Vectors

1. $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$.

   Calculate (a) $|\mathbf{b}|$ (b) $|2\mathbf{a} - \mathbf{b}|$ (c) $|3(\mathbf{a} + \mathbf{b})|$

2. (a) Find the magnitude of the vector $\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$.

   (b) Find a vector parallel to the vector $\begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}$ which has unit length.

3. A is (0,-3,5), B is (7,-6,9) and C is (21,-12,17). Show that A, B and C are collinear stating the ratio AB:BC.

4. PQRS is a parallelogram with P(3,4,0), Q(7,6,-3) and R(8,5,2). Find the coordinates of S.

5. (a) P is the point (-1,8,0) and Q is (4,-2,5). B divides PQ in the ratio 3:2. Find the coordinates of B.

   (b) A is (0,1,5) and C is (8,5,-3). Show that A, B and C are collinear.

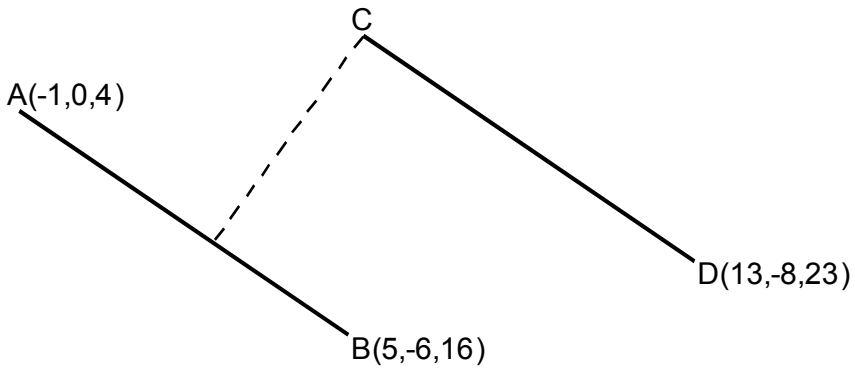
6. An aeroplane flies in a straight line at a constant speed. It takes 3 hours to fly from A to B and 4 hours to fly from B to C. Relative to coordinate axes, A is (0,-1,6) and C is (7,6,-1). Find the coordinates of B.

7. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + a\mathbf{j} + \sqrt{7}\,\mathbf{k}$. If $|\mathbf{u}| = |\mathbf{v}|$ find the value of a.

8. Show that the vectors $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 7\mathbf{j} - 6\mathbf{k}$ are perpendicular.

9. A triangle has vertices A(6,-1,9), B(3,-2,11) and C(7,-8,14). Show that this triangle is right-angled at B.

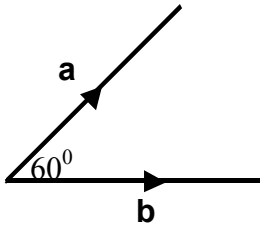
10. Three points A, B and D have coordinates as shown.



(a) Find the coordinates of C if AB is parallel and equal in length to CD.
(b) The point E divides AB in the ratio 2:1, find the coordinates of E.
(c) Prove that CE is perpendicular to AB.

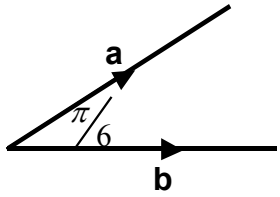
11. Use the diagrams to find the value of **a.b**.

(a)



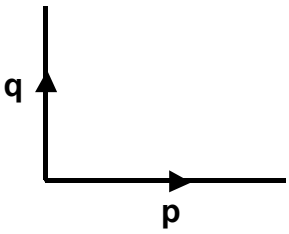
$|\mathbf{a}| = 6 \qquad |\mathbf{b}| = 7$

(b)



$|\mathbf{a}| = 4 \qquad |\mathbf{b}| = 2\sqrt{3}$

12. Write down the value of **p.q**. $|\mathbf{p}| = 8 \qquad |\mathbf{q}| = 9$



13. A triangle is formed from R(0,4,-1), S(1,5,2) and T(6,1,-2).

(a) Find the vectors $\overrightarrow{RS}$ and $\overrightarrow{RT}$.
(b) Evaluate $\overrightarrow{RS}.\overrightarrow{RT}$
(c) What can you deduce about he lines RS and RT.

14. A, B, C and D are the points (-1,3,1), (1,6,7), (0,2,5) and (1,4,10) respectively.

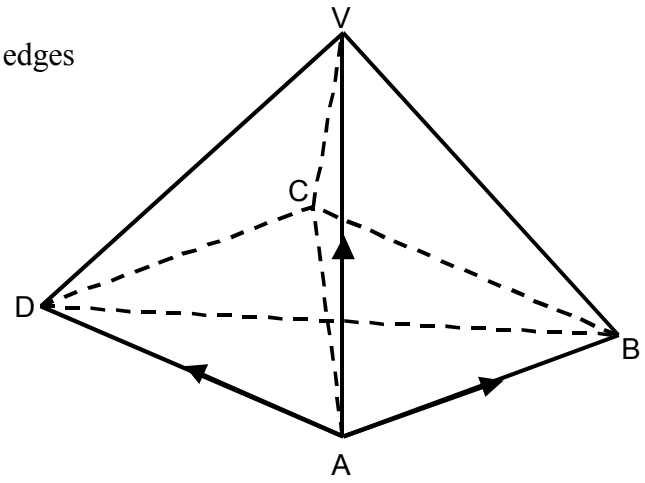
(a) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.
(b) The vector $\begin{pmatrix} p \\ q \\ 1 \end{pmatrix}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Find p and q.

15. $\mathbf{u} = \begin{pmatrix} -3 \\ 3 \\ k \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$.

(a) Write down the vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.
(b) Given that $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular find k.

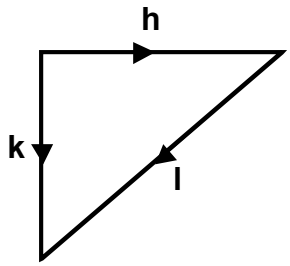
16. In the square based pyramid opposite all eight edges are of length 5 units.

Evaluate $\mathbf{p}.(\mathbf{q} + \mathbf{r})$.



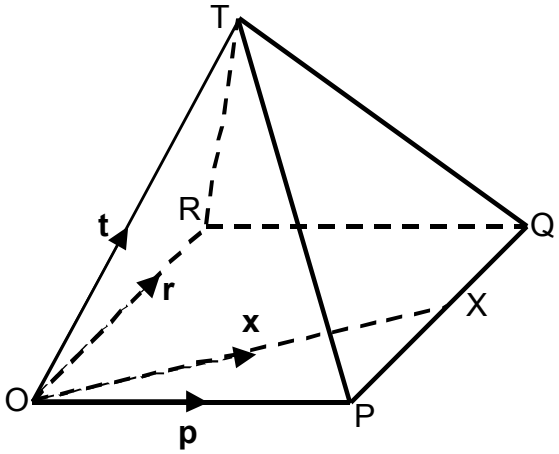
17. Shown opposite is a right-angled isosceles triangle. The two equal sides of the triangle have length 4 units.

Find the value of $\mathbf{k}.(\mathbf{h} + \mathbf{k} + \mathbf{l})$.



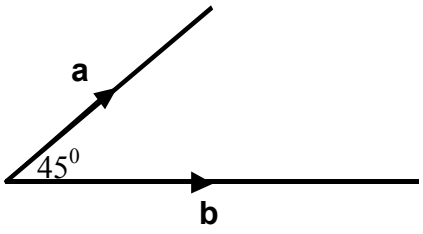
18. In the diagram opposite TOPQR is a pyramid whose base OPQR is a rhombus of length 1 unit. OPT and ORT are equilateral triangles.

(a) Evaluate $\mathbf{t}.\mathbf{r}$.
(b) Given X is the midpoint of PQ, evaluate $\mathbf{t}.\mathbf{x}$.



19. The diagram shows two vectors **a** and **b** with $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3\sqrt{3}$.

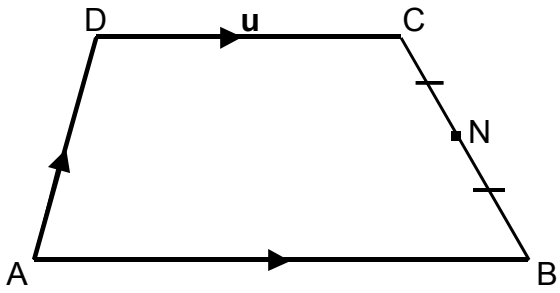
(a) Evaluate (a) $\mathbf{a}.\mathbf{a}$ (b) $\mathbf{b}.\mathbf{b}$ (c) $\mathbf{a}.\mathbf{b}$
(b) Given $\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$ evaluate $\mathbf{p}.\mathbf{p}$.



20. In the trapezium AB = 2DC and AB is paralle to DC.

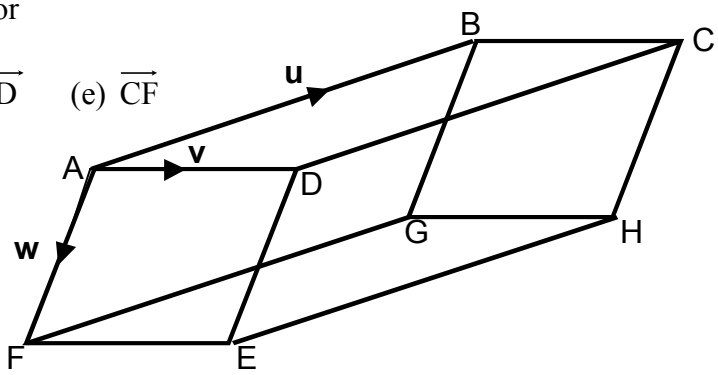
In terms of **u** and **v**, write down the vectors

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{AC}$ (c) $\overrightarrow{BC}$ (d) $\overrightarrow{AN}$

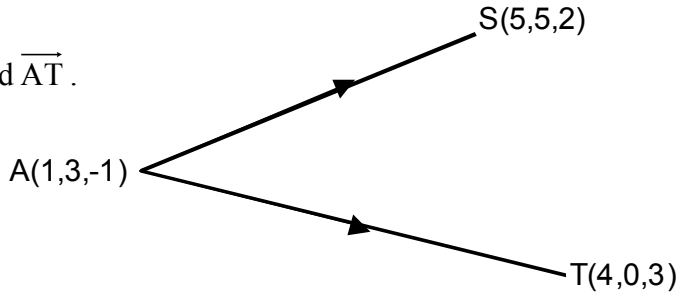


21. ABCDEFGH is a parallelipiped.
In terms of **u**, **v** and **w** find expressions for

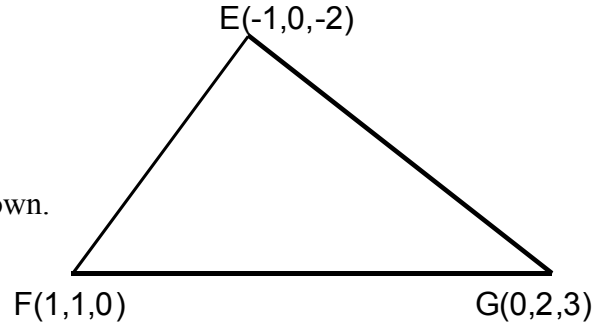
(a) $\overrightarrow{DC}$ (b) $\overrightarrow{HC}$ (c) $\overrightarrow{AC}$ (d) $\overrightarrow{FD}$ (e) $\overrightarrow{CF}$



22. (a) For the diagram opposite find $\overrightarrow{AS}$ and $\overrightarrow{AT}$.
(b) Hence calculate angle TAS.

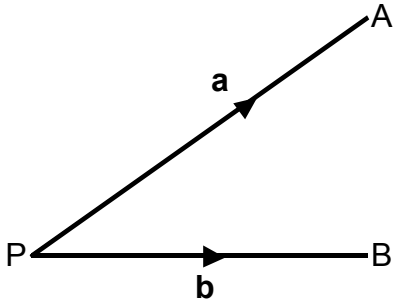


23. Calculate the size of angle FEG in the diagram shown.



24. $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are representatives of the vectors **a** and **b**.

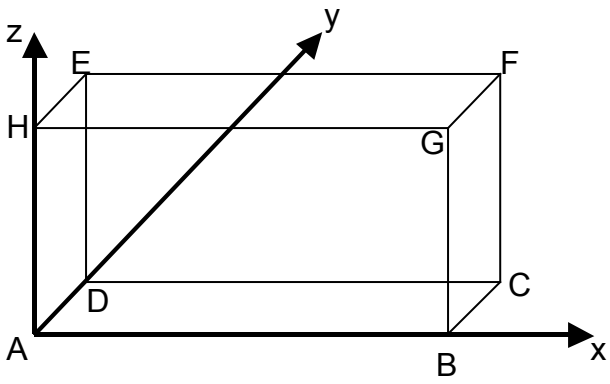
$\mathbf{a} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$ and angle APB = 2θ.



(a) Prove that $\cos 2\theta = -\frac{7}{9}$

(b) Hence find the exact value of $\cos^2\theta$.

25. In the diagram AB = 15, BC = 6 and CF = 8
    (a) Write down the coordinates of D and F
    (b) Calculate the size of angle DBF.



26. The diagram shows three cuboids placed on top of each other.
    Two of the cuboids are equal in size –
    10 cm by 3 cm by 5 cm.
    The third cuboid is centrally placed on the other two and has dimensions 6 cm by 3 cm by 5 cm.

    (a) Write down the coordinates of A, B and C.
    (b) Calculate the size of angle BAC.