Old Past Papers - Trigonometry

[SQA] 1. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \le x \le 360$. **5**

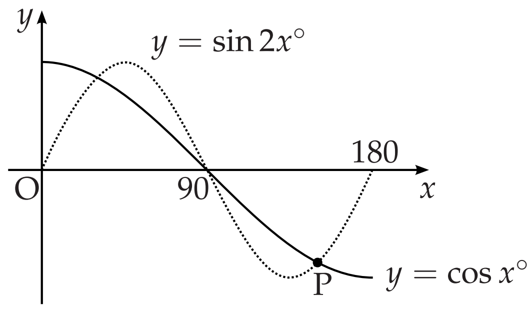
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10	$60, 131{\cdot}8, 228{\cdot}2, 300$	2000 P2 Q5

•1 ss: know to use $\cos 2x = 2\cos^2 x - 1$	•1 $3(2\cos^2 x^\circ - 1)$
•2 pd: process	•2 $6\cos^2 x^\circ + \cos x^\circ - 2 = 0$
•3 ss: know to/and factorise quadratic	•3 $(2\cos x^\circ - 1)(3\cos x^\circ + 2)$
•4 pd: process	•4 $\cos x^\circ = \frac{1}{2}, x = 60, 30$
•5 pd: process	•5 $\cos x^\circ = -\frac{2}{3}, x = 132, 228$

[SQA] 2. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \le x \le 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	NC	T10	$30, 90, 150$	2001 P1 Q5
(*b*)	1	C	NC	T3	$(150, -\frac{\sqrt{3}}{2})$	

•1 ss: use double angle formula	•1 $2\sin x^\circ \cos x^\circ$
•2 pd: factorise	•2 $\cos x^\circ(2\sin x^\circ - 1)$
•3 pd: process	•3 $\cos x^\circ = 0, \sin x^\circ = \frac{1}{2}$
•4 pd: process	•4 $90, 30, 150$
•5 ic: interpret graph	**or**
	•3 $\sin x^\circ = \frac{1}{2}$ and $x = 30, 150$
	•4 $\cos x^\circ = 0$ and $x = 90$
	•5 $\left(150, -\frac{\sqrt{3}}{2}\right)$

[SQA] 3. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	A4	(i) $\sin(2x°)$, (ii) $2\sin(x°)$	2002 P1 Q3
(*b*)	5	C	CN	T10	$0°, 60°, 180°, 300°, 360°$	

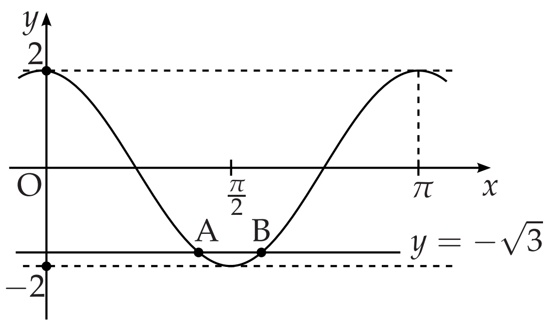
•[1] ic: interpret $f(g(x))$	•[1] $\sin(2x°)$
•[2] ic: interpret $g(f(x))$	•[2] $2\sin(x°)$
•[3] ss: equate for intersection	•[3] $2\sin(2x°) = 2\sin(x°)$
•[4] ss: substitute for $\sin 2x$	•[4] appearance of $2\sin(x°)\cos(x°)$
•[5] pd: extract a common factor	•[5] $2\sin(x°)\,(2\cos(x°) - 1)$
•[6] pd: solve a 'common factor' equation	•[6] $\sin(x°) = 0$ **and** $0, 180, 360$
•[7] pd: solve a 'linear' equation	•[7] $\cos(x°) = \frac{1}{2}$ **and** $60, 300$
	or
	•[6] $\sin(x°) = 0$ **and** $\cos(x°) = \frac{1}{2}$
	•[7] $0, 60, 180, 300, 360$

[SQA] 4. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	1	C	NC	T4	$y = 2\cos 2x$	2002 P1 Q8
(*b*)	3	C	NC	T7	$\text{B}(\frac{7\pi}{12}, -\sqrt{3})$	

•[1] ic: interpret graph	•[1] $2\cos 2x$
•[2] ss: equate equal parts	•[1] $2\cos 2x = -\sqrt{3}$
•[3] pd: solve linear trig equation in radians	•[2] $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$
•[4] ic: interpret result	•[3] $x = \frac{7\pi}{12}$

[SQA] 5. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

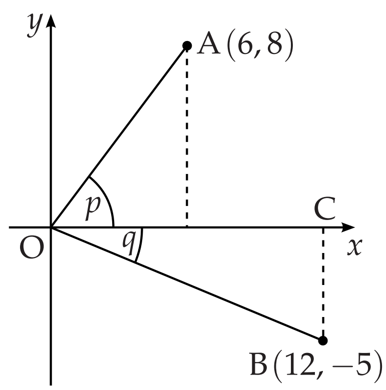
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$, (ii) $cos(x + \frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

•[1] ic: interpret composite functions	•[1] $\sin(x + \frac{\pi}{4})$
•[2] ic: interpret composite functions	•[2] $\cos(x + \frac{\pi}{4})$
•[3] ss: expand $\sin(x + \frac{\pi}{4})$	•[3] $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and complete**
•[4] ic: interpret	•[4] $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$
•[5] ic: substitute	•[5] $(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x) - (\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$
•[6] pd: start solving process	•[6] $\frac{2}{\sqrt{2}}\sin x$
•[7] pd: process	•[7] $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[SQA] 6. On the coordinate diagram shown, A is the point $(6, 8)$ and B is the point $(12, -5)$. Angle AOC $= p$ and angle COB $= q$.

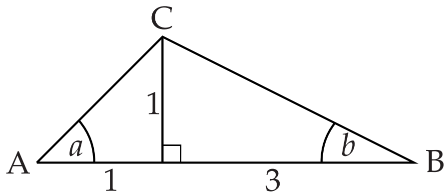
Find the exact value of $\sin(p + q)$. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	$\frac{63}{65}$	2000 P1 Q1

•[1] ss: know to use trig expansion	•[1] $\sin p \cos q + \cos p \sin q$
•[2] pd: process missing sides	•[2] 10 **and** 13
•[3] ic: interpret data	•[3] $\frac{8}{10} \cdot \frac{12}{13} + \frac{6}{10} \cdot \frac{5}{13}$
•[4] pd: process	•[4] $\frac{126}{130}$

[SQA] 7. In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	proof	2002 P1 Q5

•1 pd: process the missing sides •2 ss: expand •3 pd: substitute •4 pc: process and complete proof	•1 $AC = \sqrt{2}$ **and** $BC = \sqrt{10}$ *stated or implied by* •3 •2 $\sin(a+b) = \sin a \cos b + \cos a \sin b$ •3 $\frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ •4 $\frac{4}{\sqrt{20}} = \ldots = \frac{2}{\sqrt{5}}$

[END OF QUESTIONS]