

Old Past Papers - Sequences

[SQA] 1. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the start of two successive months. Write down a recurrence relation involving u_{n+1} and u_n .

2

(b) Find the date and the amount of the final payment.

4

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
(a)	2	C	CN	A10, A14	$u_{n+1} = 1.015u_n - 300, u_0 = 2500$	U1 P2 Q3
(b)	4	C	CR	A11, A14	1 December, £290.68	

- ¹ ic: interpret 1.5%
- ² ic: state the recurrence relation
- ³ ss: use recurrence relation
- ⁴ pd: process
- ⁵ ic: start final date
- ⁶ pd: process final payment

- ¹ 1.015 *stated or implied by the start of (b)*
- ² $u_{n+1} = 1.015u_n - 300$ and initial value
(e.g. $u_0 = 2500$) *stated or implied by the start of (b)*
- ³ u_1 i.e. £2237.50
- ⁴ u_2 **and** u_3 i.e. £1971.06, £1700.63
- ⁵ £286.38
- ⁶ £290.68 for December payment

- [SQA] 2. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \rightarrow \infty$.

Determine the value of a and evaluate the limit.

5

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
	4	C	NC	A13	$a = \frac{3}{5}, L = 25$	2000 P1 Q5
	1	A/B	NC	A12		

<ul style="list-style-type: none"> •¹ ss: know how to find limit •² pd: process •³ pd: process •⁴ ic: interpret coeff. of u_n •⁵ pd: process 	<ul style="list-style-type: none"> •¹ $L = aL + 10$ or $L = a^2L + 16$ or $L = \frac{b}{1-a}$ •² $L = \frac{10}{1-a}$ or $L = \frac{16}{1-a^2}$ •³ $\frac{10}{1-a}$ or $\frac{16}{1-a^2}$ •⁴ $10a^2 - 16a + 6 = 0$ •⁵ $a = \frac{3}{5}$ and $L = 25$
---	--

- [SQA] 3. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0.5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

(a) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run?

3

(b) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition.

3

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
(a)	3	C	CN	A13, A14	2.5 metres	2002 P2 Q4
(b)	3	C	CN	A12, A13	trim 25%	

<ul style="list-style-type: none"> •¹ ic: interpret the decay factor •² ss: strategy for limit •³ pd: process limit •⁴ ss: reverse strategy for limit •⁵ pd: process •⁶ ic: interpret scale factor 	<ul style="list-style-type: none"> •¹ 0.8 <i>stated or implied</i> •² e.g. $l = 0.8l + 0.5$ or $l = \frac{0.5}{1-0.8}$ •³ $-1 < 0.8 < 1$ so $l = 2.5$ metres •⁴ $2 = 2m + 0.5$ •⁵ $m = 0.75$ •⁶ trim 25%
--	--

[END OF QUESTIONS]