Recurrence Relations - Limits

1. Given the recurrence relation $u_{n+1} = 0.4u_n + 12$, $u_o = 22$

   (a) State why the sequence generated by it has a limit.
   (b) Calculate the value of this limit.

2. A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 4$.

   (a) Explain why this sequence has a limit as n tends to infinity.
   (b) Find the exact value of this limit.

3. Two sequences are defined by these recurrence relations

   $u_{n+1} = 2u_n - 0.2$ with $u_o = 6$ $\qquad$ $v_{n+1} = 0.25v_n + 6$ with $v_o = 10$

   (a) Explain why only one of these sequences approaches a limit as $n \to \infty$
   (b) Find algebraically the exact value of this limit.

4. A sequence is defined by the recurrence relation $u_n = 0.7u_{n-1} + 6$, $u_1 = 2$

   (a) Calculate the value of u_2 and u_3
   (b) What is the smallest value of n for which $u_n > 15$
   (c) Find the limit of this sequence as $n \to \infty$

5. A sequence is defined by the recurrence relation $V_n = 0.6V_{n-1} + 10$, $V_1 = 20$

   (a) Calculate the value of V_2
   (b) What is the smallest value of n for which $V_n > 24$
   (c) Find the limit of this sequence as $n \to \infty$

6. A recurrence relation is defined as

$$u_n = 0.25u_{n-1} - 8, u_1 = 10$$

   (a) Find the values of u_2 and u_3

   (b) Explain why this sequence has a limit as n tends to infinity and calculate the value of this limit.

7. A recurrence relation is defined as

$$u_{n+1} = 0.8u_n + 4, u_2 = 32.8$$

   (a) Find the values of u_1 and u_0

   (b) Explain why this sequence has a limit as n tends to infinity and calculate the value of this limit.

8. Two sequences are defined by the recurrence relations

   $u_{n+1} = 0.4u_n + p$ $\qquad$ $v_{n+1} = 0.8v_n + q$

   If both sequences have the same limit, express p in terms of q.