Differentiation – 2

1. Differentiate
   (a) $y = 3x^4 - 4x^2 + 2x$ (b) $f(x) = x^2(2x^3 - x)$ (c) $f(x) = 3(4x - 1)(x + 2)$

   (d) $y = \sqrt{x}(x - 4)$ (e) $f(x) = \dfrac{x^3 + 3x - 1}{x^2}$ (f) $\dfrac{3x^3 + x}{\sqrt{x}}$

2. $y = x^2(x - \sqrt{x})$. Find $f^{/}(4)$.

3. Given $f(x) = \dfrac{2x}{\sqrt[3]{x}} + x^3$, find $f^{/}(8)$.

4. Given $y = 3x - \dfrac{1}{x^2}$. Find the rate of change when $x = 2$.

5. The distance a rocket travels is calculated using the formula $d(t) = 4t^3$, where t is the time in seconds after lift-off.
   Calculate the speed of the rocket after 8 seconds.

6. Find the equation of the tangent to the curve $y = 3x^3 - 4x + 1$ at the point (1,0).

7. Find the equation of the tangent to the curve $y = \dfrac{4\sqrt{x}}{x} + 2x$ at the point where $x = 4$

8. A curve has equation $y = 3x^2 - 9x + 1$. A tangent to this curve has gradient 3. Find the equation of this tangent.

9. A curve has equation $y = x^2 + 5x + 7$. A tangent to this curve meets the positive direction of the x-axis at 45°. Find the equation of this tangent.

10. A curve has equation $y = \dfrac{x^4}{4} - 32x$. A tangent to this curve is parallel to the x-axis.
    Find the equation of this tangent.

11. Show that the curve $y = x^3 - 6x^2 + 12x - 5$ is never decreasing.

12. Show that the curve $y = 12x^2 - 6x - 8x^3$ is never increasing.

13. Show that the curve $y = x^3 - x^2 + x$ is always increasing.

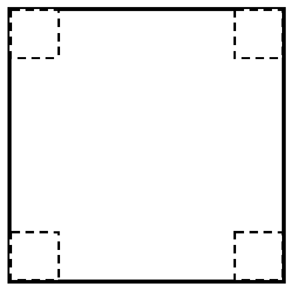
14. Find the intervals in which $y = x^3 - 6x^2 + 5$ is increasing.

15. Find the stationary points of the curve $y = x^3 - 12x + 3$ and determine their nature.

16. A curve has equation $f(x) = x^3 + 4x^2 - 3x - 18$.
    (a) Show that $(x + 3)$ is a factor of $f(x)$.
    (b) Find the points where $f(x)$ cuts the x and y axes.
    (c) Find the stationary points of $f(x)$ and determine their nature.
    (d) Make a sketch of $f(x)$.

17. A curve has equation $f(x) = 8x^3 - 3x^2$
    (a) Find the stationary points of $f(x)$ and determine their nature.
    (b) Find the maximum and minimum values of $f(x)$ in the interval $-2 \le x \le 1$.

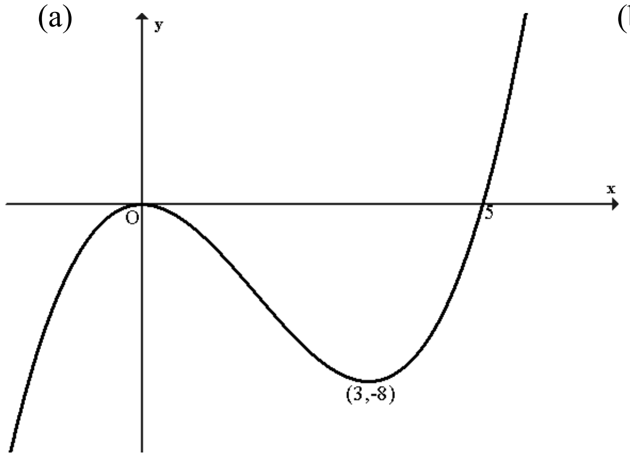
18. A square piece of card of side 20 cm has a square of side x cm cut from each corner.
    An open box is formed by turning up the sides.



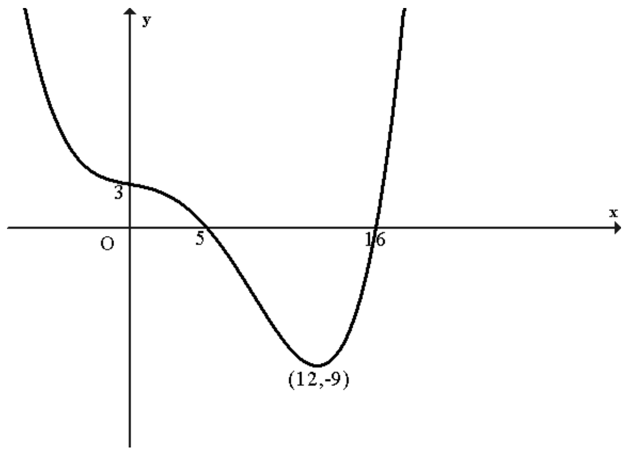
    (a) Show that the volume of the box can be written as
    $V = 400x - 80x^2 + 4x^3$
    (b) Find the maximum volume of the box.

19. For each function $f(x)$ below, sketch $f'(x)$.

(a)



(b)



20. $y = x^3 - x^2$. Show that $x\dfrac{dy}{dx} - 2y = x^3$