The Wave Function

1. Express each of the following in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a \leq 360$.

   (a) $4\cos x + 3\sin x$ (b) $\sqrt{2}\cos x + \sqrt{2}\sin x$ (c) $\cos x - \sin x$

   (d) $2\sin x - 3\cos x$

2. Express each of the following in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a \leq 360$.

   (a) $5\cos x - 12\sin x$ (b) $2\cos x - \sqrt{5}\sin x$ (c) $3\cos x + \sin x$

   (d) $\sin x + 2\cos x$

3. Express each of the following in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a \leq 360$.

   (a) $2\sin x - 2\cos x$ (b) $\sqrt{3}\sin x - \cos x$ (c) $4\cos x + 2\sin x$

4. Express each of the following in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a \leq 360$.

   (a) $6\sin x + 8\cos x$ (b) $\sin x - 4\cos x$ (c) $7\cos x - \sin x$

5.(a) Write $4\sin x + 3\cos x$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a \leq 360$.

  (b) Hence write down the maximum value of $4\sin x + 3\cos x$ and the value of x at which this maximum occurs.

6. (a) Write $2\cos x - \sin x$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a \leq 360$.

   (b) Write down the maximum value of $2\cos x - \sin x$ and determine the corresponding value of x in the interval $0 \leq x \leq 360$.

7. (a) Write $\sqrt{5}\cos x - 2\sin x$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a \leq 360$.

   (b) Hence write down the minimum value of $\sqrt{5}\cos x - 2\sin x$ and the corresponding value of x in the range $0 \leq x \leq 360$.

8. (a) Write $3\sin x + \cos x$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Hence find the maximum value of $5 + 3\sin x + \cos x$ and determine the corresponding value of x in the interval $0 \le x \le 360$.

9. (a) Write $\cos x – 7\sin x$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Hence find the minimum value of $7\sqrt{2} + \cos x – 7\sin x$ and the value of x at which this minimum occurs in the interval $0 \le x \le 360$

10. (a) Write $\sin x + \sqrt{8}\cos x$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Hence write down the maximum value of $4 + \sin x + \sqrt{8}\cos x$ and determine the value of x at which this maximum occurs in the interval $0 \le x \le 360$.

11. (a) Express $2\cos x + 3\sin x$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Hence solve the equation $2\cos x + 3\sin x = 0.5$ for $0 \le x \le 360$.

12. (a) Express $4\cos x + 3\sin x$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Hence solve the equation $4\cos x + 3\sin x = 3$ for $0 \le x \le 360$.

13. $f(x) = \sqrt{2}\cos x – 4\sin x$.

(a) Express f(x) in the form $k\cos(x + a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Solve $f(x) = \sqrt{5}$ for $0 \le x \le 360$.

14. $f(x) = 6\sin x – 2\cos x$.

(a) Express f(x) in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Solve $f(x) = \sqrt{20}$ for $0 \le x \le 360$

(c) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 6\sin x – 2\cos x$ cuts the x-axis for $0 \le x \le 360$.

15. (a) Express $\sqrt{6}\cos x + \sqrt{6}\sin x$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \le a \le 360$.

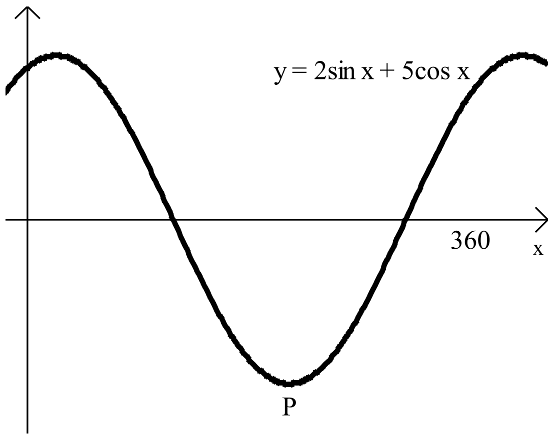
(b) Solve the equation $3 + \sqrt{6}\cos x + \sqrt{6}\sin x = 3.8$ for $0 \le x \le 360$.

(c) Find the x-coordinate of the point nearest to the origin where the graph of $y = \sqrt{6}\cos x + \sqrt{6}\sin x$ cuts the x-axis for $0 \le x \le 360$.

16. Part of the graph of $y = 2\sin x + 5\cos x$ is shown in the diagram.

(a) Express $2\sin x + 5\cos x$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \le a \le 360$.

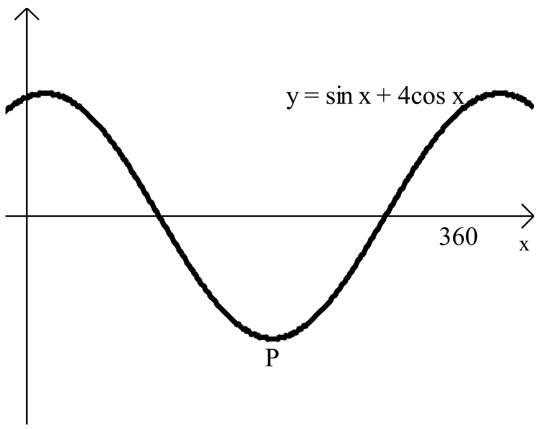
(b) Find the coordinates of the minimum turning point P.



17. Part of the graph of $y = \sin x + 4\cos x$ is shown in the diagram.

(a) Express $\sin x + 4\cos x$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \le a \le 360$.

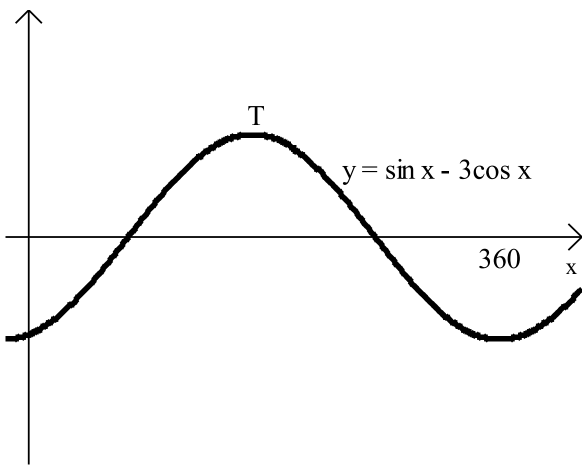
(b) Find the coordinates of the minimum turning point P.



18. Part of the graph of $y = \sin x - 3\cos x$ is shown in the diagram.

(a) Express $\sin x - 3\cos x$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a \le 360$.

(b) Find the coordinates of the maximum turning point T.



19. (a) Express sin x – cos x in the form ksin(x - a)° where k > 0 and $0 \le a \le 360$.

(b) Hence sketch the graph of y = sin x – cos x for $0 \le x \le 360$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis.

20. (a) Express $\sqrt{10}$ cos x - $\sqrt{10}$ sin x in the form kcos(x + a)° where k > 0 and $0 \le a \le 2\pi$.

(b) Hence sketch the graph of y = $\sqrt{10}$ cos x - $\sqrt{10}$ sin x for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis.

21. (a) Express sin x - $\sqrt{3}$ cos x in the form ksin(x - a)° where k > 0 and $0 \le a \le 360$.

(b) Hence, or otherwise, sketch the curve with equation y = 3 + sin x - $\sqrt{3}$ cos x in the interval $0 \le x \le 360$.

22. (a) Express $\sqrt{3}$ cos x – sin x in the form kcos(x + a)° where k > 0 and $0 \le a \le 2\pi$..

(b) Hence sketch the graph of y = $\sqrt{3}$ cos x – sin x – 5 in the interval $0 \le x \le 2\pi$.