{14}{9}</math></li> </ul>						

[SQA] 20. Find  $\int \frac{1}{(7 - 3x)^2} dx$ .

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\frac{1}{3(7 - 3x)} + c$	2000 P2 Q10
<ul style="list-style-type: none"> <li>•<sup>1</sup> pd: integrate function</li> <li>•<sup>2</sup> pd: deal with function of function</li> </ul>					<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{-1}(7 - 3x)^{-1}</math></li> <li>•<sup>2</sup> <math>\times \frac{1}{-3}</math></li> </ul>	

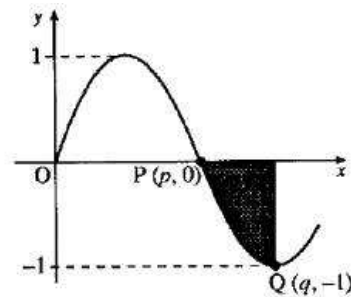


[SQA] 21. Evaluate  $\int_{-3}^0 (2x + 3)^2 dx$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	NC	C22, C15		1996 P1 Q5
• <sup>1</sup>	$\frac{1}{3}(2x+3)^3$			OR	• <sup>1</sup>	$\frac{4}{3}x^3$
• <sup>2</sup>	+2				• <sup>2</sup>	$6x^2 + 9x$
• <sup>3</sup>	$\frac{1}{6}(3)^3 - \frac{1}{6}(-6+3)^3$				• <sup>3</sup>	$ 0  - \left[ \frac{4}{3}(-3)^3 + 6(-3)^2 + 9(-3) \right]$
• <sup>4</sup>	9				• <sup>4</sup>	9

[SQA] 22. A sketch of part of the graph of  $y = \sin 2x$  is shown in the diagram.  
 The points P and Q have coordinates  $(p, 0)$  and  $(q, -1)$ .  
 (a) Write down the values of  $p$  and  $q$ .  
 (b) Find the area of the shaded region.



1  
4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	1	C	NC	T1		1998 P1 Q15
(b)	1	C	NC	C16		
(b)	3	A/B	NC	C23		
• <sup>1</sup>	$p = \frac{\pi}{2}$ and $q = \frac{3\pi}{4}$			• <sup>2</sup>	$\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (\sin 2x) dx$	
				• <sup>3</sup>	$-\frac{1}{2} \cos 2x$	
				• <sup>4</sup>	$-\frac{1}{2}$	
				• <sup>5</sup>	deal with -ve correctly giving $\frac{1}{2}$	

[SQA] 23.

(a) Evaluate  $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$ .

3

(b) Draw a sketch and explain your answer.

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	3	A/B	NC	C23		1992 P1 Q14
(b)	1	C	NC	T1, C16		
(b)	1	A/B	NC	T1, C16		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}</math></li> <li>•<sup>2</sup> <math>\sin 2x</math></li> <li>•<sup>3</sup> 0</li> <li>•<sup>4</sup> diagram</li> <li>•<sup>5</sup> +ve and -ve cancel out</li> </ul>	
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[SQA] 24. Find  $\int (6x^2 - x + \cos x) \, dx$ .

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	NC	C23		1995 P1 Q3

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x^3</math></li> <li>•<sup>2</sup> <math>-\frac{1}{2}x^2</math></li> <li>•<sup>3</sup> <math>\sin x</math></li> <li>•<sup>4</sup> <math>+c</math></li> </ul>
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[SQA] 25. The curve  $y = f(x)$  passes through the point  $(\frac{\pi}{12}, 1)$  and  $f'(x) = \cos 2x$ .Find  $f(x)$ .

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	3	A/B	NC	C23		1997 P1 Q15

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2} \sin 2x</math></li> <li>•<sup>2</sup> <math>1 = \frac{1}{2} \sin \frac{\pi}{6} + c</math></li> <li>•<sup>3</sup> <math>c = \frac{3}{4}</math></li> </ul>
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[SQA] 26.

(a) By writing  $\sin 3x$  as  $\sin(2x + x)$ , show that  $\sin 3x = 3 \sin x - 4 \sin^3 x$ . 4

(b) Hence find  $\int \sin^3 x \, dx$ . 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	2	C	NC	T8, T8		1995 P2 Q9
(a)	2	A/B	NC	T8, T8		
(b)	4	A/B	NC	C23		

<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin 2x \cos x + \cos 2x \sin x</math></li> <li>•<sup>2</sup> <math>2 \sin x \cos x \cos x + \dots\dots\dots</math></li> <li>•<sup>3</sup> <math>\dots\dots\dots + (1 - 2 \sin^2 x) \sin x</math></li> <li>•<sup>4</sup> <math>2 \sin x - 2 \sin^3 x + \sin x - 2 \sin^3 x</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>5</sup> <math>\int \frac{1}{4} (3 \sin x - \sin 3x) \, dx</math></li> <li>•<sup>6</sup> <math>-3 \cos x</math></li> <li>•<sup>7</sup> <math>+ \cos 3x</math></li> <li>•<sup>8</sup> <math>+3</math></li> </ul>
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- [SQA] 27. A function  $f$  is **EVEN** if  $f(-x) = f(x)$   
 e.g. when  $f(x) = x^2$ ,  $f$  is **EVEN** because  $f(-x) = (-x)^2 = x^2 = f(x)$ .

A function  $f$  is **ODD** if  $f(-x) = -f(x)$

e.g. when  $f(x) = x^3$ ,  $f$  is **ODD** because  $f(-x) = (-x)^3 = -x^3 = -f(x)$ .

- (a) Given that  $g(x) = \cos x$  and  $h(x) = \sin 2x$ , decide for each of the functions  $g$  and  $h$  whether it is **EVEN** or **ODD**.

Justify your decisions.

(4)

- (b) Evaluate  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$  and  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \, dx$ .

(5)

- (c) On separate diagrams, draw rough sketches of the graphs of  $y = \cos x$  and  $y = \sin 2x$  for  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ .

(2)

- (d) If  $v(x) = x \cos x$ , check whether the function  $v$  is **EVEN** or **ODD** and

suggest a value for  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$ .

(2)

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	4	C	NC	CGD		1989 P2 Q8
(b)	2	C	NC	C23, C15		
(b)	3	A/B	NC	C23, C15		
(c)	2	C	NC	T1		
(d)	2	A/B	NC	CGD		

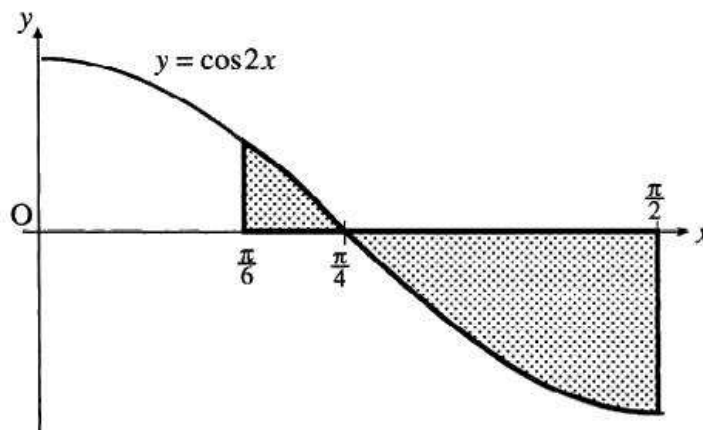
- (a) •<sup>1</sup>  $\cos(-x) = \cos x$   
 •<sup>2</sup>  $g$  is **EVEN**  
 •<sup>3</sup>  $\sin(-2x) = -\sin(2x)$   
 •<sup>4</sup>  $h$  is **ODD**

- (c) •<sup>10</sup> sketch of  $g(x) = \cos x$   
 •<sup>11</sup> sketch of  $h(x) = \sin 2x$

- (b) •<sup>5</sup>  $\sin x$   
 •<sup>6</sup>  $[\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$   
 •<sup>7</sup>  $-\cos 2x$   
 •<sup>8</sup>  $\times \frac{1}{2}$   
 •<sup>9</sup>  $[-\frac{1}{2} \cos 2x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$

- (d) •<sup>12</sup>  $v(x)$  is **ODD**  
 •<sup>13</sup> 0

- [SQA] 28. An artist has designed a 'bow' shape which he finds can be modelled by the shaded area below. Calculate the area of this shape. (6)



Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C23, C16		1996 P2 Q5
	4	A/B	NC	C23, C16		

<ul style="list-style-type: none"> <li>•<sup>1</sup> evidence of two integrals</li> <li>•<sup>2</sup> <math>\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x \, dx</math> and <math>\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \, dx</math></li> <li>•<sup>3</sup> <math>\frac{1}{2} \sin 2x</math></li> <li>•<sup>4</sup> <math>\frac{1}{2} \sin \frac{\pi}{2} - \frac{1}{2} \sin \frac{\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{4}</math></li> <li>•<sup>5</sup> <math>\frac{1}{2} \sin \pi - \frac{1}{2} \sin \frac{\pi}{2} = -\frac{1}{2}</math></li> <li>•<sup>6</sup> <math>1 - \frac{\sqrt{3}}{4}</math></li> </ul>
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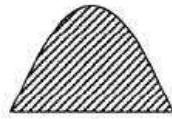
- [SQA] 29. (a) Find the derivative of the function  $f(x) = (8 - x^3)^{\frac{1}{2}}$ ,  $x < 2$ . 2
- (b) Hence write down  $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} \, dx$ . 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	2	A/B	CN	C21	$-\frac{3}{2}x^2(8 - x^3)^{-\frac{1}{2}}$	2002 P1 Q10
(b)	1	A/B	CN	C24	$-\frac{2}{3}(8 - x^3)^{\frac{1}{2}} + c$	

<ul style="list-style-type: none"> <li>•<sup>1</sup> pd: process differentiation</li> <li>•<sup>2</sup> pd: use the chain rule</li> <li>•<sup>3</sup> ic: interpret answer from (a)</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}(8 - x^3)^{-\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\dots \times -3x^2</math></li> <li>•<sup>3</sup> <math>-\frac{2}{3}f(x)</math> or <math>-\frac{2}{3}(8 - x^3)^{\frac{1}{2}}</math></li> </ul>
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[SQA] 30.

An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the same area. Three of the shapes used are drawn below.



A

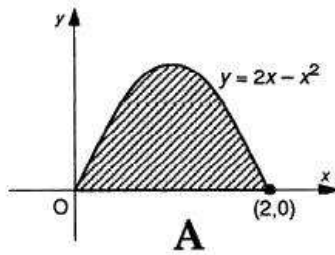


B

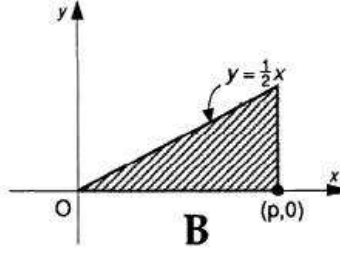


C

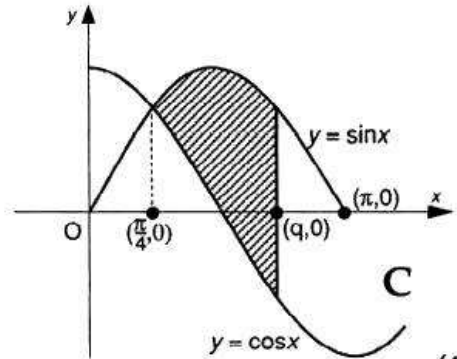
Relative to  $x,y$ -axes, the shapes are positioned as shown below.



A



B



C

- (a) Find the area shaded under  $y = 2x - x^2$ . (4)
- (b) Use the area found in part (a) to find the value of  $p$ . (2)
- (c) Prove that  $q$  satisfies the equation  $\cos q + \sin q = 0.081$  and hence find the value of  $q$  to 2 significant figures. (10)

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	C16		1992 P2 Q10
(b)	2	C	CR	A6		
(c)	2	C	CR	T16, C23, C17		
(c)	8	A/B	CR	T16, C23, C17		

<p>(a) •<sup>1</sup> strategy: know to integrate</p> <p>•<sup>2</sup> <math>\int_0^2 (2x - x^2) dx</math></p> <p>•<sup>3</sup> <math>x^2 - \frac{1}{3}x^3</math></p> <p>•<sup>4</sup> <math>1\frac{1}{3}</math> units<sup>2</sup></p> <p>(b) •<sup>5</sup> strategy: use area to find <math>p</math></p> <p>•<sup>6</sup> <math>p = \frac{4}{\sqrt{3}}</math> or equivalent</p>	<p>(c) •<sup>7</sup> <math>\int (\sin x - \cos x) dx</math></p> <p>•<sup>8</sup> for the limits <math>\frac{q}{\frac{\pi}{4}}</math></p> <p>•<sup>9</sup> <math>[-\cos x - \sin x]</math></p> <p>•<sup>10</sup> <math>-\cos q - \sin q + \sqrt{2}</math></p> <p>•<sup>11</sup> <math>\sqrt{2} - \frac{4}{3} = 0.081</math></p> <p>•<sup>12</sup> strategy: eg <math>k \cos(q - \alpha)</math></p> <p>•<sup>13</sup> <math>k = \sqrt{2}</math></p> <p>•<sup>14</sup> <math>\alpha = \frac{\pi}{4}</math></p> <p>•<sup>15</sup> <math>\cos\left(q - \frac{\pi}{4}\right) = \frac{0.081}{\sqrt{2}}</math></p> <p>•<sup>16</sup> <math>q = 2.3</math></p>
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[END OF QUESTIONS]