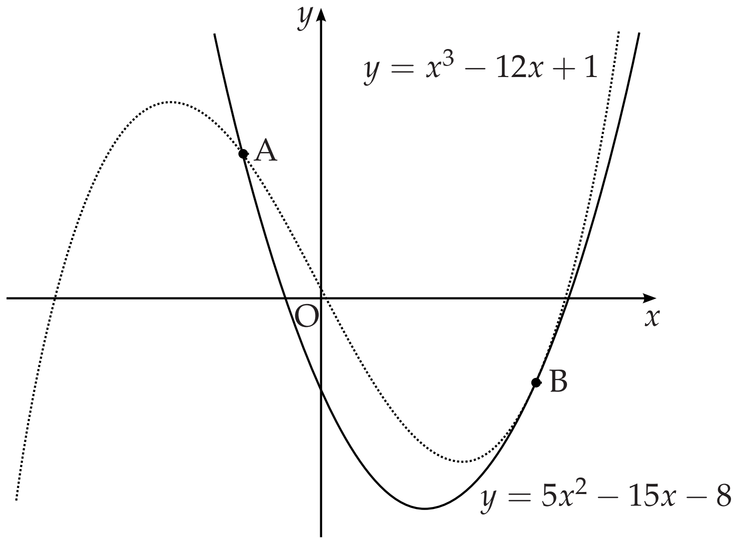
Old Past Papers - Straight Line

[SQA] 1. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

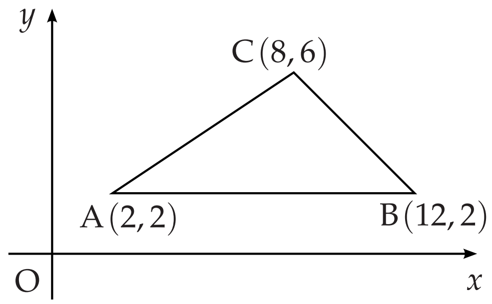
(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	CGD	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

•[1] ss: know to diff. and equate	•[1] find derivatives and equate
•[2] pd: differentiate	•[2] $3x^2 - 12$ **and** $10x - 15$
•[3] pd: form equation	•[3] $3x^2 - 10x + 3 = 0$
•[4] ic: interpret solution	•[4] $x = 3,\ x = \frac{1}{3}$
•[5] ic: interpret diagram	•[5] tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident
•[6] ss: know how to find area between curves	•[6] $\int(\text{cubic} - \text{parabola})$ **or** $\int(\text{cubic}) - \int(\text{parabola})$
•[7] ic: interpret limits	•[7] $\int_{-1}^{3} \cdots dx$
•[8] pd: form integral	•[8] $\int(x^3 - 5x^2 + 3x + 9)dx$ or equiv.
•[9] pd: process integration	•[9] $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv.
•[10] pd: process limits	•[10] $21\frac{1}{3}$

[SQA] 2. Triangle ABC has vertices A$(2,2)$, B$(12,2)$ and C$(8,6)$.



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

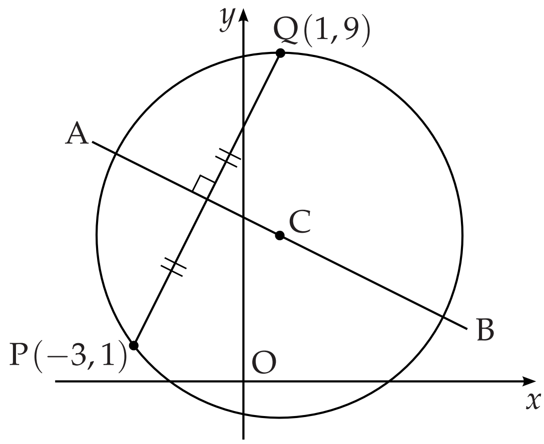
(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	1	C	CN	G3, G7	$x = 7$	2001 P2 Q7
(*b*)	4	C	CN	G7	$3x + 2y = 23$	
(*c*)	1	C	CN	G8	$(7, 1)$	
(*d*)	2	A/B	CN	G8, G9, G10	$(x-7)^2 + (y-1)^2 = 26$	

•1 ic: state equation of a vertical line	•1 $x = 7$
•2 pd: process coord. of a midpoint	•2 midpoint $= (5, 4)$
•3 ss: find gradient of AC	•3 $m_{AC} = \frac{2}{3}$
•4 ic: state gradient of perpendicular	•4 $m_{\perp} = -\frac{3}{2}$
•5 ic: state equation of straight line	•5 $y - 4 = -\frac{3}{2}(x-5)$
•6 pd: find pt of intersection	•6 $x = 7, y = 1$
•7 ss: use standard form of circle equ.	•7 $(x-7)^2 + (y-1)^2$
•8 ic: find radius and complete	•8 $(x-7)^2 + (y-1)^2 = 26$
	or
	•7 $x^2 + y^2 - 14x - 2y + c = 0$
	•8 $c = 24$

[SQA] 3. (*a*) Find the equation of AB, the perpendicular bisector of the line joing the points $P(-3,1)$ and $Q(1,9)$. **4**



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

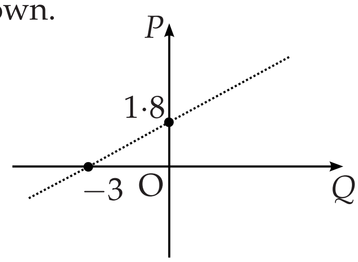
(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	4	C	CN	G7	$x+2y=9$	2000 P2 Q2
(*b*)	3	C	CN	G10	$(x-1)^2+(y-4)^2=25$	
(*c*)	2	C	CN	G11, G8	(i) $y=9$, (ii) $T(-9,9)$	

•1 ss: know to use midpoint	•1 midpoint $=(-1,5)$
•2 pd: process gradient of PQ	•2 $m_{PQ}=\frac{9-1}{1-(-1)}$
•3 ss: know how to find perp. gradient	•3 $m_{\perp}=-\frac{1}{2}$
•4 ic: state equ. of line	•4 $y-5=-\frac{1}{2}(x-(-1))$
•5 ic: interpret "parallel to y-axis"	•5 $y_C=4$ *stated or implied by* •7
•6 pd: process radius	•6 radius $=5$ or equiv. *stated or implied by* •7
•7 ic: state equ. of circle	•7 $(x-1)^2+(y-4)^2=25$
•8 ic: interpret diagram	•8 $y=9$
•9 ss: know to use equ. of AB	•9 T$=(-9,9)$

[SQA] 4. The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. **2**

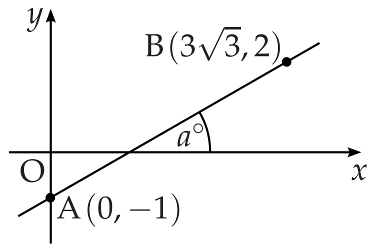
It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	2	A/B	CR	G3	$P = 0{\cdot}6Q + 1{\cdot}8$	2000 P2 Q11
(*b*)	4	A/B	CR	A33	$a = 6{\cdot}05$, $b = 0{\cdot}6$	

•1 ic: interpret gradient •2 ic: state equ. of line •3 ic: interpret straight line •4 ss: know how to deal with x of $x \log y$ •5 ss: know how to express number as log •6 ic: interpret sum of two logs	•1 $m = \frac{1{\cdot}8}{3} = 0{\cdot}6$ •2 $P = 0{\cdot}6Q + 1{\cdot}8$ Method 1 •3 $\log_e p = 0{\cdot}6 \log_e q + 1{\cdot}8$ •4 $\log_e q^{0{\cdot}6}$ •5 $\log_e 6{\cdot}05$ •6 $p = 6{\cdot}05q^{0{\cdot}6}$ Method 2 $\ln p = \ln aq^b$ •3 $\ln p = \ln a + b \ln q$ •4 $\ln p = 0{\cdot}6 \ln q + 1{\cdot}8$ *stated or implied by •5 or •6* •5 $\ln a = 1{\cdot}8$ •6 $a = 6{\cdot}05$, $b = 0{\cdot}6$

[SQA] 5. Find the size of the angle a° that the line joining the points $A(0,-1)$ and $B(3\sqrt{3},2)$ makes with the positive direction of the x-axis. **3**



Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	NC	G2	30	2000 P1 Q3

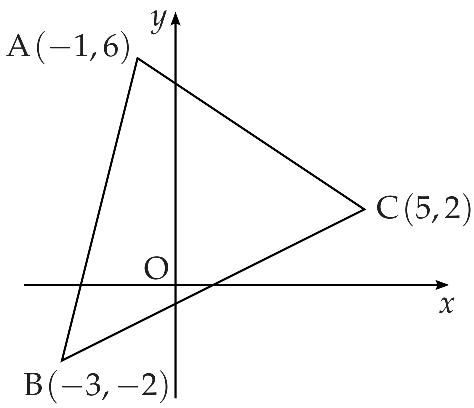
•[1] ss: know how to find gradient or equ. •[2] pd: process •[3] ic: interpret exact value	•[1] $\frac{2-(-1)}{3\sqrt{3}-0}$ •[2] $\tan a$ = gradient *stated or implied by* •[3] •[3] $a = 30$

[SQA] 6. Find the equation of the straight line which is parallel to the line with equation $2x+3y=5$ and which passes through the point $(2,-1)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	CN	G3, G2	$2x+3y=1$	2001 P1 Q1

•[1] ss: express in standard form •[2] ic: interpret gradient •[3] ic: state equation of straight line	•[1] $y = -\frac{2}{3}x + \frac{5}{3}$ *stated or implied by* •[2] •[2] $m_{\text{line}} = -\frac{2}{3}$ *stated or implied by* •[3] •[3] $y-(-1) = -\frac{2}{3}(x-2)$

[SQA] 7. Triangle ABC has vertices A$(-1,6)$, B$(-3,-2)$ and C$(5,2)$.



Find

(*a*) the equation of the line p, the median from C of triangle ABC. **3**

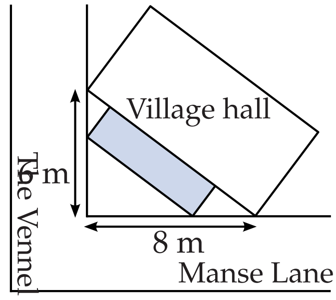
(*b*) the equation of the line q, the perpendicular bisector of BC. **4**

(*c*) the coordinates of the point of intersection of the lines p and q. **1**

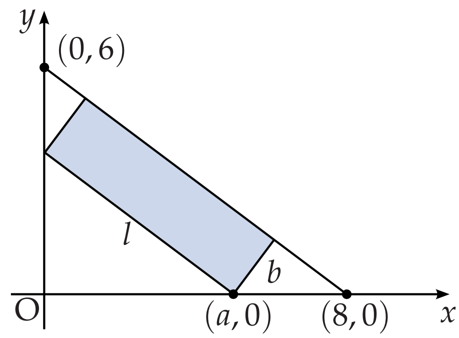
Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(*a*)	3	C	CN	G7	$y = 2$	2002 P2 Q1
(*b*)	4	C	CN	G7	$y = -2x + 2$	
(*c*)	1	C	CN	G8	$(0, 2)$	

$\bullet^1$ ss: determine midpoint coordinates	$\bullet^1$ $F = \text{mid}_{AB} = (-2, 2)$
$\bullet^2$ pd: determine gradient thro' 2 pts	$\bullet^2$ $m_{FC} = 0$ *stated or implied by* $\bullet^3$
$\bullet^3$ ic: state equation of straight line	$\bullet^3$ equ. FC is $y = 2$
$\bullet^4$ ss: determine midpoint coordinates	$\bullet^4$ $M = \text{mid}_{BC} = (1, 0)$
$\bullet^5$ pd: determine gradient thro' 2 pts	$\bullet^5$ $m_{BC} = \frac{1}{2}$
$\bullet^6$ ss: determine gradient perp. to $\bullet^5$	$\bullet^6$ $m_{\perp} = -2$
$\bullet^7$ ic: state equation of straight line	$\bullet^7$ $y - 0 = -2(x - 1)$
$\bullet^8$ pd: process intersection	$\bullet^8$ $(0, 2)$

[SQA] 8. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	A/B	CN	CGD	proof	2002 P2 Q10
(*b*)	4	A/B	CN	C11	$a = 4$	

•[1] ss: select strategy and carry through •[2] ss: select strategy and carry through •[3] ic: complete proof •[4] ss: know to set derivative to zero •[5] pd: differentiate •[6] pd: solve equation •[7] ic: justify maximum, e.g. nature table	•[1] proof of $l = \frac{5}{4}a$ •[2] $b = \frac{3}{5}(8 - a)$ •[3] complete proof leading to $A = \ldots$ •[4] $\frac{dA}{da} = \ldots = 0$ •[5] $6 - \frac{3}{2}a$ •[6] $a = 4$ •[7] e.g. nature table, comp. the square

[SQA] 9. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of $45°$ with the positive direction of the x-axis. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	G2, C4	$(2, 4)$	2002 P1 Q4

•1 sp: know to diff., and differentiate •2 pd: process gradient from angle •3 ss: equate equivalent expressions •4 pd: solve and complete	•1 $\frac{dy}{dx} = 4x - 7$ •2 $m_{\text{tang}} = \tan 45° = 1$ •3 $4x - 7 = 1$ •4 $(2, 4)$

[SQA] 10. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, CGD	proof	2002 P2 Q9

•1 ss: know to use discriminant •2 ic: pick out discriminant •3 pd: simplify to quadratic •4 ss: choose to draw table or graph •5 pd: complete proof using disc.≥ 0	•1 discriminant $= \ldots$ •2 disc $= (-5k)^2 - 4(1 - 2k)(-2k)$ •3 $9k^2 + 8k$ •4 e.g. draw a table, graph, complete the square •5 complete proof and conclusion relating to disc.≥ 0

[END OF QUESTIONS]