Further Calculus

Differentiation

1. Differentiate

    (a) $y = (4x - 2)^3$ (b) $y = \sqrt{6x - 2}$ (c) $f(x) = \dfrac{2}{5x + 2}$ (d) $y = 3\sin 2x$
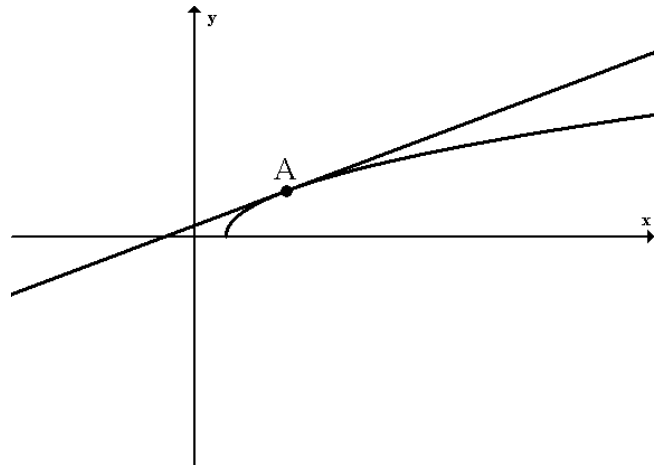
    (e) $f(x) = \cos^2 x + 2\sin 4x$

2. $f(x) = 2\cos 2x - \sin 4x$. Show that $f'\left(\frac{\pi}{6}\right) = 2(1 - \sqrt{3})$

3. $f(x) = (2 + \cos x)^3$. Show that $f'\left(\frac{\pi}{2}\right) = -12$

4. $y = \sqrt{27 + x^2}$. Find the value of $\dfrac{dy}{dx}$ when $x = 3$.

5. $f(x) = (9x - 1)^{\frac{1}{3}}$. Find $f'(1)$.

6. Find the equation of the tangent to the curve
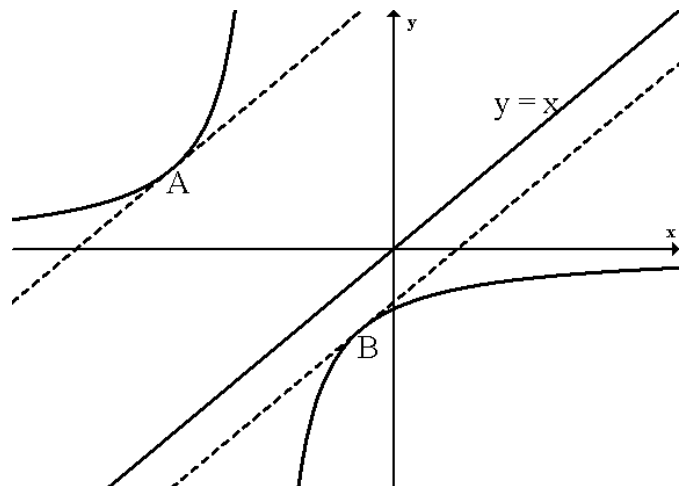 $y = \sqrt{6x - 2}$ at the point A, where $x = 1$.

7. Find the equation of the tangent to the curve $y = \dfrac{4}{3x + 4}$ at the point where $x = -2$.

8. Find the points, A and B, on the curve
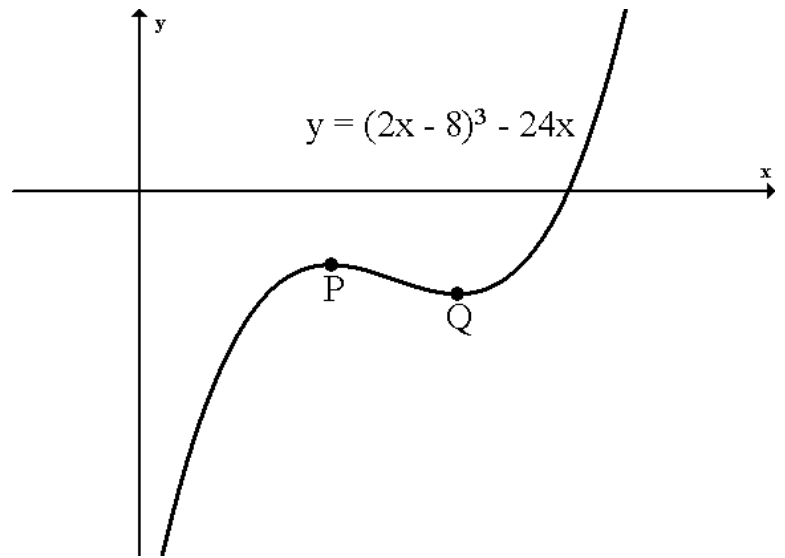$$y = \dfrac{-9}{x + 4}$$

   where the tangents to the curve are
   parallel to the line $y = x$.

9. Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.

10. Find the equation of the tangent to the curve $y = 4\cos\left(2x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{2}$

11. The curve $y = (2x - 8)^3 - 24x$
has stationary points at P and Q.
Find the coordinates of P and Q.

12. (a) Show that the curve $y = 2\sin 2x - 4x$ is never increasing.

(b) Determine the coordinates of the stationary points of this curve in the
interval $0 \le x \le 180$.

Integration

13. Integrate

(a) $\displaystyle\int (2x - 4)^3 \, dx$ (b) $\displaystyle\int \sqrt{6x + 1} \, dx$ (c) $\displaystyle\int \frac{6}{(1 - 3x)^2} \, dx$ (d) $\displaystyle\int \sin(4x - 2) \, dx$

(e) $\displaystyle\int 4\cos (2x + 1) \, dx$

14. Evaluate (a) $\displaystyle\int_0^2 \sqrt{4x + 1} \, dx$ (b) $\displaystyle\int_1^2 \frac{8}{(1 - 2x)^3} \, dx$

15. $\dfrac{dy}{dx} = 10(2x - 1)^4$ and the curve passes through the point $(1,6)$. Find a formula for y.

16. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{2x - 4}}$ and the curve passes through the point $(10,3)$. Find a formula for y.
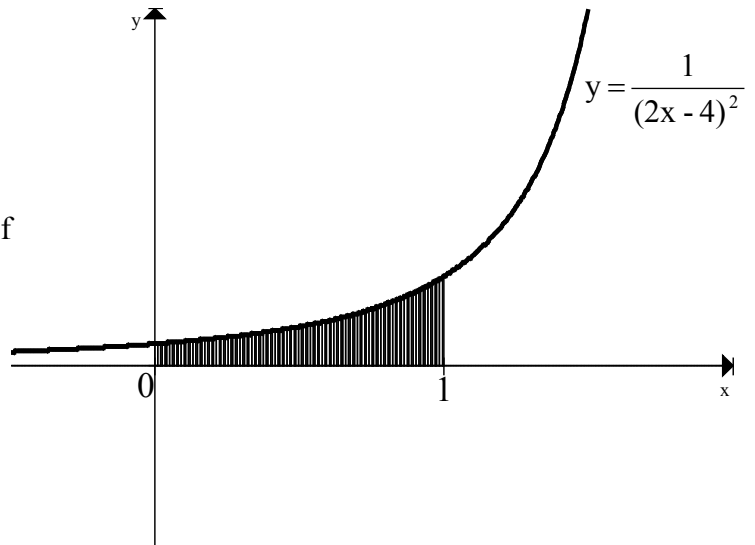
17. $\dfrac{dy}{dx} = 3\sin 2x$. This curve passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$. Find a formula for y.

18. $\dfrac{dy}{dx} = 10\cos 5x$. This curve passes through the point $\left(\frac{\pi}{6}, 4\right)$. Find y.

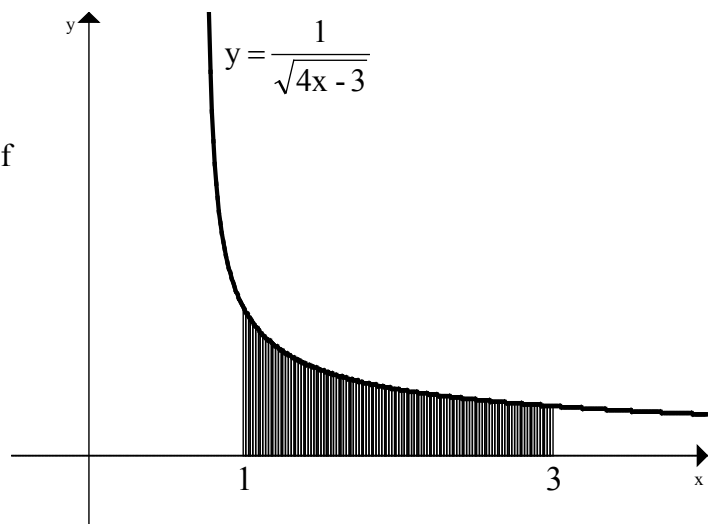19. The diagram shows part of the graph of

$$y = \dfrac{1}{(2x-4)^2}$$

Calculate the shaded area.

$$y = \dfrac{1}{(2x-4)^2}$$

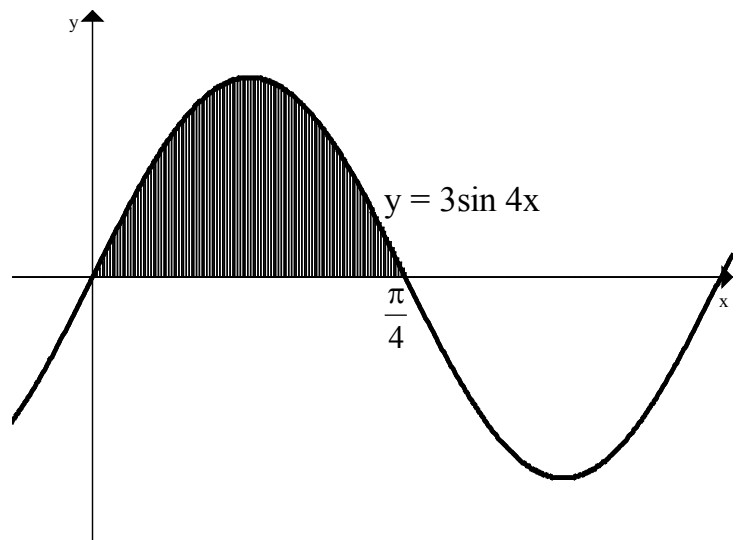20. The diagram shows part of the graph of

$$y = \dfrac{1}{\sqrt{4x-3}}$$

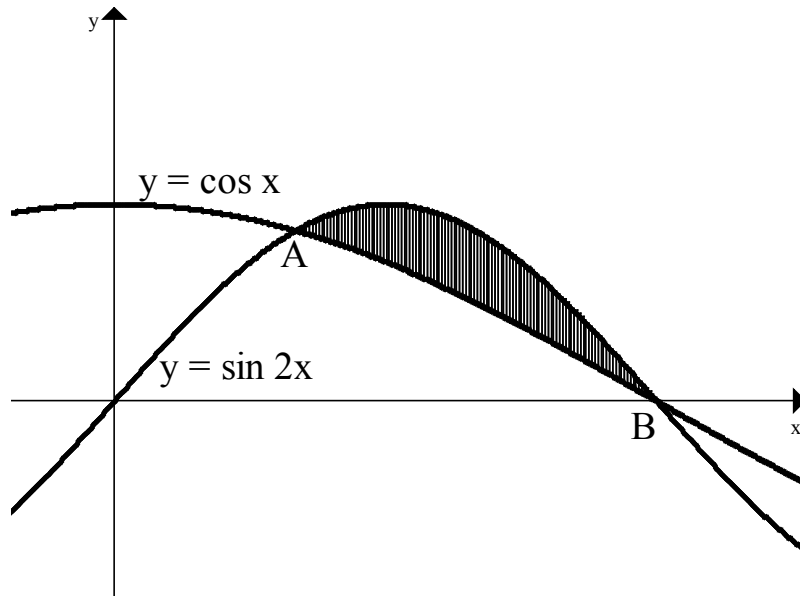Calculate the shaded area.

$$y = \dfrac{1}{\sqrt{4x-3}}$$

21. The diagram shows part of the graph of
$y = 3\sin 4x$.

Calculate the shaded area.

$y = 3\sin 4x$

22. The diagram below shows part of the graphs of $y = \sin 2x$ and $y = \cos x$.

(a) Find the x-coordinates of A and B.

(b) Calculate the shaded area.