Old Past Papers - Polynomials

[SQA] 1. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x-a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7
(*b*)	4	C	NC	A3	sketch	
(*c*)	1	C	NC	A16, A6	$-1 < x < 5$	

•1 pd: process, e.g. completing the square	•1 $a = 2$
•2 pd: process, e.g. completing the square	•2 $b = 1$
•3 ic: interpret minimum	•3 *any two from:* parabola; min. t.p. $(2,1)$; $(0,5)$
•4 ic: interpret y-intercept	•4 the remaining one from above list
•5 ss: reflect in x-axis	•5 reflecting in x-axis
•6 ss: translate parallel to y-axis	•6 translating $+10$ units, parallel to y-axis
•7 ic: interpret graph	•7 $(-1,5)$ i.e. $-1 < x < 5$

[SQA] 2. For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A18	$k = \frac{1}{4}$	2001 P1 Q2

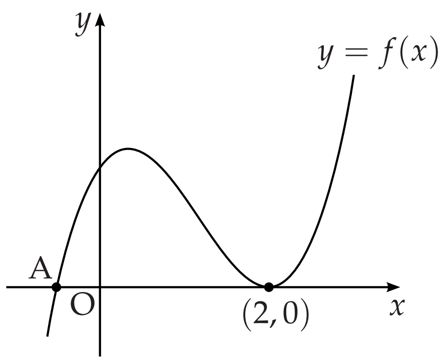
•1 ss: know to set disc. to zero	•1 $b^2 - 4ac = 0$ *stated or implied by* •2
•2 ic: substitute a, b and c into discriminant	•2 $(-5)^2 - 4 \times (k+6)$
•3 pd: process equation in k	•3 $k = \frac{1}{4}$

[SQA] 3. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, CGD	proof	2002 P2 Q9

•[1] ss: know to use discriminant	•[1] discriminant $= \dots$
•[2] ic: pick out discriminant	•[2] disc $= (-5k)^2 - 4(1-2k)(-2k)$
•[3] pd: simplify to quadratic	•[3] $9k^2 + 8k$
•[4] ss: choose to draw table or graph	•[4] e.g. draw a table, graph, complete the square
•[5] pd: complete proof using disc.≥ 0	•[5] complete proof and conclusion relating to disc.≥ 0

[SQA] 4. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x = \frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

•[1] ss: know to differentiate	•[1] $f'(x) = \dots$
•[2] pd: differentiate	•[2] $6x^2 - 14x + 4$
•[3] ss: know to set derivative to zero	•[3] $6x^2 - 14x + 4 = 0$
•[4] pd: start solving process of equation	•[4] $(3x-1)(x-2)$
•[5] pd: complete solving process	•[5] $x = \frac{1}{3}$
•[6] ss: strategy for cubic, e.g. synth. division	•[6] $\begin{array}{c\|cccc} \dots & 2 & -7 & 4 & 4 \\ & & \dots & \dots & \dots \\ \hline & \dots & \dots & \dots & \boxed{0} \end{array}$
•[7] ic: extract quadratic factor	•[7] $2x^2 - 3x - 2$
•[8] pd: complete the cubic factorisation	•[8] $(x-2)(2x+1)(x-2)$
•[9] ic: interpret the factors	•[9] $A(-\frac{1}{2}, 0)$
•[10] ic: interpret the diagram	•[10] $x < -\frac{1}{2}$

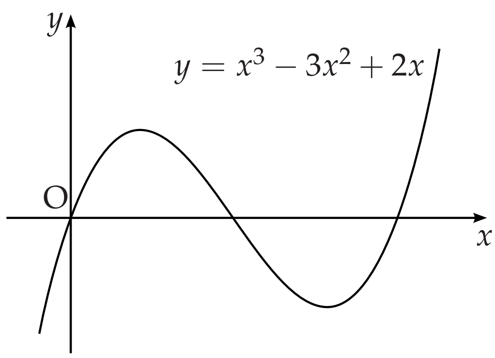
[SQA] 5. (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	CN	A21	$k = -5$	2001 P2 Q1
(*b*)	2	C	CN	A22	$x = -2, \frac{1}{2}, 1$	

•1 ss: use synth division or f(evaluation)	•1 $f(-2) = 2(-2)^3 + \cdots$
•2 pd: process	•2 $2(-2)^3 + (-2)^2 - 2k + 2$
•3 pd: process	•3 $k = -5$
•4 ss: find a quadratic factor	•4 $2x^2 - 3x + 1$ **or** $2x^2 + 3x - 2$ **or** $x^2 + x - 2$
•5 pd: process	•5 $(2x - 1)(x - 1)$ **or** $(2x - 1)(x + 2)$ **or** $(x + 2)(x - 1)$ **and** $x = -2, \frac{1}{2}, 1$

[SQA] 6. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



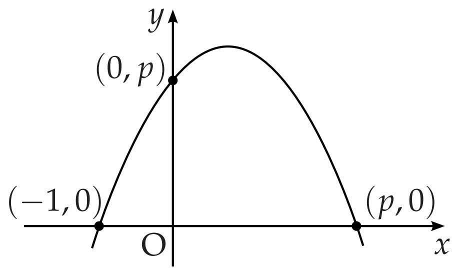
(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	C5	$x + y = 1$	2000 P2 Q1
(*b*)	5	C	CN	A23, A22, A21	$(-1, -6)$	

•1 ss: know to differentiate	•1 $y' = \ldots$
•2 pd: differentiate correctly	•2 $3x^2 - 6x + 2$
•3 ss: know that gradient $= f'(1)$	•3 $y'(1) = -1$
•4 ss: know that y-coord $= f(1)$	•4 $y(1) = 0$
•5 ic: state equ. of line	•5 $y - 0 = -1(x - 1)$
•6 ss: equate equations	•6 $2x - 4 = x^3 - 3x^2 + 2x$
•7 pd: arrange in standard form	•7 $x^3 - 3x^2 + 4 = 0$
•8 ss: know how to solve cubic	•8 $\begin{array}{c\|cccc} \cdots & 1 & -3 & 0 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \cdots \end{array}$
•9 pd: process	•9 identify $x = -1$ from working
•10 ic: interpret	•10 $(-1, -6)$

[SQA] 7. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



(*a*) Show that the equation of the parabola is $y = p + (p-1)x - x^2$. **3**

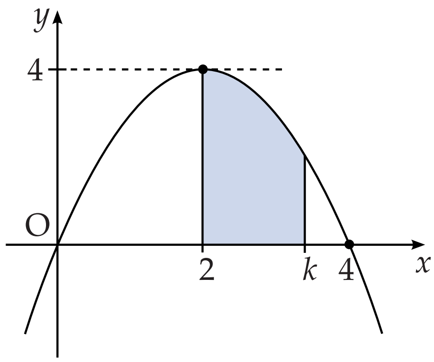
(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	A/B	CN	A19	proof	2001 P2 Q11
(*b*)	3	A/B	CN	A24	2	

•1 ss: use a standard form of parabola	•1 $y = k(x+1)(x-p)$
•2 ss: use 3rd point to determine k	•2 $k = -1$ with justification (i.e. substitute $(0, p)$)
•3 pd: complete proof	•3 $y = -1(x+1)(x-p)$ **and complete**
•4 ss: equate and simplify to zero	•4 $x^2 + 2x - px = 0$
•5 ss: use discriminant for tangency	•5 $b^2 - 4ac = (2-p)^2 = 0$ **or** $(2-p)^2 - 4 \times 0 = 0$
•6 pd: process	•6 $p = 2$

[SQA] 8. The parabola shown crosses the x-axis at $(0,0)$ and $(4,0)$, and has a maximum at $(2,4)$.

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	2	C	CN	A19	$y = 4x - x^2$	2000 P2 Q4
(*b*)	3	C	CN	C16	proof	

•[1] ic: state standard form •[2] pd: process for x^2 coeff. •[3] ss: know to integrate •[4] pd: integrate correctly •[5] pd: process limits and complete proof	•[1] $ax(x-4)$ •[2] $a = -1$ •[3] $\int_2^k$(function from (a)) •[4] $-\frac{1}{3}x^3 + 2x^2$ •[5] $-\frac{1}{3}k^3 + 2k^2 - \left(-\frac{8}{3} + 8\right)$

[SQA] 9. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	A	NC	G9, A17	for all k	2000 P1 Q6

•[1] ss: know to examine radius •[2] pd: process •[3] pd: process •[4] ic: interpret quadratic inequation •[5] ic: interpret quadratic inequation	•[1] $g = 2k,\ f = -k,\ c = -k - 2$ *stated or implied by* •[2] •[2] $r^2 = 5k^2 + k + 2$ •[3] (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (*accept* $\geq$) •[4] use discr. **or** complete sq. **or** diff. •[5] true for all k

[END OF QUESTIONS]